NAG C Library Function Document

nag_complex_polygamma (s14afc)

1 Purpose

nag_complex_polygamma (s14afc) returns the value of the $k$th derivative of the psi function $\psi(z)$, for complex $z$ and $k = 0, 1, \ldots, 4$.

2 Specification

Complex nag_complex_polygamma(Complex z, Integer k, NagError *fail)

3 Description

This routine evaluates an approximation to the $k$th derivative of the psi function $\psi(z)$ given by

$$\psi^{(k)}(z) = \frac{d^k}{dz^k} \psi(z) = \frac{d^k}{dz^k} \left( \frac{d}{dz} \log \Gamma(z) \right),$$

where $z = x + iy$ is complex provided $y \neq 0$ and $k = 0, 1, \ldots, 4$. If $y = 0$, $z$ is real and thus $\psi^{(k)}(z)$ is singular when $z = 0, -1, -2, \ldots$.

Note that $\psi^{(k)}(z)$ is also known as the polygamma function. Specifically, $\psi^{(0)}(z)$ is often referred to as the digamma function and $\psi^{(1)}(z)$ as the trigamma function in the literature. Further details can be found in Abramowitz and Stegun (1972).

nag_complex_polygamma is based on a modification of the method proposed by Kölblig K S (1972).

To obtain the value of $\psi^{(k)}(z)$ when $z$ is real, nag_real_polygamma (s14aec) can be used.

4 Parameters

1: $z$ – Complex

$\text{Input}$

On entry: the argument $z$ of the function.

Constraint: $z$.re must not be ‘too close’ (see Section 5) to a non-positive integer when $z$.im = 0.0.

2: $k$ – Integer

$\text{Input}$

On entry: the function $\psi^{(k)}(z)$ to be evaluated.

Constraint: $0 \leq k \leq 4$.

3: $\text{fail}$ – NagError*

$\text{Input/Output}$

The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

NE_INT

On entry, $k = <value>$.  
Constraint: $0 \leq k \leq 4$.

NE_COMPLEX

On entry, $z = (<value>, <value>)$.  
Constraint: $z$.re must not be ‘too close’ to a non-positive integer when $z$.im = 0.0. That is, $|z$.re $- \text{nint}(z$.re)$| \geq \text{machine precision} \times |\text{nint}(z$.re)$|$.
NE_OVERFLOWLIKELY
The evaluation has been abandoned due to the likelihood of overflow. The result is returned as zero.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments
6.1 Accuracy
Empirical tests have shown that the maximum relative error is a loss of approximately two decimal places of precision.

6.2 References
Kölblig K S (1972) Programs for computing the logarithm of the gamma function, and the digamma function, for complex arguments Comp. Phys. Comm. 4 221–226


7 See Also
None.

8 Example
The example program evaluates the psi (trigamma) function ψ^{(1)}(z) at z = −1.5 + 2.5i, and prints the results.

8.1 Program Text
/* nag_complex_polygamma (sl4afc) Example Program. *
 * Copyright 2000 Numerical Algorithms Group.
 * NAG C Library *
 * Mark 6, 2000.
 */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Complex z, z__1;
    Integer exit_status=0;
    Integer k;
    NagError fail;

    INIT_FAIL(fail);
    Vprintf("sl4afc Example Program Results\n\n");
    /* Skip heading in data file */
    Vscanf("%*[\n] ");
8.2 Program Data

s14afc Example Program Data
(1.2, 5.0)  0
(0.5, -0.2)  1
(-1.5, 2.5)  1
(8.0, 3.3)  3
(2.9, 7.5)  4  : Values of z and k

8.3 Program Results

s14afc Example Program Results

<table>
<thead>
<tr>
<th>z</th>
<th>k</th>
<th>(D'K/DZ`K)psi(z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>5.0</td>
<td>0 ( 1.6176e+00, 1.4312e+00)</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.2</td>
<td>1 ( 3.4044e+00, 2.5394e+00)</td>
</tr>
<tr>
<td>-1.5</td>
<td>2.5</td>
<td>1 ( -1.9737e-01, -2.4271e-01)</td>
</tr>
<tr>
<td>8.0</td>
<td>3.3</td>
<td>3 ( 1.1814e-03, -3.4188e-03)</td>
</tr>
<tr>
<td>2.9</td>
<td>7.5</td>
<td>4 ( -5.0227e-04, -1.4955e-03)</td>
</tr>
</tbody>
</table>