1 Purpose

nag_polygamma_fun (s14acc) returns a value of the function \( \psi(x) - \ln x \), where \( \psi \) is the psi function
\[ \psi(x) = \frac{d}{dx} \ln \Gamma(x) = \frac{\Gamma'(x)}{\Gamma(x)}. \]

2 Specification

double nag_polygamma_fun (double x, NagError *fail)

3 Description

nag_polygamma_fun (s14acc) returns a value of the function \( \psi(x) - \ln x \). The psi function is computed without the logarithmic term so that when \( x \) is large, sums or differences of psi functions may be computed without unnecessary loss of precision, by analytically combining the logarithmic terms. For example, the difference \( d = \psi(x + \frac{1}{2}) - \psi(x) \) has an asymptotic behaviour for large \( x \) given by
\[ d \sim \ln(x + \frac{1}{2}) - \ln x + O\left(\frac{1}{x^2}\right) \sim \ln\left(1 + \frac{1}{2x}\right) \sim \frac{1}{2x}. \]

Computing \( d \) directly would amount to subtracting two large numbers which are close to \( \ln(x + \frac{1}{2}) \) and \( \ln x \) to produce a small number close to \( \frac{1}{2x} \), resulting in a loss of significant digits. However, using this function to compute \( f(x) = \psi(x) - \ln x \), we can compute \( d = f(x + \frac{1}{2}) - f(x) + \ln(1 + \frac{1}{2x}) \), and the dominant logarithmic term may be computed accurately from its power series when \( x \) is large. Thus we avoid the unnecessary loss of precision.

The function is derived from the routine PSIFN in Amos (1983).

4 References


5 Parameters

1: x – double
   
   On entry: the argument \( x \) of the function.
   
   Constraint: \( x > 0.0 \).

2: fail – NagError *
   
   The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_OVERFLOW_LIKELY

Computation halted due to likelihood of overflow. \( x \) may be too small. \( x = \langle value \rangle \).
On entry, \( x = \langle \text{value} \rangle \).
Constraint: \( x > 0.0 \).

Computation halted due to likelihood of underflow. \( x \) may be too large. \( x = \langle \text{value} \rangle \).

On entry, parameter \( \langle \text{value} \rangle \) had an illegal value.

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

All constants in nag_polygamma_fun (s14acc) are given to approximately 18 digits of precision. Calling the number of digits of precision in the floating-point arithmetic being used \( t \), then clearly the maximum number of correct digits in the results obtained is limited by \( p = \min(t, 18) \).

With the above proviso, results returned by this function should be accurate almost to full precision, except at points close to the zero of \( \psi(x) \), \( x \simeq 1.461632 \), where only absolute rather than relative accuracy can be obtained.

None.

The example program reads values of the argument \( x \) from a file, evaluates the function at each value of \( x \) and prints the results.

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/* nag_polygamma_fun (s14acc) Example Program */
/* Copyright 2002 Numerical Algorithms Group. */
/* Mark 7, 2002. */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    double f, x;

    /* Skip heading in data file */
    Vscanf("%*[\n]");
    Vprintf("s14acc Example Program Results\n");
    Vprintf(" x psi(x)-log(x)\n");
    while (scanf("%lf", &x) != EOF)
    {
        f = s14acc(x, NAGERR_DEFAULT);
        Vprintf("%8.3f %14.4f\n", x, f);
    }
    return EXIT_SUCCESS;
}
9.2 Program Data

sl4acc Example Program Data
0.1
0.5
3.6
8.0

9.3 Program Results

sl4acc Example Program Results

<table>
<thead>
<tr>
<th>x</th>
<th>psi(x) - log(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.100</td>
<td>-8.1212</td>
</tr>
<tr>
<td>0.500</td>
<td>-1.2704</td>
</tr>
<tr>
<td>3.600</td>
<td>-0.1453</td>
</tr>
<tr>
<td>8.000</td>
<td>-0.0638</td>
</tr>
</tbody>
</table>