NAG C Library Function Document

nag_prob_non_central_students_t (g01gbc)

1 Purpose

nag_prob_non_central_students_t (g01gbc) returns the lower tail probability for the non-central Student’s t-distribution.

2 Specification

#include <nag.h>
#include <nag01.h>

double nag_prob_non_central_students_t (double t, double df, double delta,
        double tol, Integer max_iter, NagError *fail)

3 Description

The lower tail probability of the non-central Student’s t-distribution with \( \nu \) degrees of freedom and non-centrality parameter \( \delta \), \( P(T \leq t : \nu; \delta) \) is defined by:

\[
P(T \leq t : \nu; \delta) = C_{\nu} \int_{0}^{\infty} \left( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha \nu-\delta} e^{-x^2/2} \, dx \right) u^{\nu-1} e^{-u^2/2} \, du, \quad \nu > 0.0
\]

with

\[
C_{\nu} = \frac{1}{\Gamma(\nu/2)2^{(\nu-2)/2}}, \quad \alpha = \frac{t}{\sqrt{\nu}}
\]

The probability is computed in one of two ways,

(a) when \( t = 0.0 \), the relationship to the normal is used

\[
P(T \leq t : \nu; \delta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-u^2/2} \, du;
\]

(b) otherwise the series expansion described in Amos (1964) (equation 9) is used. This involves the sums of confluent hypergeometric functions, the terms of which are computed using recurrence relationships.

4 Parameters

1: \( t \) – double

Input

On entry: the deviate from the Student’s t-distribution with \( \nu \) degrees of freedom, \( t \).

2: \( df \) – double

Input

On entry: the degrees of freedom of the Student’s t-distribution, \( \nu \).

Constraint: \( df \geq 1.0 \).

3: \( delta \) – double

Input

On entry: the non-centrality parameter of the Students t-distribution, \( \delta \).
4:  tol – double  
   \textbf{Input}
   
   \textit{On entry}: the absolute accuracy required by the user in the results.
   
   If \texttt{nag\_prob\_non\_central\_students\_t} is entered with \texttt{tol} greater than or equal to 1.0 or less than
   \texttt{10\times machine\ precision} (see \texttt{nag\_machine\_precision} (X02AJC)), then the value of \texttt{10\times machine\ precision} is used instead.

5:  \texttt{max\_iter} – Integer  
   \textbf{Input}
   
   \textit{On entry}: the maximum number of terms that are used in each of the summations.
   
   \textit{Suggested value}: 100. See Section 6 for further comments.
   
   \textit{Constraint}: \texttt{max\_iter} \geq 1.

6:  fail – NagError *  
   \textbf{Input/Output}
   
   The NAG error parameter (see the Essential Introduction).

5  \textbf{Error Indicators and Warnings}

\textbf{NE\_REAL\_ARG\_LT}

\textit{On entry}, \texttt{df} must not be less than 1.0: \texttt{df = <value>}

\textbf{NE\_INT\_ARG\_LT}

\textit{On entry}, \texttt{max\_iter} must not be less than 1: \texttt{max\_iter = <value>}

\textbf{NE\_SERIES}

One of the series has failed to converge with \texttt{df = <value> and max\_iter = <value>}. Reconsider
the requested tolerance and/or the maximum number of iterations.

\textbf{NE\_PROBABILITY}

The probability is too small to calculate accurately.

\textbf{NE\_INTERNAL\_ERROR}

An internal error has occurred in this function. Check the function call and any array sizes. If the
call is correct then please consult NAG for assistance.

6  \textbf{Further Comments}

The rate of convergence of the series depends, in part, on the quantity: \(t^2/(t^2 + \nu)\). The smaller this
quantity the faster the convergence. Thus for large \(t\) and small \(\nu\) the convergence may be slow. If \(\nu\) is an
integer then one of the series to be summed is of finite length.

If two tail probabilities are required then the relationship of the \(t\)-distribution to the \(F\)-distribution can be
used:

\[
F = T^2, \quad \lambda = \delta^2, \quad \nu_1 = 1 \quad \text{and} \quad \nu_2 = \nu,
\]

and a call made to \texttt{nag\_prob\_non\_central\_f\_dist} (g01gdc).

\textbf{Note}: this routine only allows degrees of freedom greater than or equal to 1 although values between 0 and
1 are theoretically possible.

6.1  \textbf{Accuracy}

The series described in Amos (1964) are summed until an estimated upper bound on the contribution of
future terms to the probability is less than \texttt{tol}. There may also be some slight loss of accuracy due to
calculation of gamma functions. For large values of \(\delta > 50\) there may be significant loss of accuracy.
6.2 References
Amos D E (1964) Representations of the central and non-central \( t \)-distributions *Biometrika* **51** 451–458

7 See Also
nag_prob_non_central_students_t (g01gbc)

8 Example
Values from, and degrees of freedom for and non-centrality parameter of the non-central Student’s \( t \)-distributions are read, the lower tail probabilities calculated and all these values printed until the end of data is reached.

8.1 Program Text

```c
/* nag_prob_non_central_students_t (g01gbc) Example Program. *
 * Copyright 1999 Numerical Algorithms Group.
 * * Mark 6, 2000.
 */
#include <stdio.h>
#include <nag.h>
#include <nag01.h>

int main(void)
{
    double delta, df, prob, t, tol;
    Integer max_iter;
    Integer exit_status = 0;
    NagError fail;

    INIT_FAIL(fail);
    Vprintf("g01gbc Example Program Results\n\n");

    /* Skip heading in data file */
    Vscanf("%*\n");

    Vprintf(" t df delta prob\n\n");
    tol = 5e-6;
    max_iter = 50;
    while ( ((scanf("%lf %lf %lf %*[\n]", &t, &df, &delta)) != EOF)
    {
        prob = g01gbc(t, df, delta, tol, max_iter, &fail);
        if (fail.code == NE_NOERROR)
        Vprintf(" %8.3f%8.3f%8.3f%8.4f\n", t, df, delta, prob);
        else
        { Vprintf("Error from g01gbc.\n\n", fail.message);
            exit_status=1;
            goto END;
        }
    }

    return exit_status;
}
```

[NP3491/6] g01gbc.3
8.2 Program Data

g01gbc Example Program Data
-1.528 20.0 2.0 :t df delta
-0.188 7.5 1.0 :t df delta
1.138 45.0 0.0 :t df delta

8.3 Program Results

g01gbc Example Program Results

<table>
<thead>
<tr>
<th>t</th>
<th>df</th>
<th>delta</th>
<th>prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.528</td>
<td>20.000</td>
<td>2.000</td>
<td>0.0003</td>
</tr>
<tr>
<td>-0.188</td>
<td>7.500</td>
<td>1.000</td>
<td>0.1189</td>
</tr>
<tr>
<td>1.138</td>
<td>45.000</td>
<td>0.000</td>
<td>0.8694</td>
</tr>
</tbody>
</table>