NAG C Library Function Document

nag_tsa_multi_cross_corr (g13dmc)

1 Purpose

nag_tsa_multi_cross_corr (g13dmc) calculates the sample cross-correlation (or cross-covariance) matrices of a multivariate time series.

2 Specification

void nag_tsa_multi_cross_corr (Nag_CovOrCorr matrix, Integer k, Integer n, Integer m, const double w[], double wmean[], double r0[], double r[], NagError *fail)

3 Description

Let \( W_t = (w_{t1}, w_{t2}, \ldots, w_{tk})^T \), for \( t = 1, 2, \ldots, n \), denote \( n \) observations of a vector of \( k \) time series. The sample cross-covariance matrix at lag \( l \) is defined to be the \( k \) by \( k \) matrix \( ^\gamma CC(l) \), whose \((i,j)\)th element is given by

\[
^\gamma CC_{ij}(l) = \frac{1}{n} \sum_{t=l+1}^{n} (w_{it-l} - \bar{w}_i)(w_{jt} - \bar{w}_j), \quad l = 0, 1, 2, \ldots, m; \quad i = 1, 2, \ldots, k; \quad j = 1, 2, \ldots, k,
\]

where \( \bar{w}_i \) and \( \bar{w}_j \) denote the sample means for the \( i \)th and \( j \)th series respectively. The sample cross-correlation matrix at lag \( l \) is defined to be the \( k \) by \( k \) matrix \( ^\gamma RR(l) \), whose \((i,j)\)th element is given by

\[
^\gamma RR_{ij}(l) = \frac{^\gamma CC_{ij}(l)}{\sqrt{^\gamma CC_{ii}(0)^\gamma CC_{jj}(0)}}, \quad l = 0, 1, 2, \ldots, m; \quad i = 1, 2, \ldots, k; \quad j = 1, 2, \ldots, k.
\]

The number of lags, \( m \), is usually taken to be at most \( n/4 \).

If \( W_t \) follows a vector moving average model of order \( q \), then it can be shown that the theoretical cross-correlation matrices \((R(l))\) are zero beyond lag \( q \). In order to help spot a possible cut-off point, the elements of \( R(l) \) are usually compared to their approximate standard error of \( 1/\sqrt{n} \). For further details see, for example, Wei (1990).

The routine uses a single pass through the data to compute the means and the cross-covariance matrix at lag zero. The cross-covariance matrices at further lags are then computed on a second pass through the data.

4 References


5 Parameters

1: matrix – Nag_CovOrCorr [Input]

On entry: indicates whether the cross-covariance or cross-correlation matrices are to be computed.

If matrix = Nag_AutoCov, then the cross-covariance matrices are computed.

If matrix = Nag_AutoCorr, then the cross-correlation matrices are computed.

Constraint: matrix = Nag_AutoCov or Nag_AutoCorr.
2:  \( k \) – Integer  
   \text{Input} 
   \text{On entry: the dimension, } k, \text{ of the multivariate time series.} 
   \text{Constraint: } k \geq 1.

3:  \( n \) – Integer  
   \text{Input} 
   \text{On entry: the number of observations in the series, } n. 
   \text{Constraint: } n \geq 2.

4:  \( m \) – Integer  
   \text{Input} 
   \text{On entry: the number, } m, \text{ of cross-correlation (or cross-covariance) matrices to be computed. If in doubt set } m = 10. \text{ However it should be noted that } m \text{ is usually taken to be at most } n/4. 
   \text{Constraint: } 1 \leq m < n.

5:  \( w[\text{dim}] \) – const double  
   \text{Input} 
   \text{Note: the dimension, } \text{dim}, \text{ of the array } w \text{ must be at least } k \times n. 
   \text{On entry: } w[(t-1)k + i - 1] \text{ must contain the value for series } i \text{ at time } t, \text{ for } i = 1, 2, \ldots, k; t = 1, 2, \ldots, n.

6:  \( \text{wmean}[k] \) – double  
   \text{Output} 
   \text{On exit: the means, } \bar{w}_i, \text{ for } i = 1, 2, \ldots, k.

7:  \( r0[\text{dim}] \) – double  
   \text{Output} 
   \text{Note: the dimension, } \text{dim}, \text{ of the array } r0 \text{ must be at least } k \times k. 
   \text{On exit: if } \text{matrix} = \text{Nag_AutoCov}, \text{ } r0[(j-1)k + i - 1] \text{ contains the } (i, j)\text{th element of the sample cross-covariance matrix. If } \text{matrix} = \text{Nag_AutoCorr}, \text{ } r0[(j-1)k + i - 1], \text{ } i \neq j \text{ contains the } (i, j)\text{th element of the sample cross-correlation matrix and } r0[(i-1)k + i - 1] \text{ contains the standard deviation of the } i\text{th series.}

8:  \( r[\text{dim}] \) – double  
   \text{Output} 
   \text{Note: the dimension, } \text{dim}, \text{ of the array } r \text{ must be at least } k \times k \times m. 
   \text{On exit: if } \text{matrix} = \text{Nag_AutoCov}, \text{ } r[(l-1)k^2 + (j-1)k + i - 1] \text{ contains the } (i, j, l)\text{th element of the sample cross-covariance matrix at lag } l. \text{ If } \text{matrix} = \text{Nag_AutoCorr}, \text{ then it contains the } (i, j, l)\text{th element of the sample cross-correlation matrix lag } l, \text{ for } l = 1, 2, \ldots, m; i = 1, 2, \ldots, k; j = 1, 2, \ldots, k.

