1 Purpose

nag_rank_ci_1var (g07eac) computes a rank based (nonparametric) estimate and confidence interval for the location parameter of a single population.

2 Specification

```c
void nag_rank_ci_1var (Nag_RCIMethod method, Integer n, const double x[],
 double clevel, double *theta, double *thetal, double *thetau,
 double *estcl, double *wlower, double *wupper, NagError *fail)
```

3 Description

Consider a vector of independent observations, \( x = (x_1, x_2, \ldots, x_n)^t \) with unknown common symmetric density \( f(x_i - \theta) \). nag_rank_ci_1var (g07eac) computes the Hodges–Lehmann location estimator (see Lehmann (1975)) of the centre of symmetry \( \theta \), together with an associated confidence interval. The Hodges–Lehmann estimate is defined as

\[
\hat{\theta} = \text{median}\left\{ \frac{x_i + x_j}{2} \right\}, 1 \leq i \leq j \leq n.
\]

Let \( m = (n(n + 1))/2 \) and let \( a_k \), for \( k = 1, 2, \ldots, m \) denote the \( m \) ordered averages \( (x_i + x_j)/2 \) for \( 1 \leq i \leq j \leq n \). Then

- if \( m \) is odd, \( \hat{\theta} = a_k \) where \( k = (m + 1)/2 \),
- if \( m \) is even, \( \hat{\theta} = (a_k + a_{k+1})/2 \) where \( k = m/2 \).

This estimator arises from inverting the one-sample Wilcoxon signed-rank test statistic, \( W(x - \theta_0) \), for testing the hypothesis that \( \theta = \theta_0 \). Effectively \( W(x - \theta_0) \) is a monotonically decreasing step function of \( \theta_0 \) with

\[
\text{mean}(W) = \mu = \frac{n(n + 1)}{4},
\]

\[
\text{var}(W) = \sigma^2 = \frac{n(n + 1)(2n + 1)}{24}.
\]

The estimate \( \hat{\theta} \) is the solution to the equation \( W(x - \hat{\theta}) = \mu \); two methods are available for solving this equation. These methods avoid the computation of all the ordered averages \( a_k \); this is because for large \( n \) both the storage requirements and the computation time would be excessive.

The first is an exact method based on a set partitioning procedure on the set of all ordered averages \( (x_i + x_j)/2 \) for \( i \leq j \). This is based on the algorithm proposed by Monahan (1984).

The second is an iterative algorithm, based on the Illinois method which is a modification of the regula falsi method, see McKean and Ryan (1977). This algorithm has proved suitable for the function \( W(x - \theta_0) \) which is asymptotically linear as a function of \( \theta_0 \).

The confidence interval limits are also based on the inversion of the Wilcoxon test statistic.

Given a desired percentage for the confidence interval, \( 1 - \alpha \), expressed as a proportion between 0 and 1, initial estimates for the lower and upper confidence limits of the Wilcoxon statistic are found from

\[
W_l = \mu - 0.5 + (\sigma \Phi^{-1}(\alpha/2))
\]

and
\[ W_u = \mu + 0.5 + \left( \sigma \Phi^{-1}(1 - \alpha/2) \right), \]

where \( \Phi^{-1} \) is the inverse cumulative Normal distribution function.

\( W_l \) and \( W_u \) are rounded to the nearest integer values. These estimates are then refined using an exact method if \( n \leq 80 \), and a Normal approximation otherwise, to find \( W_l \) and \( W_u \) satisfying

\[
\begin{align*}
P(W \leq W_l) & \leq \alpha/2 \\
P(W \leq W_l + 1) & > \alpha/2
\end{align*}
\]

and

\[
\begin{align*}
P(W \geq W_u) & \leq \alpha/2 \\
P(W \geq W_u - 1) & > \alpha/2.
\end{align*}
\]

Let \( W_u = m - k; \) then \( \theta_l = a_{k+1}. \) This is the largest value \( \theta_l \) such that \( W(x - \theta_l) = W_u. \)

Let \( W_l = k; \) then \( \theta_u = a_{m-k}. \) This is the smallest value \( \theta_u \) such that \( W(x - \theta_u) = W_l. \)

As in the case of \( \hat{\theta} \), these equations may be solved using either the exact or the iterative methods to find the values \( \theta_l \) and \( \theta_u. \)

Then \( (\theta_l, \theta_u) \) is the confidence interval for \( \theta. \) The confidence interval is thus defined by those values of \( \theta_0 \) such that the null hypothesis, \( \theta = \theta_0, \) is not rejected by the Wilcoxon signed-rank test at the \((100 \times \alpha)\%\) level.

### 4 References

Lehmann E L (1975) *Nonparametrics: Statistical Methods Based on Ranks* Holden–Day


### 5 Parameters

1. **method** – Nag_RCIMethod
   
   *Input*
   
   *On entry:* specifies the method to be used.
   
