Home work week 5

1. Consider an ideal monatomic gas with N atoms ($H = E = \sum_i^N \frac{1}{2m} p_i^2$) in a closed volume V.

    - (Re)derive the partition function.
    - write down an expression for the average energy
    - write down an expression for the heat capacity. What is the heat capacity when the temperature goes to zero? What should it be, according to the third law of thermodynamics (*i.e.*, $S = 0$ JK^{-1} at $T = 0$ K)?
    - write down an expression for the entropy. What happens to the entropy when the temperature goes to 0 K? Why should the entropy at 0 K be 0 JK^{-1} ?
    - write down an expression for the Helmholtz free energy
    - write down an expresion for the pressure.

2. We have a box with two partitions of equal volume separated by a wall that we can remove (somehow). The walls of the box are such that energy can flow in from (and out to) the environment. Therefore, the temperature remains constant.

    - What is the change in entropy if initially there is an ideal gas of N_A atoms on one side and nothing on the otherside ?
    - What is the change in entropy if initially there is an ideal gas of N_A atoms of type A with mass m_A on one side, and an ideal gas of N_B atoms of type B with mass m_B on the other side?
    - What is the change in entropy if initially there is an ideal gas of N_A atoms with mass m_A on one side, and also N_A atoms of type A with mass m_A on the other side?

This problem is known as the mixing paradox.

3. write down the partition function for a gas (or fluid) of N interacting atoms in a volume V at constant temperature T. Assume that the interactions are pair-wise and approximated by a Lennard-Jones potential:

$$V = \frac{1}{2} \sum_i \sum_j \left[\left(\frac{A}{|\mathbf{r}_i - \mathbf{r}_j|} \right)^{12} - \left(\frac{B}{|\mathbf{r}_i - \mathbf{r}_j|} \right)^{6} \right] \tag{1}$$

Note: you don't need to work out this partition function. You're in for a Nobel prize if you could. Just try to simplify (integrate) as much as you can, using the results of the lecture and chapter 7 of the book. You will end up with at least one integral in the expression.