9:  \( \text{fail} \) – NagError *  
   \text{Input/Output} 
   \text{The NAG error parameter (see the Essential Introduction).}

6  \text{Error Indicators and Warnings}

\textbf{NE_INT} 
   \text{On entry, } k = \langle \text{value} \rangle. 
   \text{Constraint: } k \geq 1. 
   \text{On entry, } n = \langle \text{value} \rangle. 
   \text{Constraint: } n \geq 2. 

\textbf{NE_INT_2} 
   \text{On entry, } m < 1 \text{ or } m \geq n: \text{ } m = \langle \text{value} \rangle, n = \langle \text{value} \rangle.
NE_ZERO_VARIANCE
On entry, at least one of the series is such that all its elements are practically identical giving zero
(or near zero) variance.

NE_ALLOC_FAIL
Memory allocation failed.

NE_BAD_PARAM
On entry, parameter (value) had an illegal value.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the
call is correct then please consult NAG for assistance.

7 Accuracy
For a discussion of the accuracy of the one-pass algorithm used to compute the sample cross-covariances at
lag zero see West (1979). For the other lags a two-pass algorithm is used to compute the cross-
covariances; the accuracy of this algorithm is also discussed in West (1979). The accuracy of the cross-
correlations will depend on the accuracy of the computed cross-covariances.

8 Further Comments
The time taken is roughly proportional to $mnk^2$.

9 Example
This program computes the sample cross-correlation matrices of two time series of length 48, up to lag 10.
It also prints the cross-correlation matrices together with plots of symbols indicating which elements of the
correlation matrices are significant. Three * represent significance at the 0.5% level, two * represent
significance at the 1% level and a single * represents significance at the 5% level. The * are plotted above
or below the line depending on whether the elements are significant in the positive or negative direction.

9.1 Program Text
/* nag_tsa_multi_cross_corr (g13dmc) Example Program.
 * Copyright 2002 Numerical Algorithms Group.
 * Mark 7, 2002.
 */
#include <stdio.h>
#include <math.h>
#include <string.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagl13.h>

static void cprint(Integer, Integer, Integer, Integer,
   double *, double *);

int main(void)
{
  /* Scalars */
  Integer exit_status, i, j, k, m, n, p, w;
  NagError fail;
  Nag_CovOrCorr matrix;

  /* Arrays */

/* Allocate arrays */
if ( !(r0 = NAG_ALLOC(k * k, double)) ||
    !(r = NAG_ALLOC(k * k * m, double)) ||
    !(w = NAG_ALLOC(k * n, double)) ||
    !(wmean = NAG_ALLOC(k, double)) )
{
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Print the correlation matrices and indicator symbols. */
static void cprint(Integer k, Integer n, Integer ik, Integer m,
    double *wmean, double *r)
{
    /* Scalars */
    double c1, c2, c3, c5, c6, c7, cnst, sum;
    Integer i2, i, j, lf, llf, ii;

    /* Arrays */
    char rec[7][80];
    
    cnst = 1.0 / sqrt((double)n);
}
Vprintf("\n");
Vprintf(" THE MEANS\n");
Vprintf("  ---------\n");
Vprintf(" ");
for (i = 1; i <= k; ++i)
{
    Vprintf("%10.3f", wmean[i-1]);
    if (i % 2 == 0 || i == k)
        Vprintf("\n");
}
Vprintf("\n");
Vprintf(" CROSS-CORRELATION MATRICES\n");
Vprintf("  --------------------------\n");
for (lf = 1; lf <= m; ++lf)
{
    Vprintf("\n");
    Vprintf(" Lag = %2ld\n", lf);
    for (i = 1; i <= k; i++)
        {
            for (j = 1; j <= k; j++)
                Vprintf("%9.3f", R(i,j,lf));
            Vprintf("\n");
        }
}
/* Print indicator symbols to indicate significant elements. */
Vprintf("\n");
Vprintf(" Standard error = 1 / SQRT(N) = %5.3f\n", cnst);
Vprintf("\n");
Vprintf(" TABLES OF INDICATOR SYMBOLS\n");
Vprintf("  ---------------------------\n");
Vprintf("\n");
Vprintf(" For Lags 1 to %2ld\n", m);
Vprintf("\n");

