NAG C Library Function Document

nag_dawson (s15afc)

1 Purpose

nag_dawson (s15afc) returns a value for Dawson’s Integral, \( F(x) \), via the function name.

2 Specification

double nag_dawson (double x)

3 Description

nag_dawson (s15afc) evaluates an approximation for Dawson’s Integral

\[
F(x) = e^{-x^2} \int_0^x e^{t^2} \, dt.
\]

The function is based on two Chebyshev expansions:

For \( 0 < |x| \leq 4 \),

\[
F(x) = x \sum_{r=0}^\infty a_r T_r(t), \quad \text{where} \quad t = 2 \left( \frac{x}{4} \right)^2 - 1.
\]

For \( |x| > 4 \),

\[
F(x) = \frac{1}{x} \sum_{r=0}^\infty b_r T_r(t), \quad \text{where} \quad t = 2 \left( \frac{4}{x} \right)^2 - 1.
\]

For \( |x| \) near zero, \( F(x) \approx x \), and for \( |x| \) large, \( F(x) \approx \frac{1}{2x} \). These approximations are used for those values of \( x \) for which the result is correct to machine precision. For very large \( x \) on some machines, \( F(x) \) may underflow and then the result is set exactly to zero (see the Users’ Note for your implementation for details).

4 References


5 Parameters

1: \( x \) – double

\( \text{Input} \)

\( \text{On entry:} \) the argument \( x \) of the function.

6 Error Indicators and Warnings

None.

7 Accuracy

Let \( \delta \) and \( \epsilon \) be the relative errors in the argument and result respectively.

If \( \delta \) is considerably greater than the machine precision (i.e., if \( \delta \) is due to data errors etc.), then \( \epsilon \) and \( \delta \) are approximately related by:
The following graph shows the behaviour of the error amplification factor
\[ \frac{|x(1 - 2xF(x))|}{F(x)} \cdot \delta \].

However if \( \delta \) is of the same order as \textit{machine precision}, then rounding errors could make \( \epsilon \) somewhat larger than the above relation indicates. In fact \( \epsilon \) will be largely independent of \( x \) or \( \delta \), but will be of the order of a few times the \textit{machine precision}.

8 Further Comments

None.

9 Example

The example program reads values of the argument \( x \) from a file, evaluates the function at each value of \( x \) and prints the results.

9.1 Program Text

```c
#include <nag.h>
#include <stdio.h>
#include <nags.h>

int main(void)
{
    double x, y;
    Integer exit_status = EXIT_SUCCESS;
    ...
```

Figure 1
s – Approximations of Special Functions

9.2 Program Data

s15afc Example Program Data
-2.0
-0.5
1.0
1.5
2.0
5.0
10.0

9.3 Program Results

s15afc Example Program Results

\begin{verbatim}
x       y
-2.000e+00 -3.013e-01
-5.000e-01 -4.244e-01
1.000e+00  5.381e-01
1.500e+00  4.282e-01
2.000e+00  3.013e-01
5.000e+00  1.021e-01
1.000e+01  5.025e-02
\end{verbatim}