NAG C Library Function Document

nag_shifted_log (s01bac)

1 Purpose

nag_shifted_log (s01bac) returns a value of the shifted logarithmic function, \( \ln(1 + x) \), via the function name.

2 Specification

double nag_shifted_log (double x, NagError *fail)

3 Description

nag_shifted_log (s01bac) computes values of \( \ln(1 + x) \), retaining full relative precision even when |x| is small. The function is based on the Chebyshev expansion

\[
\ln \frac{1 + p^2 + 2px}{1 + p^2 - 2px} = 4 \sum_{k=0}^{\infty} \frac{p^{2k+1}}{2k+1} T_{2k+1}(\bar{x}).
\]

Setting \( \bar{x} = \frac{x(1 + p^2)}{2p(x + 2)} \), and choosing \( p = \frac{q - 1}{q + 1} \), \( q = \sqrt{2} \) the expansion is valid in the domain \( x \in \left[ \frac{1}{\sqrt{2}} - 1, \sqrt{2} - 1 \right] \).

Outside this domain, \( \ln(1 + x) \) is computed by the standard logarithmic function.

4 References


5 Parameters

1:  x – double  

On entry: the argument x of the function.  

Constraint: x > −1.0.

2:  fail – NagError *

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_REAL_ARG_LE

On entry, x must not be less than or equal to \(-1.0\): x = <value>.

NE_BAD_PARAM

On entry, parameter <value> had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.
7 Accuracy

The returned result should be accurate almost to machine precision, with a limit of about 20 significant figures due to the precision of internal constants. Note however that if \( x \) lies very close to \(-1.0\) and is not exact (for example if \( x \) is the result of some previous computation and has been rounded), then precision will be lost in the computation of \( 1 + x \), and hence \( \ln(1 + x) \), in nag_shifted_log (s01bac).

8 Further Comments

Empirical tests show that the time taken for a call of nag_shifted_log (s01bac) usually lies between about 1.25 and 2.5 times the time for a call to the standard logarithm function.

9 Example

The example program reads values of the argument \( x \) from a file, evaluates the function at each value of \( x \) and prints the results.

9.1 Program Text

/* nag_shifted_log (s01bac) Example Program */
/* Copyright 2002 Numerical Algorithms Group. */
/* Mark 7, 2002. */
*/
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    double x, y;

    /* Skip heading in data file */
    Vscanf("%*[^
");
    Vprintf("s01bac Example Program Results
");
    Vprintf(" x y
");
    while (scanf("%lf", &x) != EOF)
    {
        y = s01bac(x, NAGERR_DEFAULT);
        Vprintf("%12.4e %12.4e
", x, y);
    }
    return EXIT_SUCCESS;
}

9.2 Program Data

s01bac Example Program Data
2.50e+0
1.25e-1
-9.06e-1
1.29e-3
-7.86e-6
1.00e-9

9.3 Program Results

s01bac Example Program Results
 x     y
2.5000e+00 1.2528e+00
1.2500e-01 1.1778e-01
-9.0600e-01 -2.3645e+00
1.2900e-03 1.2892e-03
-7.8300e-06 -7.8300e-06
1.0000e-09 1.0000e-09