/* Set up the critical values */
c1 = cnst * 3.29;
c2 = cnst * 2.58;
c3 = cnst * 1.96;
c5 = -c3;
c6 = -c2;
c7 = -c1;
for (i = 1; i <= k; ++i)
{
    for (j = 1; j <= k; ++j)
        {
            Vprintf("\n");
            Vprintf("\n");
            if (i == j)
                Vprintf("Auto-correlation function for series %2ld\n", i);
            else
                Vprintf("Cross-correlation function for series %2ld" 
                    " and series%2ld\n", i, j);
            Vprintf("\n");
        }
    /* Clear the last plot with blanks */
    sprintf(&rec[0][0], " 0.005 :");
    sprintf(&rec[1][0], " + 0.01 :");
    sprintf(&rec[2][0], " 0.05 :");
    sprintf(&rec[3][0], " Sig. Level : - - - - - - - - - - Lags");
    sprintf(&rec[4][0], " 0.05 :");
    sprintf(&rec[5][0], " - 0.01 :");
    sprintf(&rec[6][0], " 0.005 :");
    for (i2 = 0; i2 < 7; ++i2)
    {
        for (ii = strlen(&rec[i2][0]); ii < 80; ii++)
            rec[i2][ii] = ",

for (lf = 1; lf <= m; ++lf)
{
  llf = lf * 2 + 21;
  sum = R(i, j, lf);
  /* Check for significance */
  if (sum > c1)
    rec[0][llf] = '*';
  if (sum > c2)
    rec[1][llf] = '*';
  if (sum > c3)
    rec[2][llf] = '*';
  if (sum < c5)
    rec[4][llf] = '*';
  if (sum < c6)
    rec[5][llf] = '*';
  if (sum < c7)
    rec[6][llf] = '*';
}

/* Print */
for (i2 = 0; i2 < 7; ++i2)
{
  /* Terminate the string */
  for (ii = 80; ii > 1 && rec[i2][ii-1] == ' '; ii--);
  rec[i2][ii] = '\0';
  /* Print the string */
  Vprintf("%s\n", &rec[i2][0]);
}

return;

9.2 Program Data

9.3 Program Results
<table>
<thead>
<tr>
<th>Lag</th>
<th>Auto-correlation function for series 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.456 0.076 0.069 0.260</td>
</tr>
<tr>
<td>4</td>
<td>0.379 0.014 0.026 -0.038</td>
</tr>
<tr>
<td>5</td>
<td>0.322 0.110 0.093 -0.236</td>
</tr>
<tr>
<td>6</td>
<td>0.341 0.269 0.087 -0.250</td>
</tr>
<tr>
<td>7</td>
<td>0.363 0.344 0.132 -0.227</td>
</tr>
<tr>
<td>8</td>
<td>0.280 0.425 0.207 -0.128</td>
</tr>
<tr>
<td>9</td>
<td>0.248 0.522 0.197 -0.085</td>
</tr>
<tr>
<td>10</td>
<td>0.240 0.266 0.254 0.075</td>
</tr>
</tbody>
</table>

Lag = 10
0.162 -0.020
0.267 0.005

Standard error = 1 / SQRT(N) = 0.144

TABLES OF INDICATOR SYMBOLS
-----------------------------

For Lags 1 to 10

Auto-correlation function for series 1

<table>
<thead>
<tr>
<th>Sig. Level</th>
<th>Lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05 :</td>
<td></td>
</tr>
<tr>
<td>0.01 :</td>
<td>***</td>
</tr>
<tr>
<td>0.005 :</td>
<td>**</td>
</tr>
</tbody>
</table>

Cross-correlation function for series 1 and series 2

<table>
<thead>
<tr>
<th>Sig. Level</th>
<th>Lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05 :</td>
<td></td>
</tr>
<tr>
<td>0.01 :</td>
<td>**</td>
</tr>
<tr>
<td>0.005 :</td>
<td></td>
</tr>
</tbody>
</table>

Cross-correlation function for series 2 and series 1

<table>
<thead>
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<th>Sig. Level</th>
<th>Lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05 :</td>
<td></td>
</tr>
<tr>
<td>0.01 :</td>
<td></td>
</tr>
<tr>
<td>0.005 :</td>
<td></td>
</tr>
</tbody>
</table>
Auto-correlation function for series 2

<table>
<thead>
<tr>
<th>Lags</th>
<th>Sig. Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.01</td>
<td>:</td>
</tr>
<tr>
<td>-0.05</td>
<td>:</td>
</tr>
<tr>
<td>0.005</td>
<td>:</td>
</tr>
<tr>
<td>0.01</td>
<td>: *</td>
</tr>
<tr>
<td>0.05</td>
<td>: *</td>
</tr>
<tr>
<td>0.005</td>
<td>:</td>
</tr>
</tbody>
</table>