NAG C Library Function Document

nag_prob_von_mises (g01erc)

1 Purpose

nag_prob_von_mises (g01erc) returns the probability associated with the lower tail of the von Mises distribution between \(-\pi\) and \(\pi\).

2 Specification

double nag_prob_von_mises (double t, double_vk, NagError *fail)

3 Description

The von Mises distribution is a symmetric distribution used in the analysis of circular data. The lower tail area of this distribution on the circle with mean direction \(\mu_0 = 0\) and concentration parameter \(\kappa\), can be written as

\[
\Pr(\Theta \leq \theta : \kappa) = \frac{1}{2\pi I_0(\kappa)} \int_{-\pi}^{\theta} e^{\kappa \cos \Theta} d\Theta,
\]

where \(\theta\) is reduced modulo \(2\pi\) so that \(-\pi \leq \theta < \pi\) and \(\kappa \geq 0\). Note that if \(\theta = \pi\) then nag_prob_von_mises (g01erc) returns a probability of 1. For very small \(\kappa\) the distribution is almost the uniform distribution, whereas for \(\kappa \to \infty\) all the probability is concentrated at one point.

The method of calculation for small \(\kappa\) involves backwards recursion through a series expansion in terms of modified Bessel functions, while for large \(\kappa\) an asymptotic Normal approximation is used.

In the case of small \(\kappa\) the series expansion of \(\Pr(\Theta \leq \theta : \kappa)\) can be expressed as

\[
\Pr(\Theta \leq \theta : \kappa) = \frac{1}{2} + \frac{\theta}{(2\pi)} + \frac{1}{\pi I_0(\kappa)} \sum_{n=1}^{\infty} n^{-1} I_n(\kappa) \sin n\theta,
\]

where \(I_