NAG C Library Function Document

nag_tsa_cross_corr (g13bcc)

1 Purpose

nag_tsa_cross_corr (g13bcc) calculates cross-correlations between two time series.

2 Specification

```c
void nag_tsa_cross_corr (const double x[], const double y[], Integer nxy, Integer nl, double *s, double *r0, double r[], double *stat, NagError *fail)
```

3 Description

Given two series \( x_1, x_2, \ldots, x_n \) and \( y_1, y_2, \ldots, y_n \) the routine calculates the cross-correlations between \( x_t \) and lagged values of \( y_t \):

\[
\begin{align*}
r_{xy}(l) &= \frac{\sum_{t=1}^{n-l} (x_t - \bar{x})(y_{t+l} - \bar{y})}{ns_x s_y}, \quad l = 0, 1, \ldots, L
\end{align*}
\]

where

\[
\begin{align*}
\bar{x} &= \frac{\sum_{i=1}^{n} x_i}{n} \\
\bar{y} &= \frac{\sum_{i=1}^{n} y_i}{n} \\
ns_x &= \sum_{i=1}^{n} (x_i - \bar{x})^2 \\
nxy &= \sum_{i=1}^{n} y_i
\end{align*}
\]

and similarly for \( y \).

The ratio of standard deviations \( s_y/s_x \) is also returned, and a portmanteau statistic is calculated:

\[
\text{stat} = n \sum_{l=1}^{L} r_{xy}(l)^2.
\]

Provided \( n \) is large, \( L \) much less than \( n \), and both \( x_t, y_t \) are samples of series whose true autocorrelation functions are zero, then, under the null hypothesis that the true cross-correlations between the series are zero, \( \text{stat} \) has a \( \chi^2 \) distribution with \( L \) degrees of freedom. Values of \( \text{stat} \) in the upper tail of this distribution provide evidence against the null hypothesis.

4 References


5 Parameters

1. \( x[\text{nxy}] \) – const double
   
   \textit{Input}
   
   \textit{On entry:} the \( n \) values of the \( x \) series.

2. \( y[\text{nxy}] \) – const double
   
   \textit{Input}
   
   \textit{On entry:} the \( n \) values of the \( y \) series.
3:  \textbf{\texttt{nxy} – Integer}  \hspace{1cm} \textit{Input}

\textit{On entry:} the length of the time series, \(n\).
\textit{Constraint:} \(\texttt{nxy} \geq 2\).

4:  \textbf{\texttt{nl} – Integer}  \hspace{1cm} \textit{Input}

\textit{On entry:} the maximum lag for calculating cross-correlations, \(L\).
\textit{Constraint:} \(1 \leq \texttt{nl} < \texttt{nxy}\).

5:  \textbf{\texttt{s} – double *}  \hspace{1cm} \textit{Output}

\textit{On exit:} the ratio of the standard deviation of the \(y\) series to the standard deviation of the \(x\) series, \(s_y/s_x\).

6:  \textbf{\texttt{r0} – double *}  \hspace{1cm} \textit{Output}

\textit{On exit:} the cross-correlation between the \(x\) and \(y\) series at lag zero.

7:  \textbf{\texttt{r[\texttt{nl}]} – double}  \hspace{1cm} \textit{Output}

\textit{On exit:} \(r[l-1]\) contains the cross-correlations between the \(x\) and \(y\) series at lags \(l\), \(r_{xy}(l)\), for \(l = 1, 2, \ldots, L\).

8:  \textbf{\texttt{stat} – double *}  \hspace{1cm} \textit{Output}

\textit{On exit:} the statistic for testing for absence of cross-correlation.

9:  \textbf{\texttt{fail} – NagError *}  \hspace{1cm} \textit{Input/Output}

The NAG error parameter (see the Essential Introduction).

6 \textbf{Error Indicators and Warnings}

\textbf{NE_INT}

On entry, \(\texttt{nl} = \langle\text{value}\rangle\).
\textit{Constraint:} \(\texttt{nl} > 1\).

On entry, \(\texttt{nxy} = \langle\text{value}\rangle\).
\textit{Constraint:} \(\texttt{nxy} > 1\).

\textbf{NE_INT_2}

On entry, \(\texttt{nl} \geq \texttt{nxy}: \texttt{nl} = \langle\text{value}\rangle, \texttt{nxy} = \langle\text{value}\rangle\).

