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What has caused global business cycle decoupling: Smaller shocks or reduced sensitivity?

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Abstract

According to a growing body of empirical literature, global shocks have become less important for business cycles in industrialized countries and emerging market economies since the mid-1980s. In this paper, we analyze the question of what might have caused a decoupling from the global business cycle: the smaller size of the global shocks or a reduced sensitivity of national business cycles to these shocks? To this end, we employ a large scale hierarchical dynamic factor model that decomposes the growth rates of GDP, consumption, and investment for 106 countries over 1961–2014 into a global, a group-, and a country-specific factor, as well as an idiosyncratic component. The factor loadings and conditional variances are allowed to vary over time according to random walk processes. Instead of assuming that the parameters change, we test for time variation using a Bayesian stochastic model specification search. Our results confirm a reduction in the importance of the global business cycle for the vast majority of our countries. However, the sensitivity of most countries to global or group-specific shocks as measured by the factor loadings has not changed over time. Instead, the magnitude of the global shocks relative to group-specific and country-specific shocks has decreased, resulting in a lower relevance of global shocks for national cycles.

JEL Classification: F44, C52, C32

Keywords: global business cycle, dynamic factor model, time-varying parameter, stochastic volatility, model selection

1 Introduction

Over the past decades, economic linkages between countries have increased dramatically. The reduction of trade barriers and capital controls have led to a substantial increase in world trade and international capital flows. According to the conventional wisdom, this high level of interconnectedness has resulted in a convergence of international business cycles and makes countries more vulnerable to global shocks. However, the impact of the recent global financial crisis on national business cycles differed remarkably in different regions of the world. While the industrialized countries in Europe and North America experienced the deepest and longest recession, many countries in Asia, Latin America, and Africa overcame this period with modest or no reduction in the growth rate of their GDP. This experience has raised the question of the relative importance of global versus regional or group-specific shocks in explaining national business cycles.

The empirical literature indeed finds an increasingly important role for regional or group-specific shocks and a reduced importance of global shocks for national business cycles. Using data for 106 countries from 1960 to 2008, Kose et al. (2012) (henceforth KOP) employ a dynamic factor model (DFM) to disentangle national business cycles into a global, a group-specific, and a country-specific factor.[1] They find that the fraction of the variance of output, consumption, and investment growth explained by the global factor has decreased for emerging market economies and industrial countries, but group-specific shocks have become more important for both country groups since the mid-1980s. KOP conclude that emerging economies have decoupled from industrialized countries in the globalization era. Using the same dataset, Hirata et al. (2013) estimate a DFM with seven regional factors instead of grouping countries according to their level of development. They find that regional factors have gained importance over time, particularly in regions where intra-regional trade and financial flows grew since the mid-1980s. Similarly, Mumtaz et al. (2011) use a DFM to decompose output and inflation growth for 36 countries into common, regional, and country-specific factors, and find regional cycles to have become more important over time while the importance of the global factor has declined. A somewhat different conclusion is drawn by Flood and Rose (2010), who calculate five-year correlation coefficients over rolling sub-samples for GDP growth from 1947 to 2008 for 64 countries, and find rather stable correlation coefficients of country pairs for advanced and emerging economies over time.

[1] KOP cluster countries according to their level of development, i.e., they distinguish between industrialized countries, emerging market economies and developing countries.

Ductor and Leiva-Leon (2016) estimate a DFM with time-varying factor loadings to investigate whether national business cycles for a large number of countries have exhibited changes in the sensitivity to a global business cycle. They find that the comovement of business cycles with the global factor has increased over time for emerging market economies, as measured by an increase in the loadings to the global factor.[2]

The literature using DFM to analyze changes in the synchronization of national business cycles exhibits two limitations on which we focus in this paper. First, the vast majority of papers employ a DFM with constant parameters. A potential variation over time in the comovement of business cycles is analyzed by estimating the model over different subsamples. Most of the aforementioned studies impose a break in 1985 and refer to the time prior to 1985 as the pre-globalization period, and the time since 1985 as the globalization period. However, by allowing for Markov-switching parameters, Ductor and Leiva-Leon (2016) find changes in global business cycle interdependence to have occurred in the early 2000s. Further, by assuming a single break, these studies cannot account for heterogeneity across countries, i.e., countries that adjust at different points in time. Second, the importance of the global or group factors for a country are based on decomposing the variance of the business cycle indicator considered into various components that can be attributed to global, group, country and idiosyncratic factors. The relative importance of these factors, i.e., their share of the business cycle variable's variance, can change over time for two reasons: changes in the factor loadings or changes in the factor's variance. Differentiating between these two reasons of changes in the variance decomposition is economically important. Changes in factor loadings reflect changes in how sensitive a country is to that factor. Thus, a decline in the loading to the global factor may well be interpreted as a decoupling, while an increased sensitivity to the regional or group factor, i.e., an increase in the respective factor loading, points to an increased importance of regional or group specific shocks. In contrast, changes in a factor's variance reflect changes in the size of the shocks. Allowing for heteroscedasticity is particularly important, as it is a potential reason for changes in the variance decomposition. Suppose the volatility of the global factor declines more than the business cycle volatility of a given country. The fraction of the country's variance that is explained by the global business cycle would decline too, even when its sensitivity, i.e., the factor loading to the global factor, remains constant. However, smaller global shocks cannot be interpreted as a decoupling of countries from the global factor.

[2] A related literature looks at the emergence of specific regional cycles, such as a European business cycle. For a comprehensive overview see Hirata et al. (2013).

Most of the literature restricts the innovation variance of the global and group factors to a constant. As such, the existing literature cannot account for heteroscedasticity in any common factor.[3] Observed changes in the variance decomposition, as found in, e.g., KOP, are then by construction due to changes in the factor loadings. An exception is Del Negro and Otrok (2008), who estimate a DFM with time-varying factor loadings and stochastic volatility and find a decline in the volatility for most countries in their sample of nineteen countries.

By ignoring the distinction between the source of changes in the variance decomposition, the literature may have overstated the role of factor loadings and thus overemphasized the decoupling hypothesis.

In this paper, we deal with these limitations and estimate a DFM with time-varying factor loadings and stochastic volatilities. Particular attention is paid to model uncertainty: a Bayesian model selection procedure is used to explicitly test for time variation in the factor loadings and volatilities. As such, the model allows for time variation in the parameters but does not force parameters to change. This enables us to obtain time-varying variance shares with endogenously determined time variation. Additionally, we can attribute changes in the variance decomposition to changes in either the factor loadings, volatilities, or both. We apply the model to the set of 106 countries analyzed by KOP and Hirata et al. (2013) with the sample period extended to 2014. Specifically, we follow KOP and disentangle GDP, consumption, and investment growth into a global, three group-specific, a country, and an idiosyncratic factor.

The main findings can be summarized as follows. We do find strong evidence for changes in the volatilities of the global and all three group factors: their volatility has steadily declined from 1960 until the early 2000s. Evidence for time variation in the factor loadings is much weaker. While there is some heterogeneity across countries, the overall picture is that only industrialized countries have exhibited changes in their sensitivity to the global factor. Emerging market economies and developing countries have, on average, constant factor loadings with respect to the global factor. Similarly, the sensitivity to the group-specific factors is found to be constant for the majority of countries in each country group. As a consequence, changes in the variance decomposition are primarily driven by changes in the volatilities. The importance of group-specific factors for national business cycles is confirmed. However, substantial time variation in the relative importance of the global versus group-specific factors are not found.

[3]However, there is empirical evidence that many countries experienced a decline in business cycle volatility. For instance, Blanchard and Simon (2001) show that there has been a global decline in output volatility in G7 countries. Cecchetti et al. (2006) find breaks in the volatility in most of the 25 advanced and emerging countries examined.

The remainder of this paper is organized as follows: Section 2 introduces our empirical approach, including the DFM with time-varying factor loadings and stochastic volatilities, and explains the Bayesian model selection procedure to test for time variation in the parameters. The results are presented in Section 3 and the last section concludes.

2 Empirical Approach

This section explains our econometric approach. First, it lays out a DFM with time-varying factor loadings and stochastic volatilities. Second, the Bayesian stochastic model selection approach is explained, followed by a description of the Markov Chain Monte Carlo (MCMC) algorithm employed to estimate the model.

2.1 A DFM with time-varying loadings and stochastic volatilities

We follow KOP and construct a multivariate DFM that decomposes the real GDP, private consumption, and investment growth into a global factor F_t^g, which is common to all variables in all countries, three group-specific factors, denoted F_t^{IC}, F_t^{EM}, and F_t^{DC}, which are common to all variables and all countries belonging to either the group of industrial countries (ICs), emerging market economies (EMs), or other developing countries (DCs), a country-specific factor $F_{i,t}^c$ that is common to all variables within a country i, and idiosyncratic factors $\varepsilon_{i,t}^j$, which are specific to each variable.

More specifically, let $y_{i,t}^j$ denote the annual growth rate of variable j in country i at time t. The model is given by

$$y_{i,t}^j = \alpha_{i,t}^j F_t^g + \beta_{i,t}^j F_t^r + \delta_{i,t}^j F_{i,t}^c + \varepsilon_{i,t}^j, \tag{1}$$

where the group-specific factor F_t^r equals either F_t^{IC}, F_t^{EM}, or F_t^{DC}, depending on the group affiliation of country i. All factors in Eq. (1) are assumed to follow independent AR(3) processes,

$$D_t = \sum_{l=1}^{3} \theta_l^D D_{t-l} + \exp(h_t^D)\psi_t^D, \qquad \psi_t^D \overset{iid}{\sim} \mathcal{N}(0,1), \tag{2}$$

where $D_t = \left\{F_t^g, F_t^{IC}, F_t^{EM}, F_t^{DC}, F_{i,t}^c\right\}$. Similarly, the idiosyncratic factors follow AR(3) processes,[4]

$$\varepsilon_{i,t}^{j}=\sum_{l=1}^{3}\phi_{l,i}^{j}\varepsilon_{i,t-l}^{j}+\nu_{i,t}^{j},\qquad \nu_{i,t}^{j}\overset{iid}{\sim}\mathcal{N}(0,\sigma_{\nu,i,j}^{2}). \tag{3}$$

In order to take into account possible changes in the sensitivity to the factors, we model all factor loadings as random walks,

$$\zeta_{i,t}^{j}=\zeta_{i,t-1}^{j}+\kappa_{\zeta,i,t}^{j},\qquad \kappa_{\zeta,i,t}^{j}\overset{iid}{\sim}\mathcal{N}(0,\sigma_{\kappa,\zeta,i,j}^{2}), \tag{4}$$

where $\zeta=\{\alpha,\beta,\delta\}$. The innovations to the factor loadings are orthogonal, implying that changes in the factor loadings are uncorrelated across countries. Changes in the factors' variances are accounted for by modeling the log standard deviations of the error terms pertaining to the global, the group-specific, and the country-specific factors as random walk processes,

$$h_{t}^{D}=h_{t-1}^{D}+\eta_{t}^{D},\qquad \eta_{t}^{D}\overset{iid}{\sim}\mathcal{N}(0,\sigma_{\eta,D}^{2}). \tag{5}$$

A key feature of the stochastic volatility components, $\exp\left\{h_{t}^{D}\right\}\psi_{t}^{D}$, is that they are nonlinear but can be transformed into linear components by taking the logarithm of their squares

$$\ln\left(\exp\left\{h_{t}^{D}\right\}\psi_{t}^{D}\right)^{2}=2h_{t}^{D}+\ln\left(\psi_{t}^{D}\right)^{2}, \tag{6}$$

where $ln\left(\psi_{t}^{D}\right)^{2}$ is log-chi-square distributed with expected value -1.2704 and variance 4.93. Following Kim et al. (1998), we approximate the linear model in (6) by an offset mixture time series model:

$$g_{t}^{D}=2h_{t}^{D}+\epsilon_{t}^{D}, \tag{7}$$

where $g_{t}^{D}=\ln\left(\left(\exp\left\{h_{t}^{D}\right\}\psi_{t}^{D}\right)^{2}+c\right)$ with $c=.001$ being an offset constant, and the distribution of ϵ_{t}^{D} being the following mixture of normals,

$$f\left(\epsilon_{t}^{D}\right)=\sum_{n=1}^{M}q_{n}f_{N}\left(\epsilon_{t}^{D}|m_{n}-1.2704,\vartheta_{n}^{2}\right), \tag{8}$$

[4] Given that the model is fitted to annual data, the AR(3) assumption is sufficient to capture the dynamics in output, consumption and investment growth. Furthermore, it allows us to directly compare our results to KOP, as they also model all factors as AR(3) processes.

with component probabilities q_n, means $m_n - 1.2704$, and variances ϑ_n^2. Equivalently, this mixture density can be written in terms of the component indicator variable ι_t^D as

$$\epsilon_t^D | \left(\iota_t^D = n\right) \sim \mathcal{N}\left(m_n - 1.2704, \vartheta_n^2\right), \qquad \text{with} \qquad Pr\left(\iota_t^D = n\right) = q_n. \tag{9}$$

Following Omori et al. (2007), we use a mixture of $M = 10$ normal distributions to make the approximation to the log-chi-square distribution sufficiently good. Values for $\{q_n, m_n, \vartheta_n^2\}$ are provided by Omori et al. in their table 1.

Identification

The model in Eqs. (1)–(5) exhibits two well known identification problems present in all DFM, even with constant parameters. First, we cannot separately identify the factor loadings and the factor variances, as it is possible to multiply the terms $\zeta_{i,t}^j D_t$ by any constant, which results in different decompositions of the observed time series $y_{i,t}^j$. This is referred to as the scale problem in dynamic factor models. To overcome this problem, we follow Del Negro and Otrok (2008) and fix the initial volatility h_0^D of each factor D to a constant. The second problem is that the signs of the factor loadings and the factors are not jointly identified, since the likelihood remains the same if we multiply $\zeta_{i,t}^j$ and D_t by -1. We identify the sign of the global factor by restricting the initial value of the time-varying loading to the global factor for U.S. output growth to be positive, i.e., $\alpha_{US,0}^Y > 0$. Likewise, to identify the signs of the group-specific factors, we restrict the loading for the first country listed in each group (see Appendix A) to be larger than zero for output growth. Finally, country factors are identified by means of positive loadings for the output growth of each country.

