

Predicting Image Quality of Under-Sampled Data Reconstruction in the Presence of Noise

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Introduction: The results of under-sampling reconstruction algorithms are often compared to a fully-sampled reconstruction. This comparison is overly optimistic because even if the reconstruction removes the aliasing due to under-sampling, we would still have an inherent loss of SNR due to the reduced acquisition time. We present a predictor of image quality for a given reconstruction technique and under-sampling pattern, which operates by applying the reconstruction to fully-sampled data with added noise related to reduced acquisition time. Using this prediction image as a “gold standard” enables a fair comparison for reconstruction results. The prediction image also provides an efficient means of quickly assessing the capabilities of a given set of reconstruction strategies, such as denoising with K-SVD^{1,2,3} or BM3D^{4,5}.

Theory: The acquired MRI signal is modeled as the ideal signal corrupted with complex-valued, zero-mean Gaussian noise. When under-sampling, both the MRI signal and noise variance are scaled by the sampling density, ρ , where $\rho < 1$ indicates under-sampling. The resulting SNR is scaled by $\sqrt{\rho}$, which may be simulated by scaling the noise standard deviation of the fully-sampled image, σ_{full} , by $\beta = 1/\sqrt{\rho}$. For variable density under-sampling, this β noise factor is not uniform across k-space, as seen in the left column of figure 1. Even if our reconstruction algorithm provides a perfect solution to the aliasing problem, we still have an effective k-space noise with standard deviation $\beta\sigma_{full}$. Solving $\beta\sigma_{full} = (\sigma_{full}^2 + \sigma_{add}^2)^{1/2}$ for σ_{add} , we can add the proper amount of noise to the fully-sampled k-space to simulate the result of reduced SNR without the incurred aliasing. In this ideal case, we can generate a prediction of image quality by reconstructing this noise-adjusted k-space with our reconstruction algorithm.

Methods: We illustrate the results of this process by first applying it on a digital phantom known to be sparse in the wavelet domain (see figure 2). We then demonstrate the image quality predictor effectiveness on a 1.5T, 8 channel, fully-sampled, spoiled gradient echo, axial brain dataset, using 2x-by-2x and 3x-by-3x variable density Poisson disc retrospective under-sampling and reconstructing with ESPIRiT⁶ with wavelet or K-SVD as a sparsity transform.

Results & Conclusions: When we have an explicitly sparse digital phantom, we expect to recover from the 2x-by-2x under-sampling. However, as seen in figure 2, the actual reconstruction (C) is limited by acquisition time noise simulated by the image quality prediction process (B). Likewise, figure 3 shows that reconstructing the noise-adjusted k-space (E,G,J) successfully predicts the image quality of the actual reconstruction for our wavelet (F,H) and K-SVD (K) reconstructions described above. This image quality predictor provides a better metric for evaluating the effectiveness of a reconstruction algorithm than direct comparison to a fully-sampled reconstruction. In addition, this process allows reconstruction algorithm developers to quickly evaluate more complex algorithms, such as K-SVD and BM3D, and their parameters prior to investing time to implement and compute the complete reconstruction.

References:

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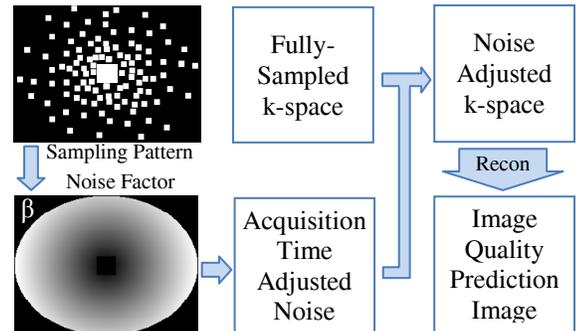


Figure 1: Process to add acquisition time adjusted noise to fully-sampled k-space and reconstruct a prediction of image quality.

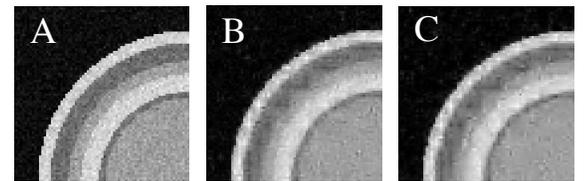


Figure 2: Sparse phantom with noise (A) followed by prediction (B) and actual reconstruction (C) for 2x2x under-sampling.

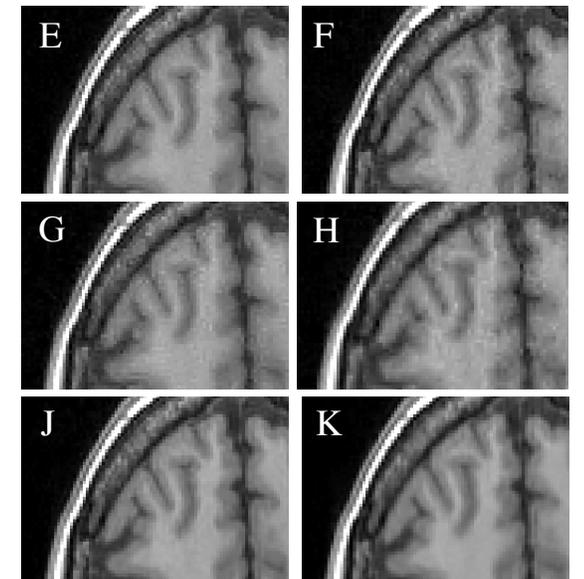
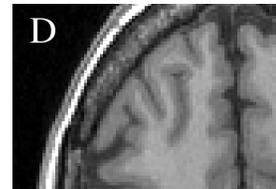


Figure 3: Fully-sampled axial brain (D) followed by prediction (left column) and actual reconstruction (right column) for three different tests: wavelet on 2x2x under-sampled data (E,F), wavelet on 3x3x under-sampled data (G,H), and K-SVD on 2x2x under-sampled data (J, K).