Compressed Sensing 4D Flow Reconstruction using Divergence-Free Wavelet Transform

Frank Ong[1], Martin Uecker[1], Umar Tariq[2], Albert Hsiao[2], Marcus Alley[2], Shreyas Vasanawala[2], and Michael Lustig[1]
[1]*Electrical Engineering and Computer Sciences, University of California, Berkeley, CA, United States,* [2]*Radiology, Stanford University, CA, United States*

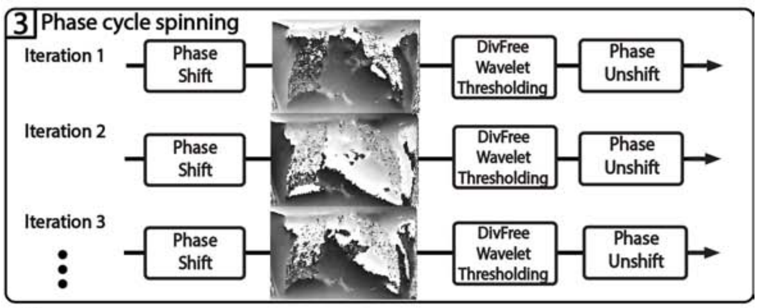
1

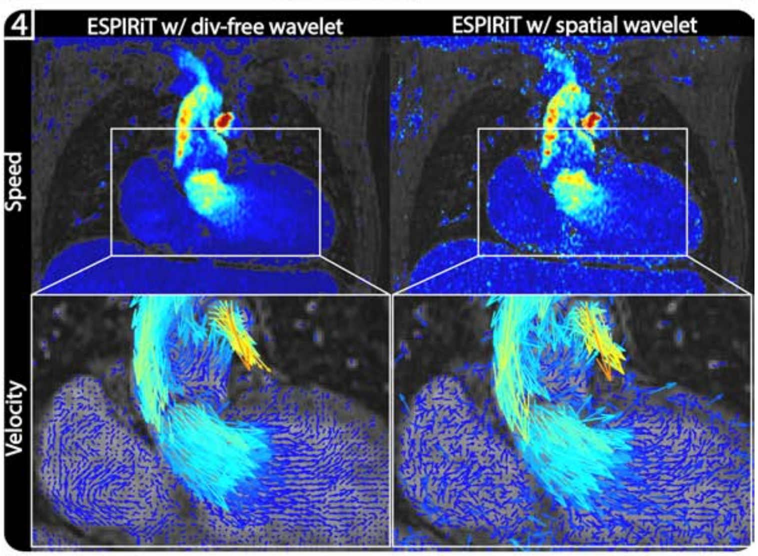
$$\min_{m\in\mathbb{R}_+,\phi\in\mathbb{R}} \frac{1}{2}\|Ame^{j\phi}-y\|_2^2+\lambda_m\|\Psi_m m\|_1+\lambda_\phi\|\Psi_\phi V^{-1}\phi\|_1$$

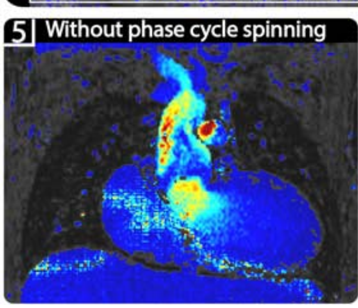
A : sensing matrix m : magnitude ϕ : phase y : kspace V : velocity encoding
Ψ_m : spatial wavelet for m Ψ_ϕ : div-free wavelet for vel./ spatial wavelet for ref. phase

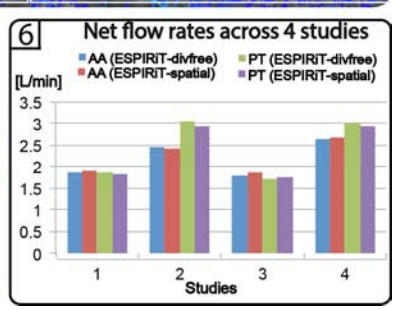
2 Update steps

$$x_n = m_n e^{j\phi_n} + A^*(y - Am_n e^{j\phi_n})$$
$$m_{n+1} = \mathrm{WavThresh}_{\lambda_m}(\mathrm{Re}_+(x_n e^{-j\phi_n}))$$
$$\phi_{n+1} = \mathrm{WavThresh}_{V,\lambda_\phi}(\phi_n + m_{n+1}|x_n|\sin(\angle x_n - \phi_n))$$









TARGET AUDIENCES: Those who are interested in phase regularized reconstruction of MR flow data.