Time-varying variance decompositions

We use variance decompositions to measure the relative importance of each factor. Since, by construction, all factors are orthogonal, the variance decompositions can be calculated based on Eq. (1). For instance, the variance share (VS) of the global factor for GDP growth (Y) in country i is given by

$$VS_{i,t}^{g,Y} = \frac{(\alpha_{i,t}^Y)^2 var_t(F_t^g)}{var_t(Y_{i,t})}, \tag{10}$$

where $var_t(Y_{i,t}) = (\alpha^Y_{i,t})^2 var_t(F^g_t) + (\beta^Y_{i,t})^2 var_t(F^r_t) + (\delta^Y_{i,t})^2 var_t(F^c_{i,t}) + \sigma^2_{\nu,i,Y}$. The factors' variances can be calculated based on their autoregressive dynamics and time-varying volatilities, i.e.,

$$var_t(F^D_t) = \left(\exp\left(h^D_t\right)\right)^2 \left[I_{3^2} - (\Theta \otimes \Theta)\right]^{-1}. \tag{11}$$

with $\otimes$ denoting the Kronecker product, I the identity matrix, and Θ the companion form of the AR coefficients in Eq. (2). The variance shares in Eq. (10) are time-varying due to the time-varying factor loadings and the stochastic volatilities. This allows us to analyze changes over time in the relative importance of factors without splitting the sample at an arbitrary point in time. Further, it allows for heterogeneity across countries since the timing of the changes in the variance shares can be different in each country. Changes in the variance share of, e.g., the global or a group factor can be caused by changes in the loadings, changes in the volatilities, or both.

A drawback of the model outlined so far is that it forces the factor loadings and the volatilities, and thus the variance shares, to change over time. The dynamics of the factor loadings and the (log) volatilities are given by random walk processes, which are driven by their innovation variance parameters, $\sigma^2_{\kappa,\zeta,i,j}$ and $\sigma^2_{\eta,D}$. Bayesian estimation techniques typically assume that the prior for a variance parameter follows an inverse Gamma distribution, which has no probability mass at zero. However, the inverse Gamma prior for $\sigma^2_{\kappa,\zeta,i,j}$ has two undesirable properties. First, consider the question whether a loading $\zeta^j_{i,t}$ is time-varying or constant. This implies testing the null hypothesis of $\sigma^2_{\kappa,\zeta,i,j} = 0$ against the alternative $\sigma^2_{\kappa,\zeta,i,j} > 0$ in Eq. (4), which is a non-regular testing problem since the null hypothesis lies at the boundary of the parameter space for the variance parameter. The same problem arises when testing whether the factors' conditional variances are time-varying or constant. As such, using the conventional inverse Gamma prior does not allow testing the null of constant parameters and variances. Second, as shown by Frühwirth-Schnatter and Wagner (2010), using an inverse Gamma prior can lead to a substantial overestimation of the variances, even in cases where the true innovation variance is positive but small. As a consequence, it can overstate changes in the variance decomposition. To deal with these problems, we rewrite the random walk processes in a non-centered parametrization form, which allows us to estimate the innovation standard errors instead of variances. Additionally, we use a stochastic model specification search to test the hypothesis of constant factor loadings and volatilities.

2.2 Stochastic model specification search

The Bayesian stochastic model specification search is based on Frühwirth-Schnatter and Wagner (2010) and extends Bayesian variable selection in standard regression models to state space models. The model selection relies on a non-centered parametrization of the model in which (i) binary stochastic indicators for each of the model components are sampled together with the parameters and (ii) the standard inverse Gamma prior for the variances of innovations to the components is replaced by a Gaussian prior centered at zero for the square root of these variances.

Non-Centered parametrization

The first piece of information on the hypothesis whether a variance parameter is zero or not can be obtained by considering a non-centered parametrization. For the variances of the innovations to the factor loadings, i.e., $\sigma^2_{\kappa,\zeta,i,j}$, this implies rearranging Eq. (4) to

$$\zeta^j_{i,t} = \zeta^j_{i,0} + \sigma^{\zeta}_{\kappa,i,j}\widetilde{\zeta}^j_{i,t}, \tag{12}$$

$$\text{with} \quad \widetilde{\zeta}^j_{i,t} = \widetilde{\zeta}^j_{i,t-1} + \widetilde{\kappa}^{j,\zeta}_{i,t}, \qquad \widetilde{\zeta}^j_{i,0} = 0, \qquad \widetilde{\kappa}^{j,\zeta}_{i,t} \overset{iid}{\sim} \mathcal{N}(0,1), \tag{13}$$

where $\zeta^j_{i,0}$ is the initial value of the level of $\zeta^j_{i,t}$. A crucial aspect of a non-centered parametrization is that it is not identified, i.e., the signs of $\sigma^{\zeta}_{\kappa,i,j}$ and $\widetilde{\zeta}^j_{i,t}$ can be changed by multiplying both with -1 without changing their product in Eq. (12). As a result of this non-identification, the likelihood is symmetric around 0 along the $\sigma^{\zeta}_{\kappa,i,j}$ dimension and therefore multimodal. If the factor loading is time-varying, i.e., $\sigma^2_{\kappa,\zeta,i,j} > 0$, the likelihood function will concentrate around the two modes $-\sigma^{\zeta}_{\kappa,i,j}$ and $\sigma^{\zeta}_{\kappa,i,j}$. For $\sigma^2_{\kappa,\zeta,i,j} = 0$, the likelihood function will become unimodal around zero. As such, allowing for a non-identification of $\sigma^{\zeta}_{\kappa,i,j}$ provides useful information on whether $\sigma^2_{\kappa,\zeta,i,j} > 0$.

Likewise, the non-centered parametrization of the stochastic volatility terms in Eq. (5) is given by

$$h^D_t = h^D_0 + \sigma_{\eta,D}\widetilde{h}^D_t, \tag{14}$$

$$\text{with} \quad \widetilde{h}^D_t = \widetilde{h}^D_{t-1} + \widetilde{\eta}^D_t, \qquad \widetilde{h}^D_0 = 0, \qquad \widetilde{\eta}^D_t \overset{iid}{\sim} \mathcal{N}(0,1), \tag{15}$$

where h_0^D is the initial value of the level of h_t^D.[5]

Parsimonious specification

A second advantage of the non-centered parametrization is that when, e.g., $\sigma^{\zeta}_{\kappa,i,j} = 0$, the transformed component $\widetilde{\zeta}^j_{i,t}$, in contrast to $\zeta^j_{i,t}$, does not degenerate to a time-invariant factor loading, as this is now represented by $\zeta^j_{i,0}$. As such, the question whether the factor loadings are time-varying or not can be expressed by a variable selection problem in Eq. (4). Consider the following parsimonious specification,

$$\zeta^j_{i,t} = \zeta^j_{i,0} + \lambda^{\zeta}_{i,j}\sigma^{\zeta}_{\kappa,i,j}\widetilde{\zeta}^j_{i,t}, \tag{16}$$

where $\lambda^{\zeta}_{i,j}$ is a binary indicator which is either 0 or 1. If $\lambda^{\zeta}_{i,j} = 0$, the component $\widetilde{\zeta}^j_{i,t}$ drops out of the model, so that $\zeta^j_{i,0}$ represents a constant factor loading. If $\lambda^{\zeta}_{i,j} = 1$, then $\widetilde{\zeta}^j_{i,t}$ is included in the model, and $\sigma^{\zeta}_{\kappa,i,j}$ is estimated from the data. In this case, $\zeta^j_{i,0}$ is the initial value of the time-varying factor loading.

Likewise, the parsimonious non-centered specification of the stochastic volatility terms in Eq. (5) is given by

$$h_t^D = h_0^D + \rho^D\sigma_{\eta,D}\widetilde{h}_t^D, \tag{17}$$

where ρ^D is again a binary indicator. If $\rho^D = 0$, the component $\widetilde{h}_t^D$ drops out of the model, so that $\left(\exp\{h_0^D\}\right)^2$ is the constant variance of ψ_t^D. If $\rho^D = 1$, then $\widetilde{h}_t^D$ is included in the model and $\sigma_{\eta,D}$ is estimated from the data. In this case, $\left(\exp\{h_0^D\}\right)^2$ is the initial value of the time-varying variance of ψ_t^D. We collect the binary indicators into the vector $\mathcal{M} = (\lambda^{\zeta}_{i,j}, \rho^D)$.

Gaussian prior centered at zero

It is well-known that when using an inverse Gamma prior distribution for the variance parameters, the choice of the shape and scale hyperparameters that define this distribution have a strong influence on the posterior when the true value of the variance is close to zero. More specifically, as the inverse Gamma distribution does not have any probability mass at zero, using it as a prior distribution tends to push the posterior density away from zero. This is of particular importance when estimating the variances of the innovations to the time-varying factor loadings and

[5] As mentioned before, h_0^D is fixed to be a constant due to an identification restriction.

to the stochastic volatilities, because for these components we want to decide whether they are relevant or not. A further important advantage of the non-centered parametrization is therefore that it allows us to replace the standard inverse Gamma prior on a variance parameter σ^2 by a Gaussian prior centered at zero on σ. Centering the prior distribution at zero makes sense, since for both $\sigma^2 = 0$ and $\sigma^2 > 0$, σ is symmetric around zero. Frühwirth-Schnatter and Wagner (2010) show that, compared to using an inverse Gamma prior for σ^2, the posterior density of σ is much less sensitive to the hyperparameters of the Gaussian distribution and is not pushed away from zero when $\sigma^2 = 0$.

As such, we choose a Gaussian prior distribution centered at zero for σ_κ and σ_η, which are the standard deviations of the innovations to the time-varying factor loadings and to the stochastic volatilities. Specifically, we choose $\mathcal{N}(0, 5^2)$ for both σ_κ and σ_η. Similarly, a flat prior is used for the time-invariant components of the factor loadings, i.e., $\zeta^j_{i,0} \sim \mathcal{N}(0.5, 10^2)$. For each binary indicator in $\mathcal{M}$, we choose a uniform prior distribution such that $p_0 = 0.5$ is the prior probability for each time-varying component to be included in the model.

For the variance parameters of the innovations to the idiosyncratic factors σ^2_ν, we use the standard inverse Gamma prior $\mathcal{IG}(c_0, C_0)$, where c_0 and C_0 are the shape and scale parameters, respectively. The calculation of c_0 and C_0 is explained in greater detail in Appendix B. Finally, the priors for the autoregressive coefficients are assumed to be Gaussian with mean zero and unit variance.

2.3 MCMC algorithm

The inclusion of time-varying factor loadings $\zeta^j_{i,t}$, stochastic volatilities h^D_t, and the use of a stochastic model specification search, confronts us with a highly non-linear estimation problem. We estimate the model using a Gibbs sampler, which is a Markov chain Monte Carlo (MCMC) method to simulate draws from the intractable joint and marginal posterior distributions of the unknown parameters and the unobserved factors and states using only tractable conditional distributions. Intuitively, this amounts to reducing the complex non-linear model to a sequence of blocks for subsets of the parameters and states that are tractable, conditional on the other blocks in the sequence.

For notational convenience, the parameters are collected into the vector $\Psi = (\theta, \phi, \zeta_0, h_0, \sigma^2_\nu, \sigma)$, where $\theta = \{\theta^D_1, \theta^D_2, \theta^D_3\}$, $\phi = \{\phi^j_{1,i}, \phi^j_{2,i}, \phi^j_{3,i}\}$, $\zeta_0 = \{\zeta^j_{i,0}\}$, and $\sigma = (\sigma^\zeta_{\kappa,i,j}, \sigma^D_\eta)$. In addition, all component indicator variables are collected into the vector $\iota_t = \{\iota^D_t\}$. Further, let

$y_{i,t} = (Y_{i,t}, C_{i,t}, I_{i,t})$ be the data vector. Stacking the observations over time and countries, we write $y = \{y_{i,t}\}_{t=1,i=1}^{T,N}$ and use a similar notation for D, ζ, ι and h.[6] Given the initial values of all factors and parameters, the sampling scheme is as follows:

1. Sample each factor from $f(D\,|D^{-}, \zeta, h, \Psi, \mathcal{M}, \iota, y)$, where D^{-} denotes the remaining factors.

2. Sample the binary indicators from $f(\mathcal{M}\,|D, \zeta, h, \iota, y)$, marginalizing over the relevant parameters in Ψ, and then sample the corresponding unrestricted parameters from $f(\Psi\,|D, \zeta, h, \mathcal{M}, \iota, y)$ while setting the restricted parameters, i.e., the elements in σ for which the time-varying component is not included in the model, equal to 0.

3. Sample the time-varying factor loadings in ζ from $f(\alpha\,|D, \beta, \delta, h, \Psi, \mathcal{M}, \iota, y)$, $f(\beta\,|D, \alpha, \delta, h, \Psi, \mathcal{M}, \iota, y)$, and $f(\delta\,|D, \alpha, \beta, h, \Psi, \mathcal{M}, \iota, y)$, respectively. Sample the mixture indicators ι from $f(\iota\,|D, \zeta, h, \Psi, \mathcal{M}, y)$ and the stochastic volatilities h from $f(h\,|D, \zeta, \Psi, \mathcal{M}, \iota, y)$.

4. Perform a random sign switch for $\sigma^{\zeta}_{\kappa,i,j}$ and $\{\widetilde{\zeta}_{i,t}\}_{t=1}^{T}$ and for σ^{D}_{η} and $\{\widetilde{h}^{D}_{t}\}_{t=1}^{T}$, where, e.g., $\sigma^{\zeta}_{\kappa,i,j}$ and $\{\widetilde{\zeta}_{i,t}\}_{t=1}^{T}$ are left unchanged with probability 0.5, while with the same probability they are replaced by $-\sigma^{\zeta}_{\kappa,i,j}$ and $\{-\widetilde{\zeta}_{i,t}\}_{t=1}^{T}$.

5. Sample the remaining hyperparameters in Ψ from $f(\Psi\,|D, \zeta, h, \mathcal{M}, \iota, y)$.

Sampling from these blocks is iterated $J = 20,000$ times and, after a sufficiently long burn-in period of $B = 10,000$, the sequence of draws $(B + 1, ..., J)$ approximates a sample from the desired posterior distribution $f(D, \zeta, h, \mathcal{M}, \iota, \Psi\,|y)$. Details of the exact implementation of the Gibbs sampler are given in Appendix B. In each iteration of the sampling process, we calculate the (potentially time-varying) variance decomposition for output, consumption, and investment growth in each country.

