NAG C Library Function Document

nag_prob_non_central_f_dist (g01gdc)

1 Purpose

nag_prob_non_central_f_dist (g01gdc) returns the probability associated with the lower tail of the non-central F or variance-ratio distribution.

2 Specification

#include <nag.h>
#include <nag01.h>

double nag_prob_non_central_f_dist(double f, double df1, double df2, 
double lambda, double tol, Integer max_iter, NagError *fail)

3 Description

The lower tail probability of the non-central F-distribution with \( \nu_1 \) and \( \nu_2 \) degrees of freedom and non-centrality parameter \( \lambda \), \( P(F \leq f : \nu_1, \nu_2; \lambda) \), is defined by

\[
P(F \leq f : \nu_1, \nu_2; \lambda) = \int_0^f p(F : \nu_1, \nu_2; \lambda) dF
\]

where

\[
P(F : \nu_1, \nu_2; \lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{(\lambda/2)^j}{j!} \times \frac{(\nu_1 + 2j)^{(n+2)/2} \nu_2^{j/2}}{B((\nu_1 + 2j)/2, \nu_2/2)}
\]

\[
\times u^{(n+2j-2)/2} [\nu_2 + (\nu_1 + 2j)u]^{-(\nu_1+2j+\nu_2)/2}
\]

and \( B(\cdot, \cdot) \) is the beta function.

The probability is computed by means of a transformation to a non-central beta distribution;

\[
P(F \leq f : \nu_1, \nu_2; \lambda) = P_\beta(X \leq x : a, b; \lambda)
\]

where \( x = \frac{nf}{\nu_1/\nu_2} \) and \( P_\beta(X \leq x : a, b; \lambda) \) is the lower tail probability integral of the non-central beta distribution with parameters \( a, b, \) and \( \lambda \).

If \( \nu_2 \) is very large, greater than \( 10^6 \), then a \( \chi^2 \) approximation is used.

4 Parameters

1: \( f \) – double

On entry: the deviate from the non-central F-distribution, \( f \).

Constraint: \( f > 0 \).

2: \( df1 \) – double

On entry: the degrees of freedom of the numerator variance, \( \nu_1 \).

Constraint: \( 0.0 < df1 \leq 1.0e6 \).

3: \( df2 \) – double

On entry: the degrees of freedom of the denominator variance, \( \nu_2 \).

Constraint: \( df2 > 0.0 \).

[NP3491/6]
4:  lambda – double
    
    Input
    
    On entry: the non-centrality parameter, \( \lambda \).
    
    Constraint: 0.0 \leq \lambda \leq -2.0 \times \log(\text{safe range parameter}) \text{ where } U \text{ is the safe range parameter as defined by nag_real_safe_small_number (X02AMC).}

5:  tol – double
    
    Input
    
    On entry: the relative accuracy required by the user in the results. If nag_prob_non_central_f_dist is entered with tol greater than or equal to 1.0 or less than 10 \times \text{machine precision} (see nag_machine_precision (X02AJC)), then the value of 10 \times \text{machine precision} is used instead.

6:  max_iter – Integer
    
    Input
    
    On entry: the maximum number of iterations to be used.
    
    Suggested value: 500. See nag_prob_non_central_chi_sq (g01gcc) and nag_prob_non_central_beta_dist (g01gec) for further details.
    
    Constraint: max_iter \geq 1.

7:  fail – NagError *
    
    Input/Output
    
    The NAG error parameter (see the Essential Introduction).

5  Error Indicators and Warnings

NE_REAL_ARG_CONS
    
    On entry, df1 = <value>.
    
    This parameter must satisfy 0.0 < df1 \leq 1.0e6.
    
    On entry, lambda = <value>.
    
    This parameter must satisfy 0.0 \leq lambda \leq -2.0 \times \log(X02AMC).

NE_REAL_ARG_LE
    
    On entry, df2 must not be less than or equal to 0.0: df2 = <value>.
    
