1 Purpose

nag_rngs_arma_time_series (g05pac) generates a realisation of a univariate time series from an autoregressive moving average (ARMA) model. The realisation may be continued or a new realisation generated at subsequent calls to nag_rngs_arma_time_series (g05pac).

2 Specification

```c
void nag_rngs_arma_time_series (Integer mode, double xmean, Integer p,
cont double phi[], Integer q, const double theta[], double avar, double var,
Integer n, double x[], Integer igen, Integer iseed[], double r[], NagError *fail)
```

3 Description

Let the vector \( x_t \), denote a time series which is assumed to follow an autoregressive moving average (ARMA) model of the form:

\[
x_t = \mu + \phi_0(x_{t-1} - \mu) + \phi_1(x_{t-2} - \mu) + \ldots + \phi_p(x_{t-p} - \mu) + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \ldots - \theta_q \epsilon_{t-q}
\]

where \( \epsilon_t \) is a residual series of independent random perturbations assumed to be Normally distributed with zero mean and variance \( \sigma^2 \). The parameters \( \{\phi_i\} \), for \( i = 1, 2, \ldots, p \), are called the autoregressive (AR) parameters, and \( \{\theta_j\} \), for \( j = 1, 2, \ldots, q \), the moving average (MA) parameters. The parameters in the model are thus the \( p \) \( \phi \)-values, the \( q \) \( \theta \)-values, the mean \( \mu \) and the residual variance \( \sigma^2 \).

nag_rngs_arma_time_series (g05pac) sets up a reference vector containing initial values corresponding to a stationary position using the method described in Tunnicliffe–Wilson (1979). The function can then return a realisation of \( x_1, x_2, \ldots, x_n \). On a successful exit, the recent history is updated and saved in the reference vector \( r \) so that nag_rngs_arma_time_series (g05pac) may be called again to generate a realisation of \( x_{n+1}, x_{n+2}, \ldots \). See the description of the parameter \( mode \) in Section 5 for details.

One of the initialisation functions nag_rngs_init_repeatable (g05kbc) (for a repeatable sequence if computed sequentially) or nag_rngs_init_nonrepeatable (g05kcc) (for a non-repeatable sequence) must be called prior to the first call to nag_rngs_arma_time_series (g05pac).

4 References


5 Parameters

1:  \( mode \) – Integer

\( Input \)

\( On \ entry: \) a code for selecting the operation to be performed by the function:

\( mode = 0 \)

Set up reference vector only.

\( mode = 1 \)

Generate terms in the time series using reference vector set up in a prior call to nag_rngs_arma_time_series (g05pac).
mode = 2
Set up reference vector and generate terms in the time series.

Constraint: $0 \leq \text{mode} \leq 2$.

2: xmean – double
On entry: the mean of the time series.

3: p – Integer
On entry: the number of autoregressive coefficients supplied, $p$.
Constraint: $p \geq 0$.

4: phi[dim] – const double
Input
Note: the dimension, dim, of the array phi must be at least max(1, p).
On entry: the autoregressive coefficients of the model, $\phi_1, \phi_2, \ldots, \phi_p$.

5: q – Integer
On entry: the number of moving average coefficients supplied, $q$.
Constraint: $q \geq 0$.

6: theta[dim] – const double
Input
Note: the dimension, dim, of the array theta must be at least max(1, q).
On entry: the moving average coefficients of the model, $\theta_1, \theta_2, \ldots, \theta_q$.

7: avar – double
On entry: the variance of the normal perturbations, $\sigma^2$.
Constraint: $avar \geq 0.0$.

8: var – double *
Output
On exit: the proportion of the variance of a term in the series that is due to the moving-average (error) terms in the model. The smaller this is, the nearer is the model to non-stationarity.

9: n – Integer
Input
On entry: the number of observations to be generated, $n$.
Constraint: $n \geq 0$.

10: x[dim] – double
Output
Note: the dimension, dim, of the array x must be at least max(1, n).
On exit: contains the next $n$ observations from the time series.

11: igen – Integer
Input
On entry: must contain the identification number for the generator to be used to return a pseudo-random number and should remain unchanged following initialisation by a prior call to one of the functions nag_rngs_init_repeatable (g05kbc) or nag_rngs_init_nonrepeatable (g05kcc).

Input/Output
On entry: contains values which define the current state of the selected generator.
On exit: contains updated values defining the new state of the selected generator.
13: \( r[dim] \) – double

Note: the dimension, \( dim \), of the array \( r \) must be at least \( p + q + 5 + \max(p, q + 1) \).

On exit: the reference vector.

14: fail – NagError *

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, \( p = \langle \text{value} \rangle \).
Constraint: \( p \geq 0 \).

On entry, \( q = \langle \text{value} \rangle \).
Constraint: \( q \geq 0 \).

On entry, \( \text{mode} = \langle \text{value} \rangle \).
Constraint: \( 0 \leq \text{mode} \leq \).

On entry, \( n = \langle \text{value} \rangle \).
Constraint: \( n \geq 0 \).

NE_REAL

On entry, \( \text{avar} = \langle \text{value} \rangle \).
Constraint: \( \text{avar} \geq 0.0 \).