3 Estimation results

3.1 Data

The data set is drawn from *Penn World Tables 9.0* and includes annual observations 1960–2014 for 106 countries. We use National Accounts data for real GDP, real private consumption, and

[6] Our MCMC scheme follows Del Negro and Otrok (2008) and Frühwirth-Schnatter and Wagner (2010) for the model selection part.

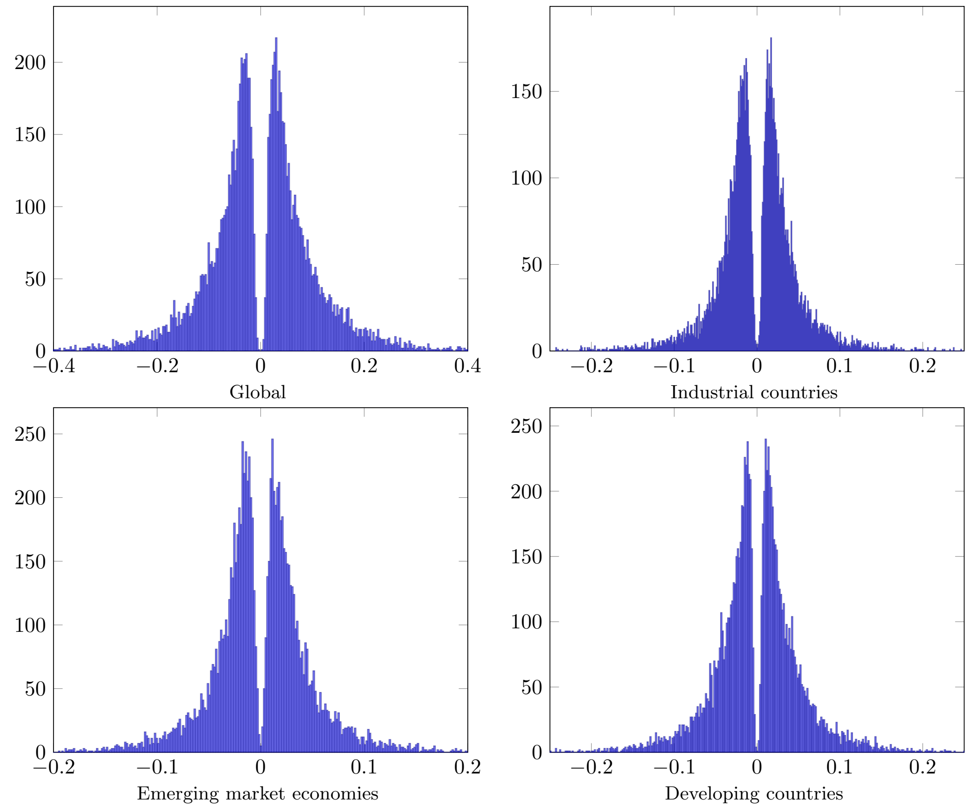
real investment at constant prices in local currencies, and compute the demeaned growth rate of each series. Following KOP, countries are grouped according to their level of development. Each country belongs either to the group of industrial countries (23 ICs), emerging market economies (24 EMs), or developing countries (59 DCs). Appendix A provides a list of the countries and their group affiliation.

3.2 Testing for time variation in the factor loadings and volatilities

3.2.1 Volatilities

This section presents the results of the stochastic model selection. Preliminary evidence for potential time variation in the factor loadings and the volatilities is obtained by estimating the model with all binary indicators set equal to one. The resulting posterior distributions for the standard deviations σ_η in the volatility equation Eq. (14) of the international factors are shown in Figure 1. The posterior distributions of these standard deviations show clear bi-modality with

Figure 1: Standard deviations of the stochastic volatilities of the global and group-specific factors

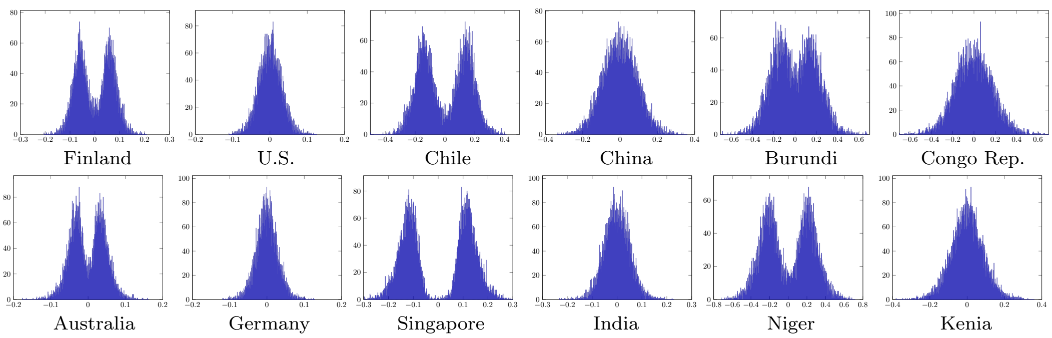


Note: Reported are the posterior distributions of the standard deviations for the non-centered stochastic volatilities in the unrestricted model, i.e., σ_η (all binary indicators set to 1).

very little probability mass at zero.[7] This is taken as preliminary evidence of time variation in the volatilities of the global and the group-specific factors, i.e., it indicates that $\sigma_\eta^2 > 0$.

Less clear cut results are obtained for the factor loadings, for which we find considerable heterogeneity across countries. The upper panel of Figure 2 shows the posterior distributions of the standard deviations of the non-centered factor loadings of output to the global factor for some randomly selected countries from all three country groups. The distributions are bi-modal for Finland, Chile, and Burundi, which suggests that the factor loadings of output to the global factor are time-varying for these countries, i.e., $\sigma_\kappa^2 > 0$, whereas the uni-modal distributions of the standard deviations for the United States, China, and Congo Republic suggest that these factor loadings did not change over time, i.e., $\sigma_\kappa^2 = 0$. Similarly, the lower panel of Figure 2 shows evidence for time-varying factor loadings of output to the group-specific factors for Australia, Singapore, and Niger, but constant factor loadings for the group factors for Germany, India, and Kenya.

Figure 2: Standard deviations of the factor loadings of output to the global (upper panel) and group-specific (lower panel) factors



Note: Reported are the posterior distributions of the standard deviations for the non-centered factor loadings of output to the global and group-specific factors for selected countries in each group in the unrestricted model, i.e., σ_κ (all binary indicators set to 1).

A more formal test for time variation in the volatilities and the factor loadings is provided by estimating the binary indicators in Eqs. (16)-(17) together with the other parameters in the model. Table 1 displays the posterior inclusion probabilities for the time-varying component in the non-centered stochastic volatilities of each factor. The inclusion probabilities are calculated as the average selection frequencies of all retained iterations of the Gibbs sampler. From Table 1, we conclude that the model clearly supports time-varying variances of the global and all group-specific factors, as the corresponding inclusion probabilities all are equal to one.

Concerning the time variation in the country factors' variances, the results are less clear,

[7] The volatilities of the country-specific factors present a more mixed picture. Graphs of the posterior distributions of the country factors are not shown, due to space constraints, but are available upon request.

Table 1: Posterior inclusion probabilities for the binary indicators for stochastic volatilities

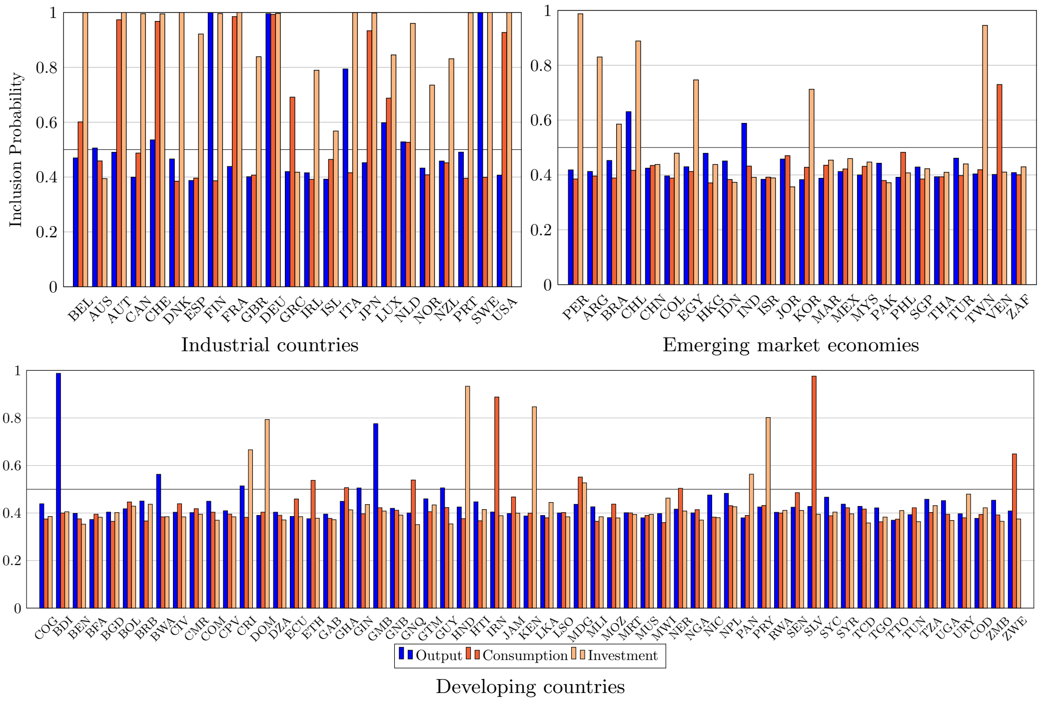
Common factors				Country factors		
Global	IC	EM	DC	IC	EM	DC
1	1	1	1	0.88	0.52	0.44

Note: The reported probabilities are calculated as the average of the binary indicators over the 10,000 iterations of the Gibbs sampler.

and differ considerably for different country groups. The average inclusion probabilities across countries in each group suggest that the volatilities of the country-specific factors are time-varying in industrial countries. The cross-country average of the inclusion probability is close to 0.5 for the remaining two groups. In particular, 21 out of the 23 industrial countries in the sample exhibit an inclusion probability above 0.5, indeed, 19 countries even have an inclusion probability larger than 0.9. In the group of emerging market economies, the fraction of countries with an inclusion probability higher than 0.5 is still 50%, but only 26 out of 59 developing countries have an inclusion probability for the binary indicator of above 0.5.

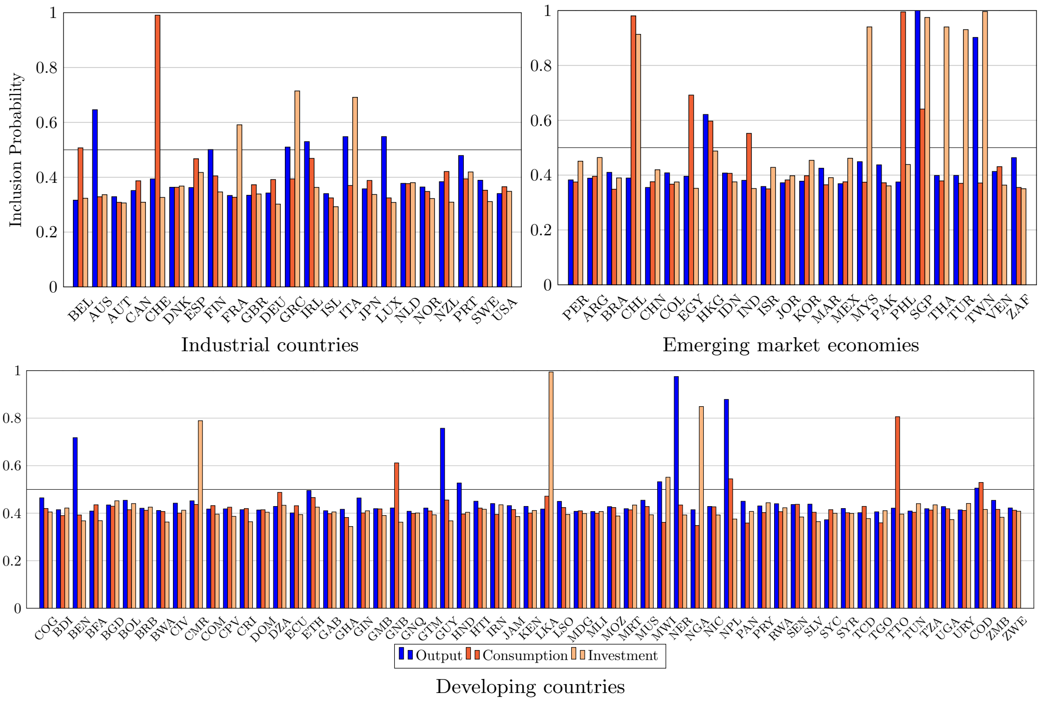
3.2.2 Factor loadings

Figure 3: Probability for time-varying loadings to the global factor



Note: Reported are the inclusion probabilities of the time-varying part in the factor loadings computed as the average selection frequencies over the retained iterations of the Gibbs sampler.

Figure 4: Probability for time-varying loadings to the group-specific factors



Note: Reported are the inclusion probabilities of the time-varying part in the factor loadings computed as the average selection frequencies over the retained iterations of the Gibbs sampler.

Turning to the tests for time variation in the factor loadings, Figure 3 displays the inclusion probabilities for the time-varying component in the loadings to the global factor. In the group of industrialized countries, only three out of 23 countries show very strong evidence of time-varying loadings of output to the global factor, i.e., an inclusion probability higher than 95%: namely, Finland, Germany, and Sweden. For Italy, the model detects time variation in about 80% of the retained Gibbs iterations. Consumption growth is found to have time-varying loadings to the global factor in nine industrialized countries. For investment growth, we find evidence that the majority of industrialized countries exhibit time variation in the loadings to the global factor, with most countries' inclusion probability being above 90%. In the group of emerging markets, evidence for time variation in the loadings to the global factor is even weaker. For output and consumption, almost all emerging market economies in our sample have a probability below 50% for including the time-varying part in their loadings to the global business cycle. For investment growth, there is evidence for time variation in seven out of 24 countries.

The picture remains the same in the group of developing countries, i.e., the model selection procedure rejects time variation in the loadings to the global factor for the vast majority of countries.

