NAG C Library Function Document

nag_real_polygamma (s14aec)

1 Purpose

nag_real_polygamma (s14aec) returns the value of the kth derivative of the psi function \( \psi(x) \) for real \( x \) and \( k = 0, 1, \ldots, 6 \).

2 Specification

double nag_real_polygamma (double x, Integer k, NagError *fail)

3 Description

This routine evaluates an approximation to the kth derivative of the psi function \( \psi(x) \) given by

\[
\psi^{(k)}(x) = \frac{d^k}{dx^k} \psi(x) = \frac{d^k}{dx^k} \left( \frac{d^{k-1}}{dx^{k-1}} \log_e \Gamma(x) \right),
\]

where \( x \) is real with \( x \neq 0, -1, -2, \ldots \) and \( k = 0, 1, \ldots, 6 \). For negative non-integer values of \( x \), the recurrence relationship

\[
\psi^{(k)}(x + 1) = \psi^{(k)}(x) + \frac{d^k}{dx^k} \left( \frac{1}{x} \right)
\]

is used. The value of \( \frac{(-1)^{k+1}\psi^{(k)}(x)}{x^k} \) is obtained by a call to a routine based on PSIFN in Amos (1983). Note that \( \psi^{(k)}(x) \) is also known as the polygamma function. Specifically, \( \psi^{(0)}(x) \) is often referred to as the digamma function and \( \psi^{(1)}(x) \) as the trigamma function in the literature. Further details can be found in Abramowitz and Stegun (1972).

4 Parameters

1: \( x \) – double

\( \text{Input} \)

On entry: the argument \( x \) of the function.

Constraint: \( x \) must not be ‘too close’ (see Section 5) to a non-positive integer.

2: \( k \) – Integer

\( \text{Input} \)

On entry: the function \( \psi^{(k)}(x) \) to be evaluated.

Constraint: \( 0 \leq k \leq 6 \).

3: \( \text{fail} \) – NagError *

\( \text{Input/Output} \)

The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

NE_INT

On entry, \( k = \langle \text{value} \rangle \).

Constraint: \( 0 \leq k \leq 6 \).
NE_REAL

On entry, \( x = \langle\text{value}\rangle \).
Constraint: \( x \) must not be ‘too close’ to a non-positive integer. That is, \( |x - \text{nint}(x)| \geq \text{machine precision} \times \text{nint}(x) \)

NE_UNDERFLOWLIKELY

The evaluation has been abandoned due to the likelihood of underflow. The result is returned as zero.

NE_OVERFLOWLIKELY

The evaluation has been abandoned due to the likelihood of overflow. The result is returned as zero.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments

6.1 Accuracy

All constants in the underlying functions are given to approximately 18 digits of precision. If \( t \) denotes the number of digits of precision in the floating-point arithmetic being used, then clearly the maximum number in the results obtained is limited by \( p = \min(t, 18) \). Empirical tests by Amos (1983) have shown that the maximum relative error is a loss of approximately two decimal places of precision. Further tests with the function \( -\psi^{(0)}(x) \) have shown somewhat improved accuracy, except at points near the positive zero of \( \psi^{(0)}(x) \) at \( x = 1.46\ldots \), where only absolute accuracy can be obtained.

6.2 References

ACM Trans. Math. Software 9 494–502

7 See Also

None.

8 Example

The example program evaluates \( \psi^{(2)}(x) \) at \( x = 2.5 \), and prints the results.

8.1 Program Text

/* nag_real_polygamma (sl4aec) Example Program.
 * *
 * Copyright 2000 Numerical Algorithms Group.
 * *
 * NAG C Library
 *
 * Mark 6, 2000.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdbib.h>
#include <nags.h>

int main(void)
{
    double x, y;
    Integer exit_status=0;
    NagError fail;
    Integer k;

    INIT_FAIL(fail);

    Vprintf("s14aec Example Program Results\n\n");
    /* Skip heading in data file */
    Vscanf("%*[\n]");
    Vprintf("\n X K (D`K/DX`K)psi(X)\n\n");

    while (scanf("%lf %ld%*[\n]", &x, &k) != EOF)
    {
        y = s14aec (x, k, &fail);
        if (fail.code == NE_NOERROR)
            Vprintf("%12.4e\n", x, k, y);
        else
        {
            Vprintf("Error from s14aec.\n\n", fail.message);
            exit_status = 1;
            goto END;
        }
    }
END:
    return exit_status;
}

8.2 Program Data

s14aec Example Program Data

<table>
<thead>
<tr>
<th>X</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>-3.6</td>
<td>2</td>
</tr>
<tr>
<td>8.0</td>
<td>3</td>
</tr>
<tr>
<td>2.9</td>
<td>4</td>
</tr>
<tr>
<td>-4.7</td>
<td>5</td>
</tr>
<tr>
<td>-5.4</td>
<td>6</td>
</tr>
</tbody>
</table>

: Values of x and k

8.3 Program Results

s14aec Example Program Results

<table>
<thead>
<tr>
<th>X</th>
<th>K</th>
<th>(D<code>K/DX</code>K)psi(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0</td>
<td>-5.7722e-01</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>4.9348e+00</td>
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<tr>
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<td>2</td>
<td>-2.2335e+01</td>
</tr>
<tr>
<td>8.0</td>
<td>3</td>
<td>4.6992e-03</td>
</tr>
<tr>
<td>2.9</td>
<td>4</td>
<td>-1.5897e-01</td>
</tr>
<tr>
<td>-4.7</td>
<td>5</td>
<td>1.6566e+05</td>
</tr>
<tr>
<td>-5.4</td>
<td>6</td>
<td>4.1378e+05</td>
</tr>
</tbody>
</table>