NE_STATIONARY_AR

\( \phi \) does not define a stationary autoregresive process.

NE_BAD_PARAM

On entry, parameter \( \langle \text{value} \rangle \) had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The errors in the initialisation process should be very much smaller than the error term; see Tunnicliffe–Wilson (1979).

8 Further Comments

The time taken by nag_rngs arma_time_series (g05pac) is essentially of order \( (p)^2 \).

Note: nag_rngs_init_repeatable (g05kbc) and nag_rngs_init_nonrepeatable (g05kcc) must be used with care if this function is used as well. The reference vector, as mentioned before, contains a copy of the recent history of the series. This will not be altered properly by calls to any of the above functions. A call to nag_rngs_init_repeatable (g05kbc) or nag_rngs_init_nonrepeatable (g05kcc) should be followed by a call to nag_rngs arma_time_series (g05pac) with \( \text{mode} = 0 \) to re-initialise the time series reference vector in use. To maintain repeatability with nag_rngs_init_repeatable (g05kbc), the calls to nag_rngs arma_time_series (g05pac) should be performed in the same order and at the same point or points in the simulation every time nag_rngs_init_repeatable (g05kbc) is used. When the generator state is saved and restored using the parameters \( \text{igen} \) and \( \text{iseed} \), the time series reference vector must be saved and restored as well.
The ARMA model for a time series can also be written as:

\[ (x_n - E) = A_1(x_{n-1} - E) + \cdots + A_{NA}(x_{n-NA} - E) + B_1a_n + \cdots + B_{NB}a_{n-NB+1} \]

where

- \( x_n \) is the observed value of the time series at time \( n \),
- \( NA \) is the number of autoregressive parameters, \( A_i \),
- \( NB \) is the number of moving average parameters, \( B_i \),
- \( E \) is the mean of the time series,

and

- \( a_i \) is a series of independent random Standard Normal perturbations.

This is related to the form given in Section 3 by:

\[ B_i^2 = \sigma^2, \]
\[ B_{i+1} = -\theta_i \sigma = -\theta_i B_i, \quad i = 1, 2, \ldots, q, \]
\[ NB = q + 1, \]
\[ E = \epsilon, \]
\[ A_i = \phi_i, \quad i = 1, 2, \ldots, p, \]
\[ NA = p. \]

9 Example

This example program calls \texttt{nag_rngs_arma_time_series (g05pac)} to set up the reference vector for an autoregressive model after initialisation by \texttt{nag_rngs_init-repeatable (g05kbc)}. The model is given by

\[ x_t = 0.4x_{t-1} + 0.2x_{t-2} + \epsilon_t \]

where \( \epsilon_t \) is a series of independent random Normal perturbations with variance 1.0. \texttt{nag_rngs_arma_time_series (g05pac)} is then called generate a sample of ten observations, which are printed.

9.1 Program Text

/* nag_rngs_arma_time_series (g05pac) Example Program. */
* Copyright 2001 Numerical Algorithms Group.
* * Mark 7, 2001. */

#include <stdio.h>
#include <nag.h>
#include <nagg05.h>

int main(void)
{
    /* Scalars */
    double avar, var, xmean;
    Integer i, igen, ip, iq, n, nr;
    NagError fail;
    /* Arrays */
    double *phi=0, *r=0, *theta=0, *x=0;
    Integer iseed[4];

    INIT_FAIL(fail);
    Vprintf("g05pac Example Program Results\n\n");
ip=2;
iq=0;
n=10;
nr=ip+iq+5+ip;

/* allocate memory */
if ( !(phi = NAG_ALLOC(ip, double)) ||
    !(r = NAG_ALLOC(nr, double)) ||
    !(theta = NAG_ALLOC(1, double)) ||
    !(x = NAG_ALLOC(n, double)) )
{
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Set the ARMA model parameters */
xmean = 0.0;
phi[0] = 0.4;
phi[1] = 0.2;
avar = 1.0;

/* Initialise the seed to a repeatable sequence */
iseed[0] = 1762543;
iseed[1] = 9324783;
iseed[2] = 4234401;
iseed[3] = 742355;
 /* igen identifies the stream. */
igen = 1;
g05kbc(&igen, iseed);

/* Set up the reference vector */
g05pac(0, xmean, ip, phi, iq, theta, avar, &var, n, x, igen,
    iseed, r, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g05pac.
    %s
", fail.message);
    exit_status = 1;
    goto END;
}

/* Generate a sample of 10 observations */
g05pac(1, xmean, ip, phi, iq, theta, avar, &var, n, x, igen,
    iseed, r, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g05pac.
    %s
", fail.message);
    exit_status = 1;
    goto END;
}
for (i = 0; i < n; ++i)
{
    Vprintf("%12.4f\n", x[i]);
}

END:
if (phi) NAG_FREE(phi);
if (r) NAG_FREE(r);
if (theta) NAG_FREE(theta);
if (x) NAG_FREE(x);
return exit_status;
}

9.2 Program Data

None.
9.3 Program Results

g05pac Example Program Results

-1.0654
-0.2828
-2.0924
-2.3304
-2.5998
-1.7143
-2.4882
-1.3882
-2.2722
-1.8806