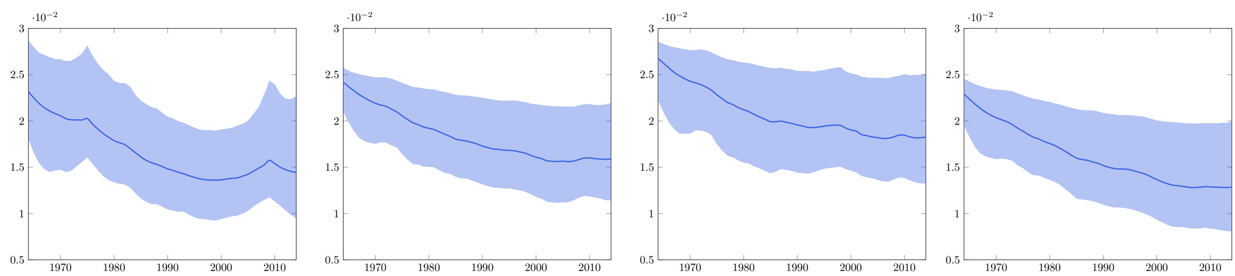
Evidence for time-varying factor loadings to the group-specific factors is even weaker, as shown in Figure 4. In most countries across all country groups, the inclusion probability of the binary indicator is well below 40% for output, consumption and investment.[8] Overall, it is striking that many of the estimated inclusion probabilities are below 50% but are still around 40%. As a robustness check, we changed the threshold for time variation to 10%, i.e., treated all loadings with an inclusion probability above 10% as time-varying. As we discuss below, the resulting variance decomposition, i.e., the shares attributed to the global, group, country and idiosyncratic factors, changes only very little. The reason is that the loadings for which the inclusion probability is below 50% hardly change in magnitude, even when we restrict the binary indicators to one, i.e., force the loading to vary over time. In sum, we find strong evidence for time-varying volatilities in the global and all three group factors. In contrast, the model selection procedure rejects time variation in the loadings for the majority of countries. The first result is therefore that the sensitivity to global and group-specific shocks has not changed over time in most countries, but the size of the shocks has changed.

3.3 Estimated factor loadings and volatilities

The estimated time-varying standard deviations for the common factors are shown in Figure 5.

The volatility of all common shocks has decreased over time, whereas periods of economic crisis are associated with a surge in volatility. For example, the volatility of the global factor increased during the first oil price shock and more recently during the Great Recession in the late 2000s. Similarly, the volatilities of the emerging market economies and the developing countries factor rose slightly during the Asian crisis in the late 1990s.

Figure 5: Stochastic volatilities of the global and group-specific factors



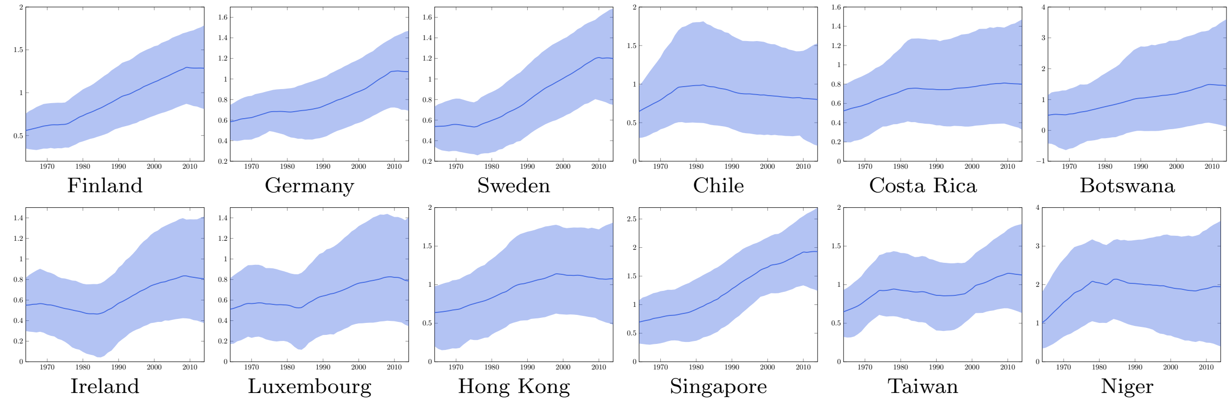
Global | Industrial countries | Emerging economies | Developing countries

Note: Reported are the median, the 5th and the 95th percentile of the posterior distribution of the time-varying standard deviations of the global and the group-specific factors.

Figure 6 shows the factor loadings for some randomly selected countries of all groups where the model selection procedure finds time variation. The upper panel displays the loadings of

[8] Inclusion probabilities for the country factors are not shown but are available upon request.

Figure 6: Factor loadings of output to the global factor



Note: Reported are the median, the 5th, and the 95th percentile of the posterior distribution of the time-varying factor loadings for output and investment to the global factor for randomly selected countries. Time-varying factor loadings are estimated given that posterior inclusion probabilities are above 0.5.

output to the global factor, which are clearly increasing over time for Finland, Germany, and Sweden and to a lesser extend for Costa Rica and Botswana. The loading for Chile increased until about 1980 but has been slightly decreasing since then. The lower panel of Figure 6 shows the loadings of output to the respective group factor. These increase moderately for Ireland, Luxembourg, Hong Kong, and Taiwan, but rather strongly for Singapore. For Niger, the loading of output to its group factor increased substantially until about 1980, but has not changed much since then. Overall, time variation in the factor loadings of output to the global and group factors is detected in 32 countries, of which only eight countries exhibit decreasing loadings over time. Thus, output growth has become more sensitive to the global and the group factor in most of the countries.[9] However, the loadings of consumption to the global factor have been decreasing over time for ten industrialized countries.

3.4 Estimated global and group-specific factors

This section presents the estimates of the global and group-specific factors obtained from estimating the model based on the results of the model selection procedure. More precisely, the factor loadings and volatilities are modeled as time-varying if the inclusion probability of the time-varying part in the stochastic model selection is above 50%. When the inclusion probability is found to be below 50%, the standard deviation σ_κ or σ_η is set to zero, so that the factor loading or the volatility is given by the time invariant part, i.e., ζ_0 for the loadings and h_0 for the volatilities.

[9]Due to the large number of individual country results, we only report randomly selected results. All remaining results not reported here are available upon request.

Figure 7 displays the posterior median of the global and the group-specific factors along with the 90% probability coverage intervals. The global business cycle factor captures some major economic events of the last decades. The first global recession is found in the mid-1970s, the time of the first oil crisis induced by a sharp increase in the price of oil in 1973/74. In 1979, the second surge in oil prices caused a global downturn in growth rates in the early 1980s, captured by the global factor, as well as the 1990s recession following a collapse of the U.S. stock market. The recent Great Recession is clearly visible in the sharp decline of the global factor in 2008/09.

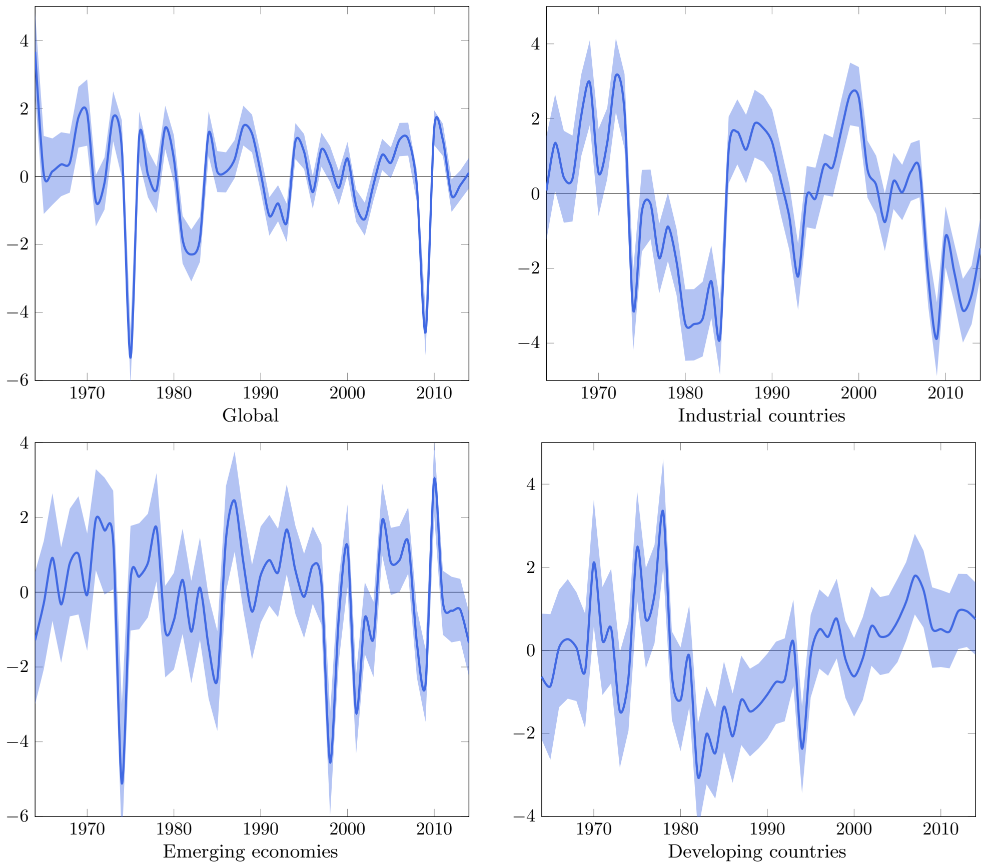
The upper right panel in Figure 7 shows the evolution of the factor specific to the group of industrial countries (ICs), which picks up both recessions associated with the oil crises. The ICs group factor accounts for a different adjustment dynamics in response to oil price shocks. In particular, the early 1980s recession is deeper and longer lasting than the perturbation of the global factor. A similar characteristic can be seen in the Great Recession, where the ICs factor shows a very slow recovery. In fact, our estimate of the ICs factor exhibits the pattern of a double dip recession, as experienced particularly in industrialized countries in Europe and North America. The estimates for the factors of the emerging market economies (EMs) and the developing countries (DCs) capture relevant economic events specific to these groups. For example, the EMs factor shown in the lower left panel picks up the Asian financial crisis in the late 1990s and the quick recovery from the recent financial crisis.

The global and the three group-specific factors are orthogonal by construction, i.e., shocks to these cycles are modeled as uncorrelated. However, specific events such as the recession in the mid-1970s and the Great Recession are visible in the global as well as in the group factors for the ICs and the EMs. When calculating the correlation of the estimated factors, we find only a very moderate correlation between these factors.

3.5 Time-varying variance decompositions

This section examines the evolution of the time-varying variance decompositions as described in Section 2. Changes over time in the variance shares attributed to each factor can be due to changes in the loadings, the variances, or both. The model selection ensures that these parameters are not forced to change, as they are in a standard time-varying parameter model with IG priors. Based on the results presented in the previous section, the time variations in the variance shares are predominantly driven by changes in the volatilities, since most of the

Figure 7: Global and group-specific factors



Note: Reported are the median, the 5th and the 95th percentile of the posterior distribution of the global and the group-specific factors.

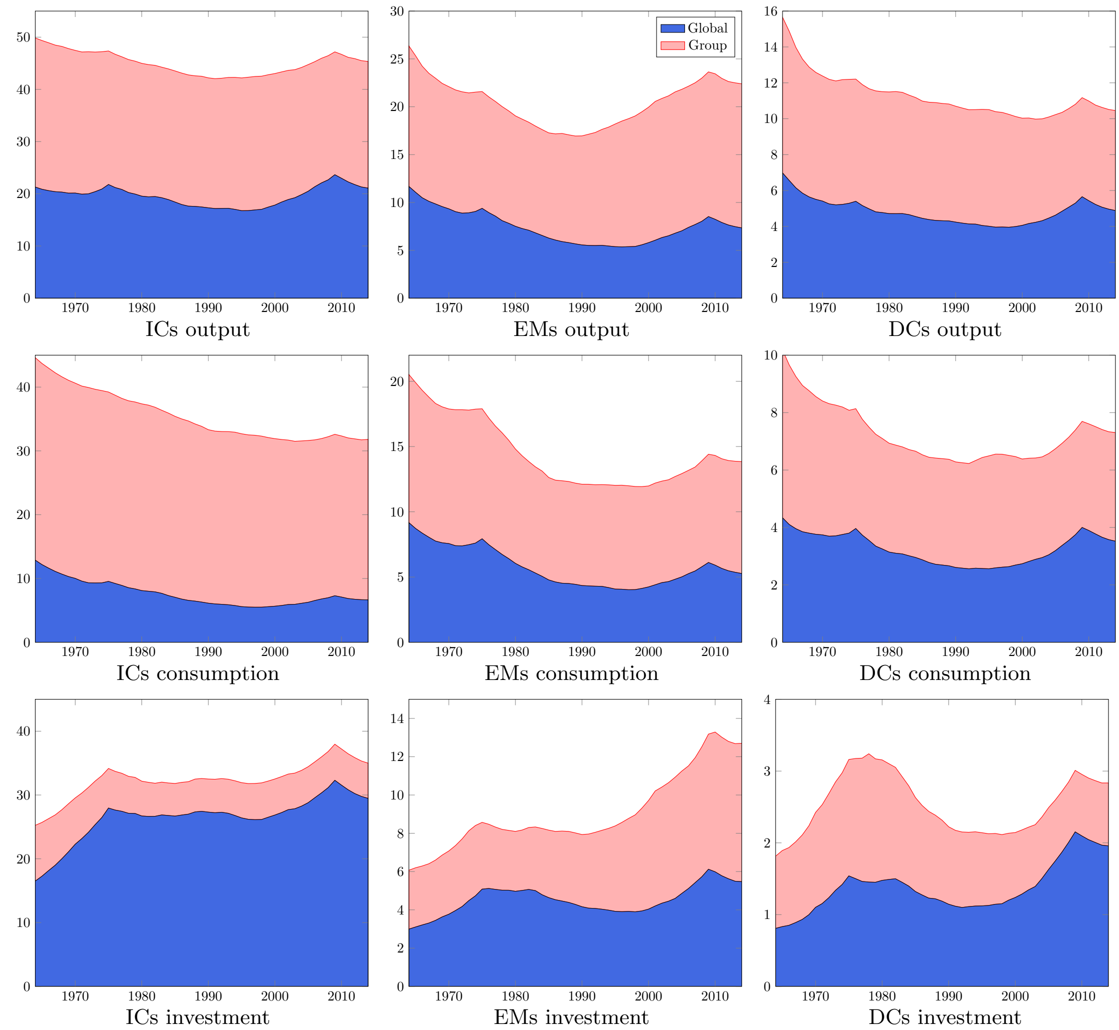
loadings are found to be constant.