PURPOSE: Time-resolved 3D phase-contrast MRI (4D flow) [1] has the potential to provide comprehensive cardiac flow quantification, yet suffers from long acquisition time and phase errors. Compressed sensing (CS) offers an effective way of speeding up scan time by exploiting sparsity and compressibility of cardiac images [2]. In this work, we extend the standard CS reconstruction to utilize the physical properties of blood flow, specifically blood flow is incompressible and therefore should have zero divergence [3-5]. We leverage our previous work [6] on flow denoising using divergence-free wavelet (DFW) and present a phase-wrap tolerant reconstruction via phase cycle spinning.

METHODS: DFW is a vector-wavelet that can provide a sparse representation of flow and enforce approximate or "soft" divergence-free conditions [7]. Prior knowledge about flow is utilized in the reconstruction by solving the optimization problem in **Fig. 1**, with DFW applied on velocities and Daubechies-4 wavelet on magnitude/reference phase. Update steps **(Fig. 2)** are derived following [8-9].

To tolerate phase wraps, we propose phase cycle spinning **(Fig. 3)**: In each update step, we rotate the phase randomly so that the phase wraps appear at different places. Similar to wavelet cycle spinning, distortions caused by thresholding phase wrap discontinuities diminish over iterations, while phase variations due to flow are preserved.

The proposed divergence-free reconstruction is combined with ESPIRiT [10], a CS and parallel imaging reconstruction algorithm and was compared with ESPIRiT with traditional spatial L1-wavelet regularization. To evaluate the performance, 4D flow data was acquired in 4 pediatric patients with 20 cardiac phases, ~140 slices, ~1.2x1.2x1.5 mm^3 resolution, ~63 ms temporal resolution, 15^o flip angle and TR/TE of ~4.94/1.91 ms. The 4D flow acquisition was performed on a 1.5T GE Signa scanner with an 8 channel cardiac array using a spoiled gradient-echo sequence with tetrahedral flow encoding. The acquisitions were undersampled by ~4-fold with variable density Poisson-disc undersampling. Eddy-current correction was performed on velocity data. Segmentations for flow calculations were done on the aorta (AA) and pulmonary trunk (PT) and net flow rate and regurgitant fraction were calculated.

RESULTS AND DISCUSSIONS: In **Fig. 4**, the proposed method reduces artifacts/noise and enforces the flow to stay within the heart. Phase cycle spinning is also shown to be effective in reducing phase wrap reconstruction artifacts that are present in **Fig. 5**, where significant distortions are seen near phase wraps. In **Fig. 6**, both the proposed method and ESPIRiT result in similar net flow rates. The maximum difference in regurgitant fractions between reconstructions is 2%. Since the measures are time-averages, they are relatively robust to zero-mean artifacts and are used here as a metric to validate there is no bias in the reconstruction.

CONCLUSION: The proposed reconstruction using DFW results in better flow data that follow boundary conditions while preserving core flow quantifications. Phase cycle spinning is also effective in tolerating phase wraps during reconstruction.

References: [1] Markl, et al. JMRI, 2003;17;499-506. [2] Lustig, et al. MRM, 2007:58,1182-95. [3] Busch, et al. MRM 2013;69;200-10. [4] Song, et al. JMRI, 1993;4;587-596. [5] Leocher, et al. ISMRM 2013;1355. [6] Ong, et al. MRM submitted. [7] Deriaz, et al. J. Turbul 2006;1. [8] Fessler, et al. ISBI 2004;209-212. [9] Funai, et al. TMI 27:10,1484-94. [10] Uecker et al. MRM, 2013 doi 10.1002/mrm.24751