    On entry, f must not be less than or equal to 0.0: f = <value>.

NE_INT_ARG_LT
    
    On entry, max_iter must not be less than 1: max_iter = <value>.

NE_CONV
    
    The solution has failed to converge in <value> iterations, consider increasing max_iter or tol.

NE_PROB_F
    
    The required probability cannot be computed accurately. This may happen if the result would be very close to zero or one. Alternatively the values of df1 and f may be too large. In the latter case the user could try using a normal approximation, see Abramowitz and Stegun (1972).

NE_PROB_F_INIT
    
    The required accuracy was not achieved when calculating the initial value of the central \( F \) or \( \chi^2 \) probability. The user should try a larger value of tol. If the \( \chi^2 \) approximation is being used then nag_prob_non_central_f_dist returns zero otherwise the value returned should be an approximation to the correct value.
NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments

When both \( \nu_1 \) and \( \nu_2 \) are large a normal approximation may be used and when only \( \nu_1 \) is large a \( \chi^2 \) approximation may be used. In both cases \( \lambda \) is required to be of the same order as \( \nu_1 \). See Abramowitz and Stegun Abramowitz and Stegun (1972) for further details.

6.1 Accuracy

The relative accuracy should be as specified by tol. For further details see nag_prob_non_central_chi_sq (g01gcc) and nag_prob_non_central_beta_dist (g01gec).

6.2 References


7 See Also

nag_prob_non_central_chi_sq (g01gcc)
nag_prob_non_central_beta_dist (g01gec)

8 Example

Values from, and degrees of freedom for \( F \)-distributions are read, the lower-tail probabilities computed, and all these values printed, until the end of data is reached.

8.1 Program Text

/* nag_prob_non_central_f_dist (g0lgdc) Example Program.
 * Copyright 2000 Numerical Algorithms Group.
 * NAG C Library
 * Mark 6, 2000.
 */

#include <stdio.h>
#include <nag.h>
#include <nag01.h>

int main(void)
{
  double df1, df2, f, prob, lambda, tol;
  Integer max_iter;
  Integer exit_status=0;
  NagError fail;

  INIT_FAIL(fail);
  Vprintf("g0lgdc Example Program Results\n");

  /* Skip heading in data file */
  Vscanf("%*[\n]"
Vprintf("\n  f  df1  df2  lambda  prob\n\n");
tol = 5e-6;
max_iter = 50;
while ((scanf("%lf %lf %lf %lf %*[\n]", &f, &df1, &df2, &lambda)) != EOF)
{
  prob = g01gdc(f, df1, df2, lambda, tol, max_iter, &fail);
  if (fail.code != NE_NOERROR)
  {
    Vprintf("Error from g01gdc.\n\n", fail.message);
    exit_status=1;
    goto END;
  }
  Vprintf("%8.3f %8.3f %8.3f %8.3f %8.4f\n", f, df1, df2, lambda, prob);
}
END:
return exit_status;

8.2 Program Data

G01gdc Example Program Data

<table>
<thead>
<tr>
<th>f</th>
<th>df1</th>
<th>df2</th>
<th>lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td>1.5</td>
<td>25.5</td>
<td>3.0</td>
</tr>
<tr>
<td>39.9</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>2.5</td>
<td>20.25</td>
<td>1.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

8.3 Program Results

G01gdc Example Program Results

<table>
<thead>
<tr>
<th>f</th>
<th>df1</th>
<th>df2</th>
<th>lambda</th>
<th>prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.500</td>
<td>1.500</td>
<td>25.500</td>
<td>3.000</td>
<td>0.8214</td>
</tr>
<tr>
<td>39.900</td>
<td>1.000</td>
<td>1.000</td>
<td>2.000</td>
<td>0.8160</td>
</tr>
<tr>
<td>2.500</td>
<td>20.25</td>
<td>1.000</td>
<td>0.000</td>
<td>0.5342</td>
</tr>
</tbody>
</table>