Figure 8 shows the group-specific average of the time-varying variance decompositions for the global and group-specific factors for all country groups and variables. Similar to previous estimates in the literature, we find the global factor to be most important for the industrialized countries and least important for the developing countries. International factors, as measured by the joint contribution of the global and the group-specific factors, explain a substantial fraction of the vast majority of variances of all three variables. In the group of industrialized countries, the global and group factors together explain roughly 50% of the output growth variances. In emerging market economies, international factors still have a share of more than 20% of the output variances for most of the sample period. The average variance share of international factors with respect to output growth in developing countries is between 10% and 12%.

Changes in the importance of the global and group-specific factors over time are shown in Table 2, where the variance shares are averaged for each decade of the sample period.

Panel M1 displays the results from the baseline model, where the loadings and volatilities

Figure 8: Time-varying variance decompositions in percent. Group-specific averages.



are time-varying when the estimated binary indicators are above 0.5. Panel M2 restricts all loadings to be constant and only allows the volatilities to vary over time. Not surprisingly, the results do not differ considerably. There are some differences in the group of industrialized countries, in particular for investment, for which time variation in the loadings is important, but there are very little differences between the two models in the variance shares in the emerging market economies and developing countries. This confirms that changes in the factor loadings are of minor importance and the estimated changes in the variance shares are predominantly attributed to changes in the size of the shocks, i.e, the volatilities.[10] Consistent with KOP, we find a decrease in the importance of the global factor for output and consumption in all country groups. The group-specific factor is more important than the global factor. However, the changes are smaller as compared to KOP. The largest increase in the importance of group-specific shocks

[10] A declining share of the global factor implies that the decline in the variance of the global factor multiplied by the squared loading, i.e., $\alpha_{i,t}^2 var_t(F_t^g)$, is larger than the decline in the aggregated variance, $var_t(y_{i,t})$ (see Eq. (10)).

Table 2: Variance decompositions by decade.

			Output				Consumption				Investment			
			'70	'80	'90	'00	'70	'80	'90	'00	'70	'80	'90	'00
M1	IC	global	20.4	19.2	17.1	18.7	9.8	7.8	5.9	5.9	23.6	26.6	26.9	27.4
		group	27.2	25.4	25.3	24.9	30.6	28.9	27.3	25.9	7.1	5.5	5.4	6.8
	EM	global	9.4	7.2	5.6	6.3	7.6	5.7	4.3	4.5	4.2	4.8	4.0	4.3
		group	12.7	11.3	11.8	14.6	10.3	8.4	7.7	7.8	3.4	3.2	3.9	5.8
	DC	global	5.4	4.6	4.2	4.3	3.8	3.1	2.6	2.9	1.2	1.4	1.2	1.4
		group	6.9	6.6	6.4	5.8	4.5	3.8	3.8	3.7	1.4	1.6	1.1	0.9
M2	IC	global	16.9	14.4	11.5	11.1	18.2	15.1	11.8	11.1	3.1	2.5	1.8	1.6
		group	25.8	23.8	23.9	24.4	15.1	13.6	13.7	14.0	8.9	7.4	7.0	6.8
	EM	global	9.1	6.9	5.2	5.2	6.5	4.8	3.6	3.6	1.1	0.8	0.6	0.5
		group	12.8	10.7	10.7	11.9	9.3	7.6	7.6	8.4	1.8	1.4	1.4	1.4
	DC	global	5.2	4.0	3.0	2.7	3.2	2.4	1.8	1.7	0.4	0.3	0.2	0.2
		group	5.8	4.6	4.0	3.7	3.6	2.8	2.5	2.3	0.4	0.3	1.4	0.2

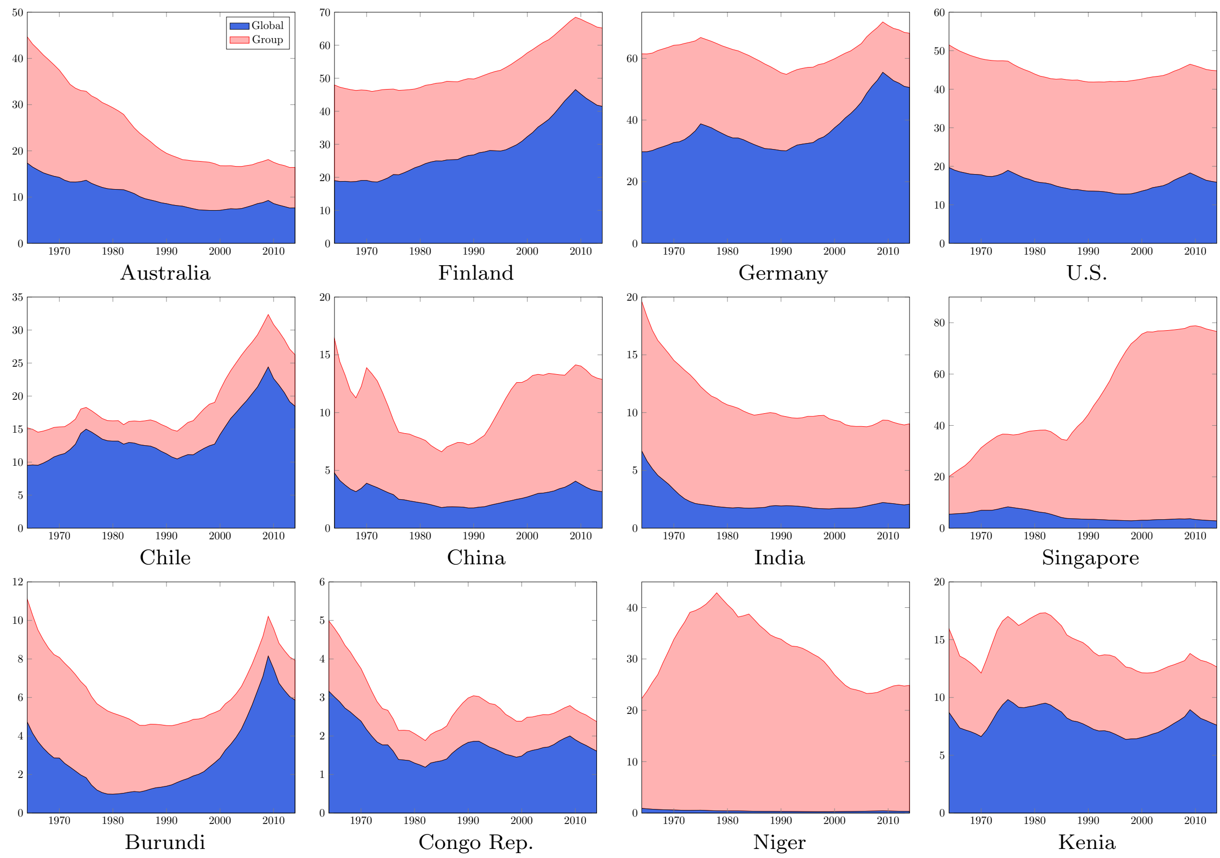
Note: Ten-year averages of the variance decompositions. **M1:** All binary indicators in the non-centered specification of factor loadings and stochastic volatilities are set according to the estimated inclusion probabilities. **M2:** All indicators of the non-centered factor loadings are equal to zero and restrict the loadings to be constant parameters.

are found for investment in emerging markets.

However, the results regarding the relative importance of the different factors are very heterogeneous across countries. Figure 9 plots the time-varying variance decompositions of output growth for the global and group-specific factors for randomly selected countries from all country groups. In the group of industrialized countries, a substantial decrease of the relative importance of international factors, as measured by the sum of the global and group factor, is found for Australia, and a moderate decrease is found for the U.S. For Finland and Germany, international factors have become more important over time, whereas the exposure to global shocks in particular has increased during the recent financial crisis. In the group of emerging market economies, an increasing importance of the global factor is only found for Chile, especially during the first oil crisis but also during the recent financial crisis. In China, India, and Singapore, the group-specific factor accounts for a relatively high fraction of the variation in output growth as compared to the global factor. Whereas international factors in all have become less important for India, a strong increase is found for Singapore, driven by the importance of the group factor. International factors are even less important in the group of developing countries, except for

Niger, for which the group factor accounts for a large fraction of the fluctuations in the growth of output. The global factor has become more important for Burundi, but less important for the Congo Republic and Kenya.

Figure 9: Time-varying variance decompositions of output growth for selected countries in percent.



Discussion

Similar to KOP and others, we find that global shocks have become less important for national business cycles over time, although the magnitude of these changes is smaller. More importantly, we find that changes in the size of the shocks are responsible for the decoupling of most countries, rather than changes in the sensitivity of countries to global or group-specific shocks.

This distinction is important, as it allows to better understand the driving forces of global business cycle decoupling. First, changes in the sensitivity to a factor as measured by the factor loadings are the result of domestic policies targeting, for example, the external sector or the financial sector of a particular country, and hence are country-specific. However, the effect of reducing trade and financial barriers on the synchronization of business cycles is ambiguous from a theoretical point of view. On the one hand, increasing trade flows should increase output comovements between countries, since both demand- and supply-side spillovers are generated. On the other hand, output comovements might decrease if stronger trade linkages result in increased specialization according to each country's comparative advantage and if industry-specific shocks are dominant (Baxter and Kouparitsas (2005)).

The effect of stronger financial linkages on output comovements could take both directions as well. For example, output synchronization between countries could decrease if stronger financial linkages facilitate the efficient reallocation of capital and hence result in an increased specialization of production according to the countries' comparative advantages. However, increased financial linkages could also result in higher international business cycle comovement if contagion effects, transmitted through financial markets, are present (Kose and Prasad (2010)).

Second, changes in the variances of the factors are affected by the magnitude of the shocks to international and national business cycles. As widely discussed for the case of the U.S. regarding the Great Moderation, there is no consensus on what has caused the reduction in the volatility of output. Hence, this phenomenon could have been the outcome of better policies (e.g., Blanchard and Simon (2001)), good luck in the sense of no major adverse shocks (e.g., Stock and Watson (2003)), or structural changes in the economy (e.g., Kahn et al. (2002)). Even though this discussion has focused on the U.S. economy, these arguments can be applied to the decreasing volatility of international business cycles as well.

Given these distinctions, we interpret global business cycle decoupling, as measured by a decreasing relative importance of the global factor, as a reduction in the magnitude of international shocks rather than in a reduction of countries' sensitivity to international shocks.

4 Conclusion

During recent years, the empirical literature that reports a decrease in the importance of global shocks to national business cycles has grown (e.g., Kose et al. (2003, 2012); Mumtaz et al. (2011)). This result is, however, at odds with the fact that countries have become more interlinked, as world trade and international capital flows have increased tremendously. At the same time, shocks dedicated to smaller sub-groups of countries, e.g., industrialized countries, emerging market economies, and developing countries, have been found to be more important in explaining national business cycles.

In this paper, we have focused on whether the changes in the relative importance of global and group-specific shocks have been driven by changes in the sensitivity to common shocks or by changes in the size of the shocks. To this end, a hierarchical DFM with time-varying loadings and stochastic volatilities for output, consumption, and investment growth, for 106 countries 1961–2014 has been estimated. A Bayesian model selection procedure has been used to explicitly test for time variation in the factor loadings and volatilities. As such, the model allows for time variation in the parameters but does not force the parameters to change.

We have found strong evidence of changes in the size of the international shocks, both at the global and the group-specific level. International shocks have become smaller over the past decades. In contrast, the sensitivity of countries to international shocks has been constant for the majority of countries. As a consequence, the finding of a decoupling of countries from the global business cycle is driven by the global shocks' having been smaller, and not by a reduction in countries' integration in the global economy.

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Appendix A List of Countries

Table A-1: List of Countries

Country Groups			
Industrial Countries	Emerging Economies	Developing Countries	
Belgium	Peru	Congo, Rep.	Sri Lanka
Australia	Argentina	Burundi	Lesotho
Austria	Brazil	Benin	Madagascar
Canada	Chile	Burkina Faso	Mali
Switzerland	China	Bangladesh	Mozambique
Denmark	Colombia	Bolivia	Mauritania
Spain	Egypt	Barbados	Mauritius
Finland	Hong Kong, China	Botswana	Malawi
France	Indonesia	Cote d'Ivoire	Niger
United Kingdom	India	Cameroon	Nigeria
Germany	Israel	Comoros	Nicaragua
Greece	Jordan	Cape Verde	Nepal
Ireland	Korea, Rep.	Costa Rica	Panama
Iceland	Morocco	Dominican Republic	Paraguay
Italy	Mexico	Algeria	Rwanda
Japan	Malaysia	Ecuador	Senegal
Luxembourg	Pakistan	Ethiopia	El Salvador
Netherlands	Philippines	Gabon	Seychelles
Norway	Singapore	Ghana	Syrian Arab Republic
New Zealand	Thailand	Guinea	Chad
Portugal	Turkey	Gambia, The	Togo
Sweden	Taiwan	Guinea-Bissau	Trinidad and Tobago
United States	Venezuela, RB	Equatorial Guinea	Tunisia
	South Africa	Guatemala	Tanzania
		Guyana	Uganda
		Honduras	Uruguay
		Haiti	Congo, Dem. Rep.
		Iran, Islamic Rep.	Zambia
		Jamaica	Zimbabwe
		Kenya	

Appendix B Gibbs sampling algorithm

In this appendix we provide details on the Gibbs sampling algorithm used in subsection 2.3 to jointly sample the binary indicators $\mathcal{M}$, the hyperparameters Ψ, the common and country-specific factors F, the time-varying factor loadings α, β, and δ, the mixture indicators ι, and the stochastic volatilities h. The structure of our Gibbs sampling approach is based on Frühwirth-Schnatter and Wagner (2010).

