NAG C Library Function Document

nag_elliptic_integral_f (s21dac)

1 Purpose

nag_elliptic_integral_f (s21dac) returns the value of the general elliptic integral of the second kind $F(z, k', a, b)$ for a complex argument $z$.

2 Specification

Complex nag_elliptic_integral_f (Complex z, double akp, double a, double b, NagError *fail)

3 Description

This routine evaluates an approximation to the general elliptic integral of the second kind $F(z, k', a, b)$ given by

$$F(z, k', a, b) = \int_0^z \frac{a + b\zeta^2}{(1 + \zeta^2)(1 + k'^2\zeta^2)} \, d\zeta,$$

where $a$ and $b$ are real parameters, $z$ is a complex argument whose real part is non-negative and $k'$ is a real parameter (the complementary modulus). The evaluation of $F$ is based on the Gauss transformation. Further details, in particular for the conformal mapping provided by $F$, can be found in Bulirsch (1965).

Special values include

$$F(z, k', 1, 1) = \int_0^z \frac{d\zeta}{\sqrt{(1 + \zeta^2)(1 + k'^2\zeta^2)}},$$

or $F_1(z, k')$ (the elliptic integral of the first kind) and

$$F(z, k', 1, k^2) = \int_0^z \frac{\sqrt{1 + k'^2\zeta^2}}{(1 + \zeta^2)\sqrt{1 + \zeta^2}} \, d\zeta,$$

or $F_2(z, k')$ (the elliptic integral of the second kind). Note that the values of $F_1(z, k')$ and $F_2(z, k')$ are equal to $\tan^{-1}(z)$ in the trivial case $k' = 1$.

nag_elliptic_integral_f is derived from a procedure given by Bulirsch (1965).

Constraints are placed on the values of $z$ and $k'$ in order to avoid the possibility of machine overflow.

4 Parameters

1: $z$ – Complex  
   $\text{Input}$  
   $\text{On entry:}$ the argument $z$ of the function.  
   $\text{Constraints:}$  
   $0.0 \leq z.\text{re} \leq \lambda$,  
   $|z.\text{im}| \leq \lambda$, where $\lambda^6 = 1/X02AMC$.

2: $akp$ – double  
   $\text{Input}$  
   $\text{On entry:}$ the argument $k'$ of the function.  
   $\text{Constraint: } |akp| \leq \lambda$.  

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3: a – double
   \(\text{Input}\)
   \(\text{On entry:}\) the argument \(a\) of the function.

4: b – double
   \(\text{Input}\)
   \(\text{On entry:}\) the argument \(b\) of the function.

5: fail – NagError *
   \(\text{Input/Output}\)
   \(\text{The NAG error parameter (see the Essential Introduction).}\)

5 \ Error Indicators and Warnings

NE_COMPLEX
   \(\text{On entry,}\ z = (\text{<value>}, \text{<value>}).\)
   \(\text{Constraints:}\)
   \[0.0 \leq z.\text{re} \leq \lambda,\]
   \[|z.\text{im}| \leq \lambda, \text{where}\ \lambda = 1/X02AMC.\]

NE_S21_CONV
   \(\text{The iterative procedure used to evaluate the integral has failed to converge. The result is returned as zero.}\)

NE_INTERNAL_ERROR
   \(\text{An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.}\)

6 \ Further Comments

6.1 \ Accuracy
   \(\text{In principle the routine is capable of achieving full relative precision in the computed values. However, the accuracy obtainable in practice depends on the accuracy of the C standard library elementary functions such as atan2 and log.}\)

6.2 \ References

7 \ See Also
   \(\text{None.}\)

8 \ Example
   \(\text{The example program evaluates the elliptic integral of the first kind}\ F_1(z, k') \text{ given by}\)
   \[F_1(z, k') = \int_0^z \frac{d\zeta}{\sqrt{(1 + \zeta^2)(1 + k'^2\zeta^2)}},\]
   \(\text{where}\ z = 1.2 + 3.7i\ \text{and}\ k' = 0.5, \text{and prints the results.}\)
8.1 Program Text

/* nag_elliptic_integral_f (s21dac) Example Program. */
/* Copyright 2000 Numerical Algorithms Group. */
/* NAG C Library */
/* Mark 6, 2000. */
*/
#include <stdio.h>
#include <nag.h>
#include <nag_stdbib.h>
#include <nags.h>

int main(void)
{
    const char fmt_99999[] = " (%4.1f, %4.1f) %7.1f %7.1f %7.1f (%12.4e, %12.4e)\n";
    Complex y, z, z__1;
    double a, akp, b;
    Integer exit_status=0;
    NagError fail;

    INIT_FAIL(fail);
    Vprintf("s21dac Example Program Results\n\n");

    /* Skip heading in data file */
    Vscanf("%*[^
] ");
    Vprintf(" z akp a b y\n\n");
    while(scanf(" (%lf,%lf) %lf %lf %lf%*[^
] ", &z.re, &z.im, &akp, &a, &b) != EOF )
    {
        z__1 = s21dac (z, akp, a, b, &fail);
        y.re = z__1.re, y.im = z__1.im;
        if (fail.code == NE_NOERROR)
            { Vprintf(fmt_99999,z.re,z.im,akp,a,b,y.re,y.im);
            }
        else
            { Vprintf("Error from s21dac.\n\n", fail.message);
                exit_status = 1;
                goto END;
            }
    }
    END:
    return exit_status;
}

8.2 Program Data

s21dac Example Program Data
(1.2, 3.7) 0.5 1.0 1.0
(9.2,-3.4) 0.8 0.4 2.7 : Values of z, akp, a and b
### 8.3 Program Results

#### s21dac Example Program Results

<table>
<thead>
<tr>
<th>z</th>
<th>akp</th>
<th>a</th>
<th>b</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.2, 3.7)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>(1.9713e+00, 5.0538e-01)</td>
</tr>
<tr>
<td>(9.2, -3.4)</td>
<td>0.8</td>
<td>0.4</td>
<td>2.7</td>
<td>(2.5042e+00, -1.1709e-01)</td>
</tr>
</tbody>
</table>