\textbf{NE_ZERO_VARIANCE}

One or both of the \(x\) and \(y\) series have zero variance.

\textbf{NE_BAD_PARAM}

On entry, parameter \(\langle\text{value}\rangle\) had an illegal value.

\textbf{NE_INTERNAL_ERROR}

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 \textbf{Accuracy}

All computations are believed to be stable.
8 Further Comments

The time taken by the routine is approximately proportional to \( nL \).

9 Example

The example program reads two time series of length 20. It calculates and prints the cross-correlations up to lag 15 for the first series leading the second series and then for the second series leading the first series.

9.1 Program Text

```c
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
    double r0xy, r0yx, statxy, statyx, sxy, syx;
    Integer exit_status, i, nl, nxy;
    NagError fail;

    double *rxy = 0, *ryx = 0, *x = 0, *y = 0;
    INIT_FAIL(fail);
    exit_status = 0;
    Vprintf("g13bcc Example Program Results\n");
    /* Skip heading in data file */
    Vscanf("%*\[\[^\n\]\] ");
    /* Read series length and number of lags */
    Vscanf("%ld%ld%*\[\[^\n\]\] ", &nxy, &nl);
    /* Allocate memory */
    if ( !(rxy = NAG_ALLOC(nl, double)) ||
        !(ryx = NAG_ALLOC(nl, double)) ||
        !(x = NAG_ALLOC(nxy, double)) ||
        !(y = NAG_ALLOC(nxy, double)) )
    {
        Vprintf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    /* Read series */
    for (i = 1; i <= nxy; ++i)
        Vscanf("%lf", &x[i-1]);
    Vscanf("%*\[\[^\n\]\] ");
    for (i = 1; i <= nxy; ++i)
        Vscanf("%lf", &y[i-1]);
    Vscanf("%*\[\[^\n\]\] ");
    /* Call routine to calculate cross correlations between X and Y */
    g13bcc(x, y, nxy, nl, &sxy, &r0xy, rxy, &statxy, &fail);
    if (fail.code != NE_NOERROR)
    {
```

```
/* Call routine to calculate cross correlations between Y and X */
g13bcc(y, x, nxy, nl, &syx, &r0yx, ryx, &statyx, &fail);
  if (fail.code != NE_NOERROR)
  {
    Vprintf("Error from g13bcc, 2nd call.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }
Vprintf("\n");
Vprintf(" Between Between\n");
Vprintf(" X and Y Y and X\n");
Vprintf("\n");
Vprintf("Standard deviation ratio%10.4f%15.4f\n", sxy, syx);
Vprintf("\n");
Vprintf("Cross correlation at lag\n");
Vprintf(" 0\n");
Vprintf("%10.4f%15.4f\n", r0xy, r0yx);
  for (i = 1; i <= nl; ++i)
    Vprintf(" %4ld%10.4f%15.4f\n", i, rxy[i-1], ryx[i-1]);
Vprintf("\n");
Vprintf("Test statistic %10.4f%15.4f\n", statxy, statyx);
END:
if (rxy) NAG_FREE(rxy);
if (ryx) NAG_FREE(ryx);
if (x) NAG_FREE(x);
if (y) NAG_FREE(y);
return exit_status;
}

9.2 Program Data
g13bcc Example Program Data
  20 15
  0.02 0.05 0.08 0.03 -0.05 0.11 -0.01 -0.08 -0.08 -0.11
-0.18 -0.19 -0.09 0.03 0.10 0.15 -0.14 0.07 0.09 0.16
  3.18 3.21 3.26 3.25 3.08 3.01 3.06 3.17 3.12 3.04
  3.26 3.45 3.33 3.70 3.31 3.81 3.33 2.96 3.28 3.10

9.3 Program Results
g13bcc Example Program Results
  Between Between
  X and Y Y and X

  Standard deviation ratio 2.0053 0.4987

  Cross correlation at lag
  0  0.0568  0.0568
  1  0.0438  -0.0151
  2 -0.3762  0.3955
  3 -0.4864  0.3417
  4 -0.6294  0.5486
  5 -0.3871  0.2291
  6 -0.1690  0.3190
  7 -0.0678  0.1980
  8  0.0962  0.0438
  9  0.0788  -0.1428
 10  0.2910  -0.1376
## Test statistic

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Test statistic: 22.1269, 17.2917