Block 1: Filtering and sampling the state vectors F

In this block we use the general forward-filtering and backward-sampling approach for all common and country-specific factors F based on a state space model of the general form

$$y_t^* = Z_t s_t + e_t, \quad e_t \overset{iid}{\sim} \mathcal{N}(0, H), \tag{A-1}$$

$$s_t = T s_{t-1} + K_t v_t, \quad v_t \overset{iid}{\sim} \mathcal{N}(0, Q), \quad s_1 \overset{iid}{\sim} \mathcal{N}(a_1, A_1), \tag{A-2}$$

where y_t^* is a vector of observations and s_t an unobserved state vector. The matrices Z_t, T, K_t, H, Q and the expected value a_1 and variance A_1 of the initial state vector s_1 are assumed to be known (conditioned upon) and the error terms e_t and ν_t are assumed to be serially uncorrelated and independent of each other at all points in time. As Eqs. (A-1)–(A-2) constitute a linear Gaussian state space model, the unknown state variables in s_t can be filtered using the Kalman filter. In particular, we filter and draw the unobserved common and country-specific factors $D = \left\{F_t^g, F_t^{IC}, F_t^{EM}, F_t^{DC}, F_{i,t}^c\right\}$ conditionally on the time-varying factor loadings $\zeta = \left\{\alpha_{i,t}^j, \beta_{i,t}^j, \delta_{i,t}^j\right\}$, the stochastic volatility terms $h = \left\{h_t^D\right\}$ and the hyperparameters collected in Ψ.

Block 1(a): Sampling the global factor F_t^g

In this step of the Gibbs sampler, we filter and sample the common global factor F_t^g conditioning on the group-specific factors $F_t^r = (F_t^{IC}, F_t^{EM}, F_t^{DC})$, the country-specific factors $F_{i,t}^c$, the time-varying factor loadings ζ, the stochastic volatilities h, and the hyperparameters. As we treat the country- and variable-specific idiosyncratic disturbances $\varepsilon_{i,t}^j$ as autocorrelated processes of order three, we transform the endogenous variables $Y_{i,t}$, $C_{i,t}$ and $I_{i,t}$ and express these variables as functions of the global factor F_t^g. The state space representation for the conditional model in this block is given by

$$
\underbrace{\begin{bmatrix} Y^*_{1,t} \\ \vdots \\ Y^*_{N,t} \\ C^*_{1,t} \\ \vdots \\ C^*_{N,t} \\ I^*_{1,t} \\ \vdots \\ I^*_{N,t} \end{bmatrix}}_{y^*_t} = \underbrace{\begin{bmatrix} \alpha^Y_{1,t} & -\alpha^Y_{1,t}\phi^Y_{1,1} & -\alpha^Y_{1,t}\phi^Y_{2,1} & -\alpha^Y_{1,t}\phi^Y_{3,1} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha^Y_{N,t} & -\alpha^Y_{N,t}\phi^Y_{1,N} & -\alpha^Y_{N,t}\phi^Y_{2,N} & -\alpha^Y_{N,t}\phi^Y_{3,N} \\ \alpha^C_{1,t} & -\alpha^C_{1,t}\phi^C_{1,1} & -\alpha^C_{1,t}\phi^C_{2,1} & -\alpha^C_{1,t}\phi^C_{3,1} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha^C_{N,t} & -\alpha^C_{N,t}\phi^C_{1,N} & -\alpha^C_{N,t}\phi^C_{2,N} & -\alpha^C_{N,t}\phi^C_{3,N} \\ \alpha^I_{1,t} & -\alpha^I_{1,t}\phi^I_{1,1} & -\alpha^I_{1,t}\phi^I_{2,1} & -\alpha^I_{1,t}\phi^I_{3,1} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha^I_{N,t} & -\alpha^I_{N,t}\phi^I_{1,N} & -\alpha^I_{N,t}\phi^I_{2,N} & -\alpha^I_{N,t}\phi^I_{3,N} \end{bmatrix}}_{Z_t} \underbrace{\begin{bmatrix} F^g_t \\ F^g_{t-1} \\ F^g_{t-2} \\ F^g_{t-3} \end{bmatrix}}_{s_t} + \underbrace{\begin{bmatrix} \nu^Y_{1,t} \\ \vdots \\ \nu^Y_{N,t} \\ \nu^C_{1,t} \\ \vdots \\ \nu^C_{N,t} \\ \nu^I_{1,t} \\ \vdots \\ \nu^I_{N,t} \end{bmatrix}}_{e_t}, \quad \text{(A-3)}
$$

$$
\underbrace{\begin{bmatrix} F^g_t \\ F^g_{t-1} \\ F^g_{t-2} \\ F^g_{t-3} \end{bmatrix}}_{s_t} = \underbrace{\begin{bmatrix} \theta^g_1 & \theta^g_2 & \theta^g_3 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} F^g_{t-1} \\ F^g_{t-2} \\ F^g_{t-3} \\ F^g_{t-4} \end{bmatrix}}_{s_{t-1}} + \underbrace{\begin{bmatrix} \exp(h^g_t) \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{K_t} \psi^g_t, \quad \text{(A-4)}
$$

where $H = \text{diag}(\sigma^2_{\nu,i,Y}, \sigma^2_{\nu,i,C}, \sigma^2_{\nu,i,I})$, $Q = 1$, and $Y^*_{i,t} = \phi^Y_i(L)\left[Y_{i,t} - \beta^Y_{i,t}F^r_t - \delta^Y_{i,t}F^c_{i,t}\right]$, $C^*_{i,t} = \phi^C_i(L)\left[C_{i,t} - \beta^C_{i,t}F^r_t - \delta^C_{i,t}F^c_{i,t}\right]$ and $I^*_{i,t} = \phi^I_i(L)\left[I_{i,t} - \beta^I_{i,t}F^r_t - \delta^I_{i,t}F^c_{i,t}\right]$ with $\phi^j_i(L) = (1 - \phi^j_{1,i}L - \phi^j_{2,i}L^2 - \phi^j_{3,i}L^3)$ and $r = IC, EM, DC$ depending on the group affiliation of country i. Premultiplying each term in the observation equation by the lag polynomial $\phi^j_i(L)$ is necessary to take into account the autoregressive structure of the variable specific idiosyncratic error terms $\varepsilon^j_{i,t}$ in Eq. (3).

The unobserved state vector s_t is extracted using standard forward filtering and backward sampling. Instead of taking the entire observational vector y^*_t as the item for analysis, we follow the univariate treatment of the multivariate series approach of Durbin and Koopman (2012), in which each of the elements $y^*_{i,t}$ in y^*_t is brought into the analysis one at a time. This not only offers significant computational gains, it also avoids the risk that the prediction error variance matrix becomes nonsingular during the Kalman filter procedure.

Block 1(b): Sampling the group-specific factors F_t^r

In order to filter and sample the group-specific factors $F_t^r = (F_t^{IC}, F_t^{EM}, F_t^{DC})$, we split our entire sample into smaller sub-samples, each including the countries belonging to either the group of industrial countries (IC), the emerging market economies (EM), or the developing countries (DC). More specifically, the group of industrial countries includes those indexed from $i = 1, \ldots, 23$, the group of emerging market economies includes countries indexed from $i = 24, \ldots, 47$, and the group of developing countries includes countries indexed from $i = 48, \ldots, 106$. For these smaller subgroups, we sample each group-specific factor separately, similarly to the procedure outlined in Block 1(a), conditioning on the global factor F_t^g, the country-specific factors $F_{i,t}^c$, the time-varying factor loadings α, β, and δ, the stochastic volatilities h, and the corresponding hyperparameters in Ψ. We transform the endogenous variables and express them as functions of the group-specific factor F_t^r, which gives us $Y_{i,t}^* = \phi_i^Y(L)\left[Y_{i,t} - \alpha_{i,t}^Y F_t^g - \delta_{i,t}^Y F_{i,t}^c\right]$, $C_{i,t}^* = \phi_i^C(L)\left[C_{i,t} - \alpha_{i,t}^C F_t^g - \delta_{i,t}^C F_{i,t}^c\right]$ and $I_{i,t}^* = \phi_i^I(L)\left[I_{i,t} - \alpha_{i,t}^I F_t^g - \delta_{i,t}^I F_{i,t}^c\right]$.

Block 1(c): Sampling the country-specific factors $F_{i,t}^c$

In this block of the Gibbs sampler, we filter and sample the country-specific factors $F_{i,t}^c$ separately for each country i, conditioning on the global factor F_t^g, the group-specific factors F_t^r, the time-varying factor loadings α, β, and δ, the stochastic volatilities h, and the corresponding hyperparameters in Ψ. Again, we transform each endogenous variable as in Block 1(a) and (b), and express them as functions of the country-specific factor $F_{i,t}^c$, which gives us $Y_{i,t}^* = \phi_i^Y(L)\left[Y_{i,t} - \alpha_{i,t}^Y F_t^g - \beta_{i,t}^Y F_t^r\right]$, $C_{i,t}^* = \phi_i^C(L)\left[C_{i,t} - \alpha_{i,t}^C F_t^g - \beta_{i,t}^C F_t^r\right]$ and $I_{i,t}^* = \phi_i^I(L)\left[I_{i,t} - \alpha_{i,t}^I F_t^g - \beta_{i,t}^I F_t^r\right]$.

Block 2: Sampling the binary indicators $\mathcal{M}$ and the parameters σ, ζ_0 and h_0

For notational convenience, let us define a general regression model,

$$w = z^{\mathcal{M}} b^{\mathcal{M}} + e, \qquad e \sim \mathcal{N}(0, \Sigma), \tag{A-5}$$

with w a vector including observations on a dependent variable w_t, and z an unrestricted predictor matrix with rows z_t that contain the state processes from the vectors D_t, ζ_t and h_t that are relevant for explaining w_t. The corresponding unrestricted parameter vector with the relevant elements from Ψ is denoted by b. Then, $z^{\mathcal{M}}$ and $b^{\mathcal{M}}$ are the restricted predictor matrix and restricted parameter vector excluding those elements in z and b for which the corresponding indicator in $\mathcal{M}$ is 0. Furthermore, Σ is a diagonal matrix with elements $\sigma_{e,t}^2$ that may vary over time to allow for heteroskedasticity of a known form.

A naive implementation of the Gibbs sampler would be to sample $\mathcal{M}$ from $f(\mathcal{M} \,|D, \zeta, h, \Psi, w)$ and Ψ from $f(\Psi \,|D, \zeta, h, \mathcal{M}, w)$. However, this approach does not result in an irreducible Markov chain, since whenever an indicator in $\mathcal{M}$ equals zero, the corresponding coefficient in Ψ is also zero, which implies that the chain has absorbing states. Therefore, as in Frühwirth-Schnatter and Wagner (2010), we marginalize over the parameters in Ψ for which variable selection is carried out when sampling $\mathcal{M}$, and then draw the respective parameters in Ψ conditional on the indicators $\mathcal{M}$. The posterior distribution $f(\mathcal{M} \,|D, \zeta, h, w)$ can be obtained using Bayes' Theorem as

$$f(\mathcal{M} \,|D, \zeta, h, w) \propto f(w \,|\mathcal{M}, D, \zeta, h)\, p(\mathcal{M}), \tag{A-6}$$

with $p(\mathcal{M})$ being the prior probability of $\mathcal{M}$ and $f(w \,|\mathcal{M}, D, \zeta, h)$ being the marginal likelihood of the regression model (A-5) where the effect of the parameters $b^{\mathcal{M}}$ and σ_e^2 has been integrated out. The closed form solution of the marginal likelihood depends on whether the error term e_t is homoskedastic or heteroskedastic. More specifically:

- In the homoskedastic case, $\Sigma = \sigma_e^2 I_T$, under the normal-inverse gamma conjugate prior

$$b^{\mathcal{M}} \sim \mathcal{N}\left(a_0^{\mathcal{M}}, A_0^{\mathcal{M}} \sigma_e^2\right), \qquad \sigma_e^2 \sim IG(c_0, C_0), \tag{A-7}$$

the closed form solution for $f(w \,|\mathcal{M}, D, \zeta, h)$ is

$$f(w \,|\mathcal{M}, D, \zeta, h) \propto \frac{\left|A_T^{\mathcal{M}}\right|^{0.5}}{\left|A_0^{\mathcal{M}}\right|^{0.5}} \frac{\Gamma(c_T)\, C_0^{c_0}}{\Gamma(c_0)\left(C_T^{\mathcal{M}}\right)^{c_T}}, \tag{A-8}$$

and the posterior moments $a_T^{\mathcal{M}}$, $A_T^{\mathcal{M}}$, c_T and $C_T^{\mathcal{M}}$ of $b^{\mathcal{M}}$ and σ_e^2 can be calculated as

$$a_T^{\mathcal{M}} = A_T^{\mathcal{M}} \left(\left(z^{\mathcal{M}}\right)' w + \left(A_0^{\mathcal{M}}\right)^{-1} a_0^{\mathcal{M}}\right), \tag{A-9}$$

$$A_T^{\mathcal{M}} = \left(\left(z^{\mathcal{M}}\right)' z^{\mathcal{M}} + \left(A_0^{\mathcal{M}}\right)^{-1}\right)^{-1}, \tag{A-10}$$

$$c_T = c_0 + T/2\,, \tag{A-11}$$

$$C_T^{\mathcal{M}} = C_0 + 0.5\left(w'w + \left(a_0^{\mathcal{M}}\right)' \left(A_0^{\mathcal{M}}\right)^{-1} a_0^{\mathcal{M}} - \left(a_T^{\mathcal{M}}\right)' \left(A_T^{\mathcal{M}}\right)^{-1} a_T^{\mathcal{M}}\right). \tag{A-12}$$

