# Three-Nearest-Neighbor Alignment for Smooth ESPIRiT Maps

Dara Bahri[1], Martin Uecker[1], and Michael Lustig[1]
[1]*Electrical Engineering and Computer Sciences, University of California, Berkeley, California, United States*

**PURPOSE**: ESPIRiT (1) is a subspace-based method that estimates coil sensitivity maps from just calibration data. Like other methods (2), it produces maps where only the relative phase among the coils is known. Often, the phase of a single coil is used as reference. In some cases the resulting phase can be non-smooth (4). This is problematic for phase-sensitive applications and for compressed sensing as it reduces the sparsity of the image. In this work we propose a technique for determining basis vectors for ESPIRiT maps which are smoothly aligned in space. In addition to producing smooth phase in the maps, it allows us to prevent mixing of signal components in the case of multiple sets of maps, as might occur when imaging a tight FOV. We then demonstrate how image-space interpolation can be used to significantly accelerate ESPIRiT computation as well as the aforementioned method with little error.

**METHODS:** While the sets of sensitivity maps form an orthonormal basis for a particular subspace at every voxel, there is no guarantee that these basis vectors are spatially smooth. Our method, which we call "Three-Nearest-Neighbor Alignment" and which is formalized in Figure 1 enables us to transform the sets of maps so that each set is spatially smooth while preserving the subspace they describe. We derive the transformed maps at a particular voxel by considering the already transformed maps at the three voxels spatially closest to it. In this way, the aligned maps can be grown from the center voxel outwards. A technique of similar flavor but in 1D was used in (3) for coil compression. The posed optimization problem can be cast as an instance of the *orthogonal Procrustes Problem*, which admits a closed-form solution via singular value decomposition. The unitary constraint in the problem enforces that no vital information about the maps is lost while the objective measures smoothness, with the weights controlling smoothness among voxels. Here we choose the weight matrix (as defined in Fig. 1) for a particular voxel to be a diagonal matrix consisting of the corresponding thresholded eigenvalues produced by ESPIRiT. We test how well our method can produce smooth phase on a fully-sampled, 32-channel brain dataset (3D FLASH, TE/TR = 11/4.9 ms, $B_0$ = 3T, size = 192x192). Additionally, we test how well our method can separate a signal into smooth components on an aliased, tight-FOV, 8-channel brain dataset (Spin-Echo, TE/TR = 550/14 ms, $B_0$ = 1.5T, size = 320x168). Computing full-size maps with ESPIRiT and then aligning them is costly. To this end, we first compute downsampled maps of a size much smaller than the image, align, and then bilinear-interpolate them to full-size.

**Figure 1**

For each voxel $r$ :

(1) Find optimal transformation $P_r$ :

$$P_r = \underset{Q_r}{\operatorname{argmin}} \sum_{i=1}^{3} \alpha_{(r,r_i)} \left\| A_r W_r Q_r - \tilde{A}_{r_i} W_{r_i} \right\|_F^2$$

$$\text{s.t.} \quad Q_r Q_r^H = Q_r^H Q_r = I$$

(2) Align :

$$\tilde{A}_r = A_r P_r$$

$A_r(i,j)$ = sensitivity for coil $i$ in set $j$ for voxel $r$

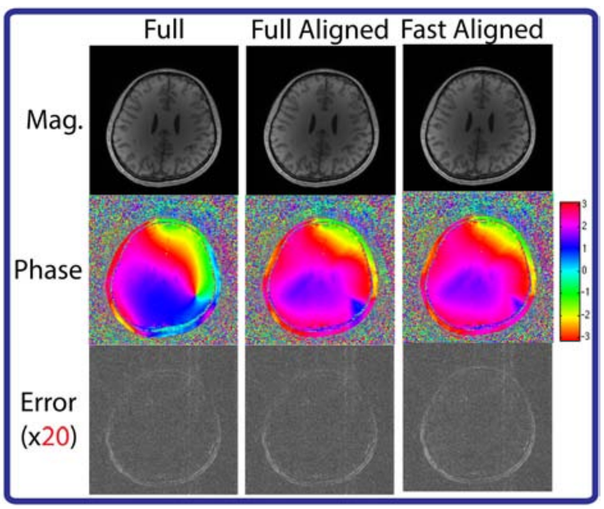
$\tilde{A}_r(i,j)$ = aligned sensitivity for coil $i$ in set $j$ for voxel $r$

$\{r_1, r_2, r_3\}$ = three nearest voxels to $r$

$\alpha_{(r,r_i)}$ = "distance" between voxels $r$ and $r_i$

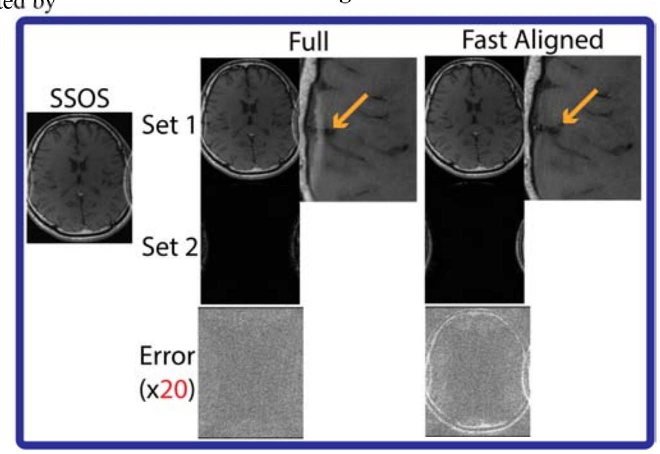
$W_r$ = weight matrix for voxel $r$

**Figure 2**




**RESULTS & DISCUSSION:** In Figure 2 we compare channel combinations of the 32-channel dataset with 1) full-size maps with no alignment and with phase referenced to the first coil ("Full"), 2) full-size maps with alignment ("Full Aligned"), and 3) maps downsampled by a factor of 2 in each dimension, aligned, and then interpolated to full-size ("Fast Aligned"). To estimate how faithful our maps represent the signal subspace, we compute a square-root-sum-of-squares-combined projection error of the coil images on the subspace defined by the sets of maps. First, we observe that using the phase of the first coil as a reference causes a phase singularity, which is prevented by alignment. Secondly, we see that interpolating downsampled maps does not degrade the quality of the image. In Figure 3, we compare channel combinations of the tight-FOV dataset with each set of 1) full-size maps with alignment and 2) maps downsampled by a factor of 10 in each dimension, aligned, and then interpolated. We observe that in this case, alignment is able to successfully separate the aliased and unaliased components of the signal. Again, the interpolation of the downsampled, aligned maps does not significantly alter the subspace described by full-size maps.

**Figure 3**




**CONCLUSION:** We presented an algorithm that finds a spatially smooth basis for ESPIRiT coil sensitivity maps. We demonstrated how alignment can be used to produce smooth phase in the maps and to prevent signal mixing in the case of multiple maps. Furthermore, we showed how interpolation can be used to significantly accelerate both map computation and alignment by up to a factor of 100.

**References:** [1] Uecker et al. MRM (2013), doi: 10.1002/mrm.24751. [2] Walsh et al. MRM (2000), 43:682. [3] Zhang et al. MRM (2013), doi: 10.1002/mrm.24767. [4] Inati et al., ISMRM 2013:2672.