   If **method** = Nag_RCI_Exact, the exact algorithm is used.
   
   If **method** = Nag_RCI_Approx, the iterative algorithm is used.
   
   *Constraint:* **method** = Nag_RCI_Exact or Nag_RCI_Approx.

2. **n** – Integer
   
   *Input*
   
   *On entry:* the sample size, \( n. \)
   
   *Constraint:* \( n \geq 2. \)

3. **x[n]** – const double
   
   *Input*
   
   *On entry:* the sample observations, \( x_i \) for \( i = 1, 2, \ldots, n. \)

4. **clevel** – double
   
   *Input*
   
   *On entry:* the confidence interval desired.
For example, for a 95% confidence interval set \texttt{clevel} = 0.95.

\textit{Constraint:} $0.0 < \texttt{clevel} < 1.0.$

5: \texttt{theta} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} the estimate of the location, \( \hat{\theta} \).

6: \texttt{thetal} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} the estimate of the lower limit of the confidence interval, \( \theta_l \).

7: \texttt{thetau} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} the estimate of the upper limit of the confidence interval, \( \theta_u \).

8: \texttt{estcl} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} an estimate of the actual percentage confidence of the interval found, as a proportion between (0.0,1.0).

9: \texttt{wlower} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} the upper value of the Wilcoxon test statistic, \( W_u \), corresponding to the lower limit of the confidence interval.

10: \texttt{wupper} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} the lower value of the Wilcoxon test statistic, \( W_l \), corresponding to the upper limit of the confidence interval.

11: \texttt{fail} – NagError * \hspace{1cm} \textit{Input/Output}

The NAG error parameter (see the Essential Introduction).

6 \hspace{1cm} \textbf{Error Indicators and Warnings}

\textbf{NE\_INT}

On entry, \texttt{n} = (value).

\textit{Constraint:} \texttt{n} \geq 2.

\textbf{NE\_CONVERGENCE}

Warning. The iterative procedure to find an estimate of the upper confidence point had not converged in 100 iterations.

Warning. The iterative procedure to find an estimate of the lower confidence point had not converged in 100 iterations.

Warning. The iterative procedure to find an estimate of Theta had not converged in 100 iterations.

\textbf{NE\_REAL}

On entry \texttt{clevel} is out of range: \texttt{clevel} = (value).

\textbf{NE\_SAMPLE\_IDEN}

Not enough information to compute an interval estimate since the whole sample is identical. The common value is returned in \texttt{theta}, \texttt{thetal} and \texttt{thetau}.

\textbf{NE\_ALLOC\_FAIL}

Memory allocation failed.
NE_BAD_PARAM
On entry, parameter \( value \) had an illegal value.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy
The function should produce results accurate to 5 significant figures in the width of the confidence interval; that is the error for any one of the three estimates should be less than \( 0.00001 \times (\text{thetau} - \text{thetal}) \).

8 Further Comments
The time taken increases with the sample size \( n \).

9 Example
The following program calculates a 95% confidence interval for \( \theta \), a measure of symmetry of the sample of 50 observations.

9.1 Program Text
/* nag_rank_ci_1var (g07eac) Example Program. */
/* Copyright 2001 Numerical Algorithms Group. */
/* Mark 7, 2001. */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg07.h>

int main(void)
{
  /* Scalars */
  double clevel, estcl, theta, thetal, thetau, wlower, wupper;
  Integer exit_status, i, n;
  NagError fail;
  /* Arrays */
  double *x = 0;

  INIT_FAIL(fail);
  exit_status = 0;
  Vprintf("g07eac Example Program Results\n");

  /* Skip heading in data file */
  Vscanf("%*[\n] ");
  Vscanf("%ld%*[\n] ", &n);

  /* Allocate memory */
  if ( !(x = NAG_ALLOC(n, double)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  for (i = 1; i <= n; ++i)
Vscanf("%lf", &x[i - 1]);
Vscanf("%*[\n] ");
Vscanf("%lf%*[\n] ", &clevel);
g07eac(Nag_RCI_Exact, n, x, clevel, &theta, &thetal, &thetau, &estcl,
&wlower, &wupper, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g07eac.
%s
", fail.message);
    exit_status = 1;
    goto END;
}
Vprintf("\n");
Vprintf(" Location estimator Confidence Interval\n");
Vprintf("\n");
Vprintf("%10.4f ( %6.4f , %6.4f )\n", theta, thetal, thetau);
Vprintf("\n");
Vprintf(" Corresponding Wilcoxon statistics\n");
Vprintf("\n");
Vprintf(" Lower : %8.2f\n", wlower);
Vprintf(" Upper : %8.2f\n", wupper);
END:
if (x) NAG_FREE(x);
return exit_status;
}

9.2 Program Data

9.3 Program Results