- In the heteroskedastic case, $\Sigma = \text{diag}\left(\sigma^2_{e,1}, ..., \sigma^2_{e,T}\right)$, under the normal conjugate prior $b^{\mathcal{M}} \sim \mathcal{N}\left(a_0^{\mathcal{M}}, A_0^{\mathcal{M}}\right)$, the closed form solution for the marginal likelihood $f\left(w\,|\mathcal{M}, D, \zeta, h\right)$ is

$$f\left(w\,|\mathcal{M}, D, \zeta, h\right) \propto \frac{|\Sigma|^{-0.5}\left|A_T^{\mathcal{M}}\right|^{0.5}}{\left|A_0^{\mathcal{M}}\right|^{0.5}} \exp\left(-\frac{1}{2}\left(w'\Sigma^{-1}w + \left(a_0^{\mathcal{M}}\right)'\left(A_0^{\mathcal{M}}\right)^{-1} a_0^{\mathcal{M}} - \left(a_T^{\mathcal{M}}\right)'\left(A_T^{\mathcal{M}}\right)^{-1} a_T^{\mathcal{M}}\right)\right), \tag{A-13}$$

with

$$a_T^{\mathcal{M}} = A_T^{\mathcal{M}}\left(\left(z^{\mathcal{M}}\right)'\Sigma^{-1}w + \left(A_0^{\mathcal{M}}\right)^{-1} a_0^{\mathcal{M}}\right), \tag{A-14}$$

$$A_T^{\mathcal{M}} = \left(\left(z^{\mathcal{M}}\right)'\Sigma^{-1}z^{\mathcal{M}} + \left(A_0^{\mathcal{M}}\right)^{-1}\right)^{-1}. \tag{A-15}$$

Following George and McCulloch (1993), instead of using a multi-move sampler in which all the elements in $\mathcal{M}$ are sampled simultaneously, we use a single-move sampler in which each of the binary indicators λ and ρ in $\mathcal{M}$ is sampled from $f\left(\lambda^{\alpha}_{i,j}|\lambda^{\beta}_{i,j}, \lambda^{\delta}_{i,j}, \rho^D, D, \zeta^j_i, h, w\right)$, $f\left(\lambda^{\beta}_{i,j}|\lambda^{\alpha}_{i,j}, \lambda^{\delta}_{i,j}, \rho^D, D, \zeta^j_i, h, w\right)$, $f\left(\lambda^{\delta}_{i,j}|\lambda^{\alpha}_{i,j}, \lambda^{\beta}_{i,j}, \rho^D, D, \zeta^j_i, h, w\right)$, and $f\left(\rho^k|\rho^{\backslash k}, \lambda^{\zeta}, D, \zeta, h, w\right)$ respectively. Block 2 is therefore split up into the following subblocks:

Block 2(a): Sampling the binary indicators λ and the parameters ζ_0 and σ_κ

In this block we sample the binary indicators $\lambda = \left\{\lambda^{\zeta}_{i,j}\right\}$ and the parameters $\zeta^j_{i,0}$, and $\sigma_\kappa = \left\{\sigma^{\zeta}_{\kappa,i,j}\right\}$ conditional on the states D, ζ, and h.

In order to sample the binary indicators and parameters corresponding to the time-varying loadings to the global factor, we use (16) and rewrite each equation in model (1) in the general linear regression format of (A-5) as

$$\overbrace{y^{j*}_{i,t}}^{w_t} = \overbrace{\begin{bmatrix} F^{g*}_t & \lambda^{\alpha}_{i,j}\widetilde{\alpha}^j_{i,t}F^{g*}_t \end{bmatrix}}^{z^{\mathcal{M}}_t} \overbrace{\begin{bmatrix} \alpha^j_{i,0} \\ \sigma^{\alpha}_{\kappa,i,j} \end{bmatrix}}^{b^{\mathcal{M}}} + \overbrace{\nu^j_{i,t}}^{e_t}, \tag{A-16}$$

with $y^{j*}_{i,t} = \phi^j_i(L)\left[y^j_{i,t} - \beta^j_{i,t}F^r_t - \delta^j_{i,t}F^c_{i,t}\right]$ and $F^{g*}_t = \phi^j_i(L)F^g_t$ for $j = (Y, C, I)$, where r = IC, EM, or DC depending on the corresponding group membership of country i.

Likewise, in order to sample the binary indicators and parameters corresponding to the time-varying loadings to the group factors, we use (16) and rewrite each equation in (1) in the general linear regression format of (A-5) as

$$\overbrace{y_{i,t}^{j*}}^{w_t} = \overbrace{\left[\begin{array}{cc} F_t^{r*} & \lambda_{i,j}^{\beta}\widetilde{\beta}_{i,t}^{j}F_t^{r*} \end{array}\right]}^{z_t^{\mathcal{M}}} \overbrace{\left[\begin{array}{c} \beta_{i,0}^{j} \\ \sigma_{\kappa,i,j}^{\beta} \end{array}\right]}^{b^{\mathcal{M}}} + \overbrace{\nu_{i,t}^{j}}^{e_t}, \tag{A-17}$$

with $y_{i,t}^{j*} = \phi_i^j(L)\left[y_{i,t}^j - \alpha_{i,t}^j F_t^g - \delta_{i,t}^j F_{i,t}^c\right]$ and $F_t^{r*} = \phi_i^j(L)F_t^r$.

Lastly, in order to sample the binary indicators and parameters corresponding to the time-varying loadings to the country-specific factors, we use (16) and rewrite each equation in (1) in the general linear regression format of (A-5) as

$$\overbrace{y_{i,t}^{j*}}^{w_t} = \overbrace{\left[\begin{array}{cc} F_{i,t}^{c*} & \lambda_{i,j}^{\delta}\widetilde{\delta}_{i,t}^{j}F_{i,t}^{c*} \end{array}\right]}^{z_t^{\mathcal{M}}} \overbrace{\left[\begin{array}{c} \delta_{i,0}^{j} \\ \sigma_{\kappa,i,j}^{\delta} \end{array}\right]}^{b^{\mathcal{M}}} + \overbrace{\nu_{i,t}^{j}}^{e_t}, \tag{A-18}$$

with $y_{i,t}^{j*} = \phi_i^j(L)\left[y_{i,t}^j - \alpha_{i,t}^j F_t^g - \beta_{i,t}^j F_t^r\right]$ and $F_{i,t}^{c*} = \phi_i^j(L)F_{i,t}^c$.

In all three equations, Eqs. (A-16)–(A-18), the second term in both the restricted vector $z_t^{\mathcal{M}}$ and the restricted parameter vector $b^{\mathcal{M}}$ is excluded when $\lambda_{i,j}^{\zeta} = 0$. Note that next to the parameters in $b^{\mathcal{M}}$ and σ_e^2, each of the specifications (A-16), (A-17), and (A-18) depends only on the transformed data $y_{i,t}^{j*}$, on some of the transformed factors in D_t^*, on the time-varying loadings $\zeta_{i,t}^j$, and on the corresponding binary indicator $\lambda_{i,j}^{\zeta}$. Hence, we can simplify the specification of the posterior from $f\left(\lambda_{i,j}^{\zeta}\middle|\lambda_{\backslash i,\backslash j}^{\backslash\zeta}, \rho^D, D, \zeta, h, x\right)$ to $f\left(\lambda_{i,j}^{\zeta}\middle|D^*, \zeta_i^j, y_i^{j*}\right)$, for which we have $f\left(\lambda_{i,j}^{\zeta}\middle|D^*, \zeta_i^j, y_i^{j*}\right) \propto f\left(y_i^{j*}\middle|\lambda_{i,j}^{\zeta}, D^*, \zeta_i^j\right) p\left(\lambda_{i,j}^{\zeta}\right)$.

As the error terms $\nu_{i,t}^j$ in Eqs. (A-16)–(A-18) are homoskedastic, we have $\Sigma = \sigma_e^2 I_T$ in the general notation of Eq. (A-5), so that the marginal likelihood $f\left(\lambda_{i,j}^{\zeta}\middle|D^*, \zeta_i^j, y_i^{j*}\right)$ can be calculated as in Eq. (A-8). The binary indicator $\lambda_{i,j}^{\zeta}$ can then be sampled from the Bernoulli distribution with probability

$$p\left(\lambda_{i,j}^{\zeta} = 1\middle|D^*, \zeta_i^j, y_i^{j*}\right) = \frac{f\left(\lambda_{i,j}^{\zeta} = 1\middle|D^*, \zeta_i^j, y_i^{j*}\right)}{f\left(\lambda_{i,j}^{\zeta} = 0\middle|D^*, \zeta_i^j, y_i^{j*}\right) + f\left(\lambda_{i,j}^{\zeta} = 1\middle|D^*, \zeta_i^j, y_i^{j*}\right)}, \tag{A-19}$$

while $b^{\mathcal{M}}$ can be sampled from $\mathcal{N}\left(a_T^{\mathcal{M}}, A_T^{\mathcal{M}}\sigma_{\nu,i,j}^2\right)$ with $a_T^{\mathcal{M}}$ and $A_T^{\mathcal{M}}$ as defined in Eqs. (A-9)–(A-15). Note that $b^{\mathcal{M}} = \left(\zeta_{i,0}^j, \sigma_{\kappa,i,j}^{\zeta}\right)'$ when $\lambda_{i,j}^{\zeta} = 1$ and $b^{\mathcal{M}} = \zeta_{i,0}^j$ when $\lambda_{i,j}^{\zeta} = 0$. In the former case, $\sigma^{\zeta}_{\kappa,i,j}$ is sampled from the posterior, whereas in the latter case we set $\sigma^{\zeta}_{\kappa,i,j} = 0$.

Block 2(b): Sampling the binary indicators ρ and the parameters h_0 and σ_η

In this block we sample the binary indicators $\rho^D = \{\rho^g, \rho^{IC}, \rho^{EM}, \rho^{DC}, \rho^c_i\}$ and the parameters $h_0 = \left\{h^g_0, h^{IC}_0, h^{EM}_0, h^{DC}_0, h^c_{i,0}\right\}$ and $\sigma_\eta = \left\{\sigma^g_\eta, \sigma^{IC}_\eta, \sigma^{EM}_\eta, \sigma^{DC}_\eta, \sigma^c_{\eta,i}\right\}$ conditional on the states D and h. Using Eq. (17), Eq. (7) can be rewritten in the general linear regression format of (A-5) as

$$\overbrace{g^D_t - \left(m_{\iota^D_t} - 1.2704\right)}^{w_t} = \overbrace{2\begin{bmatrix} 1 & \rho^D \widetilde{h}^D_t \end{bmatrix}}^{z^{\mathcal{M}}_t} \overbrace{\begin{bmatrix} h^D_0 \\ \sigma_{\eta,D} \end{bmatrix}}^{b^{\mathcal{M}}} + \overbrace{\widetilde{\epsilon}^D_t}^{e_t}, \tag{A-20}$$

with $\widetilde{\epsilon}^D_t = \epsilon^D_t - \left(m_{\iota^D_t} - 1.2704\right)$ is ϵ^D_t recentered around zero, and where using Eq. (2), $g^D_t = \ln\left(\left(\exp\{h^D_t\}\psi^D_t\right)^2 + .001\right)$ can be calculated as

$$g^D_t = \ln\left(\left(D_t - \theta^D_1 D_{t-1} - \theta^D_2 D_{t-2} - \theta^D_3 D_{t-3}\right)^2 + .001\right), \tag{A-21}$$

As specification (A-20) depends only on the data w_t, on the stochastic volatility term h^D_t, and on ρ^D, we can simplify the specification of the posterior from $f\left(\rho^D|\rho^{\backslash D}, \lambda, \zeta, h, w\right)$ to $f\left(\rho^D\left|h^D, w\right.\right)$. Using Bayes' Theorem, we have $f\left(\rho^D\left|h^D, w\right.\right) \propto f\left(w\left|\rho^D, h^D\right.\right) p\left(\rho^D\right)$. Given the mixture distribution of ϵ^D_t defined in Eq. (9), the error term $\widetilde{\epsilon}^D_t$ in Eq. (A-20) has a heteroskedastic variance $v^2_{\iota^D_t}$ such that $\Sigma = \text{diag}\left(v^2_{\iota^D_1}, ..., v^2_{\iota^D_T}\right)$ in the general notation of Eq. (A-5). In this case, the marginal likelihood $f\left(w\left|\rho^D, h^D\right.\right)$ can be calculated as in Eq. (A-13). The binary indicator ρ^D can then be sampled from the Bernoulli distribution with probability $p\left(\rho^D = 1\left|h^D, w\right.\right)$ calculated from an equation similar to Eq. (A-19). Next, $b^{\mathcal{M}}$ can be sampled from $\mathcal{N}\left(a^{\mathcal{M}}_T, A^{\mathcal{M}}_T\right)$ with $a^{\mathcal{M}}_T$ and $A^{\mathcal{M}}_T$ as defined in Eqs. (A-14) and (A-15). Note that $b^{\mathcal{M}} = \left(h^D_0, \sigma_{\eta,D}\right)'$ when $\rho^D = 1$ and $b^{\mathcal{M}} = h^D_0$ when $\rho^D = 0$. In the latter case, we set $\sigma_{\eta,D} = 0$.

Block 3: Sampling the state vectors ζ and h, and the mixture indicators ι

In this block we use the forward-filtering and backward-sampling approach of Carter and Kohn (1994) and De Jong and Shephard (1995) to sample the states ζ and h based on a general state space model of the form

$$w_t = Z_t s_t + e_t, \qquad e_t \overset{iid}{\sim} \mathcal{N}(0, H_t), \tag{A-22}$$

$$s_t = R_1 s_{t-1} + K_t \mu_t, \qquad \mu_t \overset{iid}{\sim} \mathcal{N}(0, Q_t), \qquad s_1 \overset{iid}{\sim} \mathcal{N}(a_1, A_1), \tag{A-23}$$

where w_t is now a vector of observations and s_t an unobserved state vector. The matrices Z_t, R_1, K_t, H_t, Q_t and the expected value a_1 and variance A_1 of the initial state vector s_1 are assumed to be known (conditioned upon). The error terms e_t and μ_t are assumed to be serially uncorrelated and independent of each other at all points in time. As Eqs. (A-22)–(A-23) constitute a linear Gaussian state space model, the unknown state variables in s_t can be filtered using the standard Kalman filter. Sampling $s = [s_1, \ldots, s_T]$ from its conditional distribution can then be done using the multimove Gibbs sampler of Carter and Kohn (1994) and De Jong and Shephard (1995).

