Robust Low-rank Matrix Completion for sparse motion correction in auto calibration PI

Zhongyuan Bi[1], Martin Uecker[2], Dengrong Jiang[3], Michael Lustig[2], and Kui Ying[3]

[1]Biomedical Engineering, Tsinghua University, Beijing, Beijing, China, [2]Electrical Engineering and Computer Science, University of California Berkeley, Berkeley, California, United States, [3]Engineering Physics, Tsinghua University, Beijing, Beijing, China

Purpose

Auto-calibration parallel imaging (acPI) [1] is based on local correlations in k-space. It is known to perform robustly in practice, especially when accurate sensitivity information is hard to obtain. Also, acPI renders new opportunities for motion correction [2]. However, the reconstruction quality of acPI methods is highly dependent on the accuracy of the interpolation kernels calculated from the auto calibration signal (ACS). Corruption of ACS data, e.g. by motion, often leads to serious artifacts in the reconstructed images. In this work, we propose to exploit the redundancy in k-space to detect and correct sparse corruptions in ACS data, which could result from random, time-limited motion in clinical practice (e.g. swallowing, jerk, etc). Our work is based on low-rank matrix completion with sparse errors, and is an extension of our calibrationless parallel imaging reconstruction methods [3,4] .

Methods

Low-rank matrix completion: In general, missing entries of a matrix can be completed if the matrix is low-rank. This could be efficiently achieved by singular-value thresholding (SVT) [5]. In acPI, the calibration matrix should have low-rank [3,4]. However, if corrupted by random and sparse errors, the correlation within the calibration matrix will be reduced, leading to higher rank values. Intuitively, we could enforce low-rank regularization onto the corrupted calibration matrix to detect and correct these errors. For consistency with the uncorrupted acquired data, we use a soft-thresholding scheme to control the difference between original k-space and newly synthesized k-space by low-rank approximation. The flowchart of our proposed method is shown in Figure 1 and can be described as: (1) Construct matrix A from over-lapping blocks; (2) Compute $[U,\Sigma,V] = svd(A)$; Threshold the singular values $\Sigma = S(\Sigma,\lambda)$; (3) Compute $A = U\Sigma V'$; (4) Reconstruct k-space from A; (5) Soft-threshold the difference between original k-space and the synthesized k-space, the resulting k-space can be expressed point-wise as:

$$k(i,j) = k_{ori}(i,j) + shrink[(k_{syn}(i,j) - k_{ori}(i,j)),\gamma]$$

, where k_{ori}, k_{syn}, k stand for the original k-space, the synthesized k-space, and the resulting k-space by soft-thresholding, respectively. γ is the threshold for soft-thresholding, and $shrink(x,\gamma) = sign(x)\bullet\max(|x|-\gamma,0)$; (6) Repeat 1-5 till convergence.

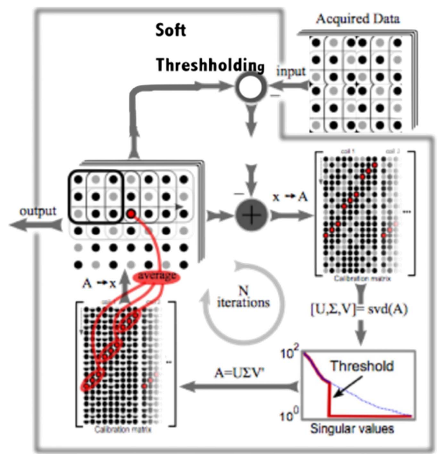

Figure 1. The flowchart of the algorithm based on singular-value thresholding and soft-thresholding of the residual.

Experimental setup: T1-weighted axial head data were acquired on a 1.5T GE system with an 8-channel head coil, using RF-spoiled gradient-echo sequence (inversion-recovery prepared 3D, TR=12.2ms, TE=5.2ms, TI=450ms, FA=20°, matrix size= $256\times180\times230$). To simulate sparse and random corruptions by time-limited motions, random phase shift was added to 20% of the fully-acquired k-space. Corrupted points distributed randomly in k-space, with identical positions in every coil.

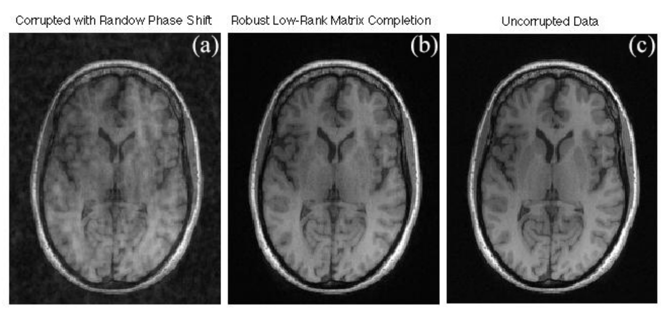

Figure 2. Reconstructed images from: (a) corrupted k-space; (b) corrected *k*-space by Robust Low-rank Matrix Completion; (c) uncorrupted k-space;

Results and Discussion

Figure 2 demonstrates the simulation results. Incoherent artifacts appear in the corrupted image, but after correction, these artifacts are greatly reduced while the boundaries are still well-preserved. Figure 3 verifies the assumption that corrupted data matrix has higher rank. After correction, the rank has been lowered and the singular values are almost identical to the original uncorrupted data matrix. In this initial work, SVT-based correction was done to the whole k-space to demonstrate its efficiency. Further work will be to apply this method to correct the calibration area only, and use other methods like GRAPPA to reconstruct the rest of k-space. Thus, we can reduce computational costs and improve image quality in situations where motion exists.

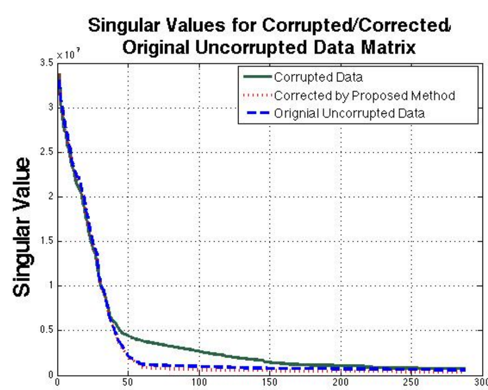

Figure 3. Singular values for corrupted, corrected and original uncorrupted data matrix.

Conclusion

We present a new motion-correction method based on low-rank matrix completion. Simulation results demonstrate that it could be used to correct random and sparse corruptions in calibration area in acPI. Experiments on in-vivo data is underway to verify the potential of proposed method to solve motion problems caused by swallow or jerk in clinical practice, especially in 3-D imaging.

References :

[1] Griswold et.al MRM 2002; 47(6): 1202-10
[2] Lin et.al, ISMRM'09 pp.757
[3] Lustig et.al, ISMRM'10 pp. 2870
[4] Lustig et.al, ISMRM'11 pp. 483
[5] Cai et.al SIAM J. OPTIM 2010;20(4): 1956-1982