NAG C Library Function Document

nag_prob_non_central_chisq (g01gcc)

1 Purpose

nag_prob_non_central_chisq (g01gcc) returns the probability associated with the lower tail of the non-central $\chi^2$ distribution.

2 Specification

#include <nag.h>
#include <nag01.h>

double nag_prob_non_central_chisq (double x, double df, double lambda,
       double tol, Integer max_iter, NagError *fail)

3 Description

The lower tail probability of the non-central $\chi^2$ distribution with $\nu$ degrees of freedom and non-centrality parameter $\lambda$, $P(X \leq x : \nu; \lambda)$, is defined by

$$P(X \leq x : \nu; \lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{\lambda^j}{j!} P(X \leq x + 2j; 0)$$

(1)

where $P(X \leq x + 2j; 0)$ is a central $\chi^2$ with $\nu + 2j$ degrees of freedom.

The value of $j$ at which the Poisson weight, $e^{-\lambda/2} \frac{(\lambda/2)^j}{j!}$, is greatest is determined and the summation (1) is made forward and backward from that value of $j$.

The recursive relationship:

$$P(X \leq x : a + 2; 0) = P(X \leq x : a; 0) - \frac{(x^a/2)e^{-x/2}}{\Gamma(a + 1)}$$

(2)

is used during the summation in (1).

4 Parameters

1: \textbf{x} – double \hspace{1cm} \textit{Input}

\textit{On entry:} the deviate from the non-central $\chi^2$ distribution with $\nu$ degrees of freedom and non-centrality parameter $\lambda$.

\textit{Constraint:} x $\geq$ 0.0.

2: \textbf{df} – double \hspace{1cm} \textit{Input}

\textit{On entry:} the degrees of freedom, $\nu$, of the non-central $\chi^2$ distribution.

\textit{Constraint:} df $\geq$ 0.0.

3: \textbf{lambda} – double \hspace{1cm} \textit{Input}

\textit{On entry:} the non-centrality parameter, $\lambda$, of the non-central $\chi^2$ distribution.

\textit{Constraint:} lambda $\geq$ 0.0 if df > 0.0 or lambda > 0.0 if df = 0.0.
4:   tol – double

    Input
    On entry: the required accuracy of the solution. If nag_prob_non_central_chisq is entered with toler greater than or equal to 1.0 or less than $10 \times \text{machine precision}$ (see nag_machine_precision (X02AJC)), then the value of $10 \times \text{machine precision}$ is used instead.

5:   max_iter – Integer

    Input
    On entry: the maximum number of iterations to be performed.
    Suggested value: 100. See Section 6 for further discussion.
    Constraint: $\text{max_iter} \geq 1$.

6:   fail – NagError *

    Input/Output
    The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

NE_REAL_ARG_LT

    On entry, df must not be less than 0.0: $df = <value>$.
    On entry, lambda must not be less than 0.0: $\lambda = <value>$.
    On entry, $x$ must not be less than 0.0: $x = <value>$.

NE_2_REAL_ARG_CONS

    On entry, $df = <value>$ while $\lambda = <value>$.
    These parameters must satisfy $\lambda > 0.0$ if $df = 0.0$.

NE_INT_ARG_LT

    On entry, $\text{max_iter}$ must not be less than 1: $\text{max_iter} = <value>$.

NE_POISSON_WEIGHT

    The initial value of the Poisson weight used in the summation of (1) (see Section 3) was too small to be calculated. The computed probability is likely to be zero.

NE_CONV

    The solution has failed to converge in $<value>$ iterations, consider increasing $\text{max_iter}$ or $\text{tol}$.

NE_TERM_LARGE

    The value of a term required in (2) (see Section 3) is too large to be evaluated accurately. The most likely cause of this error is both $x$ and $\lambda$ are too large.

NE_CHI_PROB

    The calculations for the central chi-square probability has failed to converge. A larger value of $\text{tol}$ should be used.

NE_INTERNAL_ERROR

    An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.
6 Further Comments
The number of terms in (1) (see Section 3) required for a given accuracy will depend on the following factors:
(i) The rate at which the Poisson weights tend to zero. This will be slower for larger values of \( \lambda \).
(ii) The rate at which the central \( \chi^2 \) probabilities, tend to zero. This will be slower for larger values of \( \nu \) and \( x \).

6.1 Accuracy
The summations described in Section 3 are made until an upper bound on the truncation error relative to the current summation value is less than tol.

6.2 References

7 See Also
None.

8 Example
Values from various non-central \( \chi^2 \) distributions are read, the lower-tail probabilities calculated, and all these values printed out, until the end of data is reached.

8.1 Program Text
/* nag_prob_non_central_chisq (g01gcc) Example Program. */
/* Copyright 1999 Numerical Algorithms Group. */
/* Mark 6, 2000. */

#include <stdio.h>
#include <nag.h>
#include <nag01.h>

int main(void)
{

double df, prob, lambda, tol, x;
Integer max_iter;
Integer exit_status=0;
NagError fail;

INIT_FAIL(fail);
Vprintf("g01gcc Example Program Results\n\n");

/* Skip heading in data file */
Vscanf("%*[\n]";

Vprintf("\n x df lambda prob\n\n");
tol = 5e-6;
max_iter = 50;

while ((scanf(" %lf %lf %lf *\n", &x, &df, &lambda)) != EOF)
{
    prob = g01gcc(x, df, lambda, tol, max_iter, &fail);
    if (fail.code == NE_NOERROR)
    {
        Vprintf("%8.3f %8.3f %8.3f %8.4f\n", x, df, lambda, prob);
    }
    else
    {
        Vprintf("Error from g01gcc.\n\n", fail.message);
        exit_status = 1;
        goto END;
    }
}
END:
return exit_status;

8.2 Program Data

g01gcc Example Program Data

<table>
<thead>
<tr>
<th>x</th>
<th>df</th>
<th>lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.26</td>
<td>20.0</td>
<td>3.5</td>
</tr>
<tr>
<td>6.2</td>
<td>7.5</td>
<td>2.0</td>
</tr>
<tr>
<td>55.76</td>
<td>45.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

8.3 Program Results

g01gcc Example Program Results

<table>
<thead>
<tr>
<th>x</th>
<th>df</th>
<th>lambda</th>
<th>prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.260</td>
<td>20.000</td>
<td>3.500</td>
<td>0.0032</td>
</tr>
<tr>
<td>6.200</td>
<td>7.500</td>
<td>2.000</td>
<td>0.2699</td>
</tr>
<tr>
<td>55.760</td>
<td>45.000</td>
<td>1.000</td>
<td>0.8443</td>
</tr>
</tbody>
</table>