Block 3(a): Sampling the time-varying parameters ζ

We first filter and draw the time-varying parameters $\zeta_t = (\alpha_{i,t}^j, \beta_{i,t}^j, \delta_{i,t}^j)$ conditionally on all factors D, the stochastic volatility terms h, the hyperparameters Ψ, and the binary indicators $\mathcal{M}$. More specifically, using Eq. (16) for each equation in (1), the unrestricted (i.e., $\lambda = 1$) conditional state space representations for the time-varying factor loadings $\widetilde{\zeta}_{i,t}^j$ are given by

$$\overbrace{\left[\, y_{i,t}^{j*} - \alpha_{i,0}^j F_t^{g*} \,\right]}^{w_t} = \overbrace{\left[\, \sigma_{\kappa,i,j}^{\alpha} F_t^{g*} \,\right]}^{Z_t^{\mathcal{M}}} \overbrace{\left[\, \widetilde{\alpha}_{i,t}^j \,\right]}^{s_t^{\mathcal{M}}} + \overbrace{\left[\, \nu_{i,t}^j \,\right]}^{e_t}, \tag{A-24}$$

$$\underbrace{\left[\, \widetilde{\alpha}_{i,t}^j \,\right]}_{s_t} = \underbrace{\left[\, 1 \,\right]}_{R_1} \underbrace{\left[\, \widetilde{\alpha}_{i,t-1}^j \,\right]}_{s_{t-1}} + \underbrace{\left[\, 1 \,\right]}_{K_t} \underbrace{\left[\, \widetilde{\kappa}_{i,t}^{j,\alpha} \,\right]}_{\mu_t}, \tag{A-25}$$

with $H_t = \sigma_{\nu,i,j}^2$ and $Q_t = 1$, $y_{i,t}^{j*} = \phi_i^j(L)\left[y_{i,t}^j - \beta_{i,t}^j F_t^r - \delta_{i,t}^j F_{i,t}^c\right]$ and $F_t^{g*} = \phi_i^j(L) F_t^g$ and

$$\overbrace{\left[\, y_{i,t}^{j*} - \beta_{i,0}^j F_t^{r*} \,\right]}^{w_t} = \overbrace{\left[\, \sigma_{\kappa,i,j}^{\beta} F_t^{r*} \,\right]}^{Z_t^{\mathcal{M}}} \overbrace{\left[\, \widetilde{\beta}_{i,t}^j \,\right]}^{s_t^{\mathcal{M}}} + \overbrace{\left[\, \nu_{i,t}^j \,\right]}^{e_t}, \tag{A-26}$$

$$\underbrace{\left[\, \widetilde{\beta}_{i,t}^j \,\right]}_{s_t} = \underbrace{\left[\, 1 \,\right]}_{R_1} \underbrace{\left[\, \widetilde{\beta}_{i,t-1}^j \,\right]}_{s_{t-1}} + \underbrace{\left[\, 1 \,\right]}_{K_t} \underbrace{\left[\, \widetilde{\kappa}_{i,t}^{j,\beta} \,\right]}_{\mu_t}, \tag{A-27}$$

with $H_t = \sigma^2_{\nu,i,j}$ and $Q_t = 1$, $y^{j*}_{i,t} = \phi^j_i(L)\left[y^j_{i,t} - \alpha^j_{i,t}F^g_t - \delta^j_{i,t}F^c_{i,t}\right]$ and $F^r_t{}^* = \phi^j_i(L)F^r_t$ and

$$\overbrace{\left[\; y^{j*}_{i,t} - \delta^j_{i,0}F^{c*}_{i,t} \;\right]}^{w_t} = \overbrace{\left[\; \sigma^\delta_{\kappa,i,j}F^{c*}_{i,t} \;\right]}^{Z^{\mathcal{M}}_t}\overbrace{\left[\; \widetilde{\delta}^j_{i,t} \;\right]}^{s^{\mathcal{M}}_t} + \overbrace{\left[\; \nu^j_{i,t} \;\right]}^{e_t}, \tag{A-28}$$

$$\underbrace{\left[\; \widetilde{\delta}^j_{i,t} \;\right]}_{s_t} = \underbrace{\left[\; 1 \;\right]}_{R_1}\underbrace{\left[\; \widetilde{\delta}^j_{i,t-1} \;\right]}_{s_{t-1}} + \underbrace{\left[\; 1 \;\right]}_{K_t}\underbrace{\left[\; \widetilde{\kappa}^{j,\delta}_{i,t} \;\right]}_{\mu_t}, \tag{A-29}$$

with $H_t = \sigma^2_{\nu,i,j}$ and $Q_t = 1$, $y^{j*}_{i,t} = \phi^j_i(L)\left[y^j_{i,t} - \alpha^j_{i,t}F^g_t - \beta^j_{i,t}F^r_t\right]$ and $F^{c*}_{i,t} = \phi^j_i(L)F^c_{i,t}$. All random walk components $\widetilde{\alpha}^j_{i,t}$, $\widetilde{\beta}^j_{i,t}$ and $\widetilde{\delta}^j_{i,t}$ are initialized by setting $a_1 = 0$ and $A_1 = 1000$.

In the restricted model (i.e., $\lambda = 0$), $Z^{\mathcal{M}}$ and $s^{\mathcal{M}}$ are empty. In this case, no forward-filtering and backward-sampling is needed, and $\widetilde{\zeta}^j_{i,t}$ can be sampled directly from the prior using Eq. (13). Note that the sampling of the common and country-specific factors in block 1 depends on ζ rather than on $\widetilde{\zeta}$. We therefore reconstruct $\zeta^j_{i,t}$ from Eqs. (12) by using the corresponding parameters from ζ_0 and σ_κ.

Block 3(b): Sampling the mixture indicators ι and the stochastic volatilities h

In this block we draw the mixture indicators $\iota = (\iota^g, \iota^{IC}, \iota^{EM}, \iota^{DC}, \iota^c_i)$ and the stochastic volatilities $h = \left(h^g, h^{IC}, h^{EM}, h^{DC}, h^c_i\right)$ conditionally on the state vector D, the time-varying parameters ζ, the hyperparameters Ψ, and the binary indicators $\mathcal{M}$. Following Del Negro and Primiceri (2014), in this block we first sample the mixture indicator ι^D_t from its conditional probability mass

$$p\left(\iota^D_t = n | h^D_t, \epsilon^D_t\right) \propto q_n f_{\mathcal{N}}\left(\epsilon^D_t | 2h^D_t + m_n - 1.2704, \nu^2_n\right), \tag{A-30}$$

with values for $\{q_n, m_n, \nu^2_n\}$ taken from Table 1 in Omori et al. (2007).

Next, we filter and sample the stochastic volatility terms $\widetilde{h}^D_t$ conditioning on the transformed states g^D_t defined in Eq. (A-21), on the mixture indicators ι^D_t, and on the parameters Ψ. More specifically, the unrestricted (i.e., $\rho^D = 1$) conditional state space representation is given by

$$\overbrace{\left[\; g_t^D - \left(m_{\iota_t^D} - 1.2704\right) - 2h_0^D \;\right]}^{w_t} = \overbrace{\left[\; 2\rho^D \sigma_{\eta,D} \;\right]}^{Z_t^{\mathcal{M}}} \overbrace{\left[\; \widetilde{h}_t^D \;\right]}^{s_t^{\mathcal{M}}} + \overbrace{\left[\; \widetilde{\epsilon}_t^D \;\right]}^{e_t}, \tag{A-31}$$

$$\underbrace{\left[\; \widetilde{h}_t^D \;\right]}_{s_t} = \underbrace{\left[\; 1 \;\right]}_{R_1} \underbrace{\left[\; \widetilde{h}_{t-1}^D \;\right]}_{s_{t-1}} + \underbrace{\left[\; 1 \;\right]}_{K_t} \underbrace{\left[\; \widetilde{\eta}_t^D \;\right]}_{\mu_t}, \tag{A-32}$$

with $H_t = v^2_{\iota_t^D}$, $Q_t = 1$ and where $\widetilde{\epsilon}_t^D = \epsilon_t^D - \left(m_{\iota_t^D} - 1.2704\right)$ is ϵ_t^D recentered around zero. The random walk components $\widetilde{h}_t^D$ are initialized by setting $a_1 = 0$ and $A_1 = 1000$.

In the restricted model (i.e., $\rho^D = 0$), $Z^{\mathcal{M}}$ and $s^{\mathcal{M}}$ are empty. In this case, no forward-filtering and backward-sampling is needed and $\widetilde{h}_t^D$ can be sampled directly from its prior using Eq. (15). Note that the sampling of the common and country-specific factors F_t in block 1 depends on h_t^D rather than on $\widetilde{h}_t^D$. Using h_0^D, $\sigma_{\eta,D}$ and $\widetilde{h}_t^D$, h_t^D can easily be reconstructed from Eq. (14).

Block 4: Estimating and sampling the parameters θ, ϕ, and σ^2_ν

In the final block of the Gibbs sampler we estimate and draw the autoregressive parameters θ and ϕ and the variances of the idiosyncratic error terms σ^2_ν conditioning on the factors D, the time-varying factor loadings ζ and the stochastic volatilities h, and the binary indicators $\mathcal{M}$. We estimate and draw the variances and the AR parameters separately. Therefore, block 4 is split up in the following subblocks:

Block 4(a): Estimating and sampling the variances σ^2_ν

First we estimate and draw the variances $\sigma^2_{\nu,i,j}$ of the iid shocks to the idiosyncratic error terms in Eq. (3) for each country i and variable j separately. We follow the approach of Kim and Nelson (1999) (pp. 175–177) and draw the variance of the shocks to idiosyncratic errors from an inverted-gamma distribution with prior information

$$\sigma^2|\beta \sim \mathcal{IG}\left(c_0, C_0\right), \tag{A-33}$$

where β is a vector of known parameters and is conditioned on. We obtain prior information $c_0 = T * str_0/2$ on the shape parameter, where T (observations) is the prior number of degrees of freedom and str_0 is the strength of the belief about the value of the variance σ^2. Prior information on the scale parameter is given by $C_0 = bel_0 * c_0$, where bel_0 denotes the corresponding prior belief about the value of the variance parameter σ^2. The posterior distribution can be represented by

$$\sigma^2|\beta \sim \mathcal{IG}\left(c_1, C_1\right), \tag{A-34}$$

where $c_1 = c_0 + T/2$ and $C_1 = C_0 + (w' * w)/2$, and $w' * w$ are the residual sum of squares yielding an estimate of the variance based on the data. Therefore, the posterior parameters of the inverse gamma distribution consist of prior information and the information in the data.

We sample the variances $\sigma^2_{\nu,i,j}$ of the shocks to the idiosyncratic errors conditioning on the factors D, the time-varying loadings α, β, and δ, and the AR coefficients ϕ from the posterior given in (A-34) for each country i and variable j separately. Therefore, we first compute the residuals for each equation in (1) as $\varepsilon^j_{i,t} = y^j_{i,t} - \alpha^j_{i,t}F^g_t - \beta^j_{i,t}F^r_t - \delta^j_{i,t}F^c_{i,t}$ and set $w^j_{i,t} = \varepsilon^j_{i,t} - \phi^j_{1,i}\varepsilon^j_{i,t-1} - \phi^j_{2,i}\varepsilon^j_{i,t-2} - \phi^j_{3,i}\varepsilon^j_{i,t-3}$ to take into account the autoregressive structure of the error terms.

Block 4(b): Estimating and sampling the AR parameters ϕ and θ

Next we estimate and draw the autoregressive parameters ϕ^j_i of the idiosyncratic error terms $\varepsilon^j_{i,t}$. Conditioning on the factors D, the time-varying loadings α, β, and δ, and the error variance of the idiosyncratic error terms σ^2_ν known from the previous step, these are all unknown parameters in the standard linear regression model

$$y = X\beta + e, \quad e \sim \mathcal{N}(0, \sigma^2 I_T), \tag{A-35}$$

where y and e are $T \times 1$ vectors, and X is a $T \times K$ matrix containing the fixed regressors $X_1, ..., X_K$. β is the unknown vector of coefficients and the error variance σ^2 is assumed to be known. We follow the approach outlined in Kim and Nelson (1999) (pp. 173–174). The prior information of β follows a normal distribution

$$\beta|\sigma^2 \sim \mathcal{N}(\beta_0, \Sigma_0), \tag{A-36}$$

where β_0 is the prior belief of β and Σ_0 is the prior variance regarding this belief indicating the degree of uncertainty on β_0. The parameters β can then be sampled from the posterior distribution

$$\beta|\sigma^2 \sim \mathcal{N}(\beta_1, \Sigma_1), \tag{A-37}$$

with hyperparameters defined by

$$\Sigma_1 = \left(\frac{X'X}{\sigma^2} + \Sigma_0^{-1}\right)^{-1} \tag{A-38}$$

$$\beta_1 = \left(\frac{X'y}{\sigma^2} + \Sigma_0^{-1}\beta_0\right)\Sigma_1, \tag{A-39}$$

where the prior information is combined with the information in the data given by the sample estimate.[11]

The autoregressive parameters can now be sampled as follows:

- We obtain the posterior distribution of $\phi^j_{p,i}$ for each country i and each variable j separately (for $p = 1, 2, 3$ lags). First we compute the variable specific idiosyncratic error terms for each country i:

$$\varepsilon^j_{i,t} = y^j_{i,t} - \alpha^j_{i,t}F^g_t - \beta^j_{i,t}F^r_t - \delta^j_{i,t}F^c_{i,t}. \tag{A-40}$$

  We then set $y = \varepsilon^j_{i,t}$, $X_1 = \varepsilon^j_{i,t-1}$, $X_2 = \varepsilon^j_{i,t-2}$, and $X_3 = \varepsilon^j_{i,t-3}$ for each country i and variable j in (A-35) and sample the AR coefficients $\phi^j_{p,i}$ from (A-37) correspondingly. We accept the draw if $\left|\Sigma^3_{p=1}\left(\phi^j_{p,i}\right)\right| < 1$, ensuring the stationarity of the autoregressive processes.

- We obtain the posterior distribution of the AR coefficients $\theta_{p,D}$ for each factor separately, conditioning on D_t and h^D_t by using (A-37) and setting $y = D_t/\exp(h^D_t)$, $X_1 = D_{t-1}/\exp(h^D_{t-1})$, $X_2 = D_{t-2}/\exp(h^D_{t-2})$, and $X_3 = D_{t-3}/\exp(h^D_{t-3})$, so that (A-35) becomes a GLS-type regression model since the errors in (2) are heteroskedastic due to the stochastic volatilities. Again, we then sample the AR coefficients θ^D_p from (A-37). We accept the draw if $\left|\Sigma^3_{p=1}\left(\theta^D_p\right)\right| < 1$, ensuring the stationarity of the autoregressive processes.

[11] We refer to Kim and Nelson (1999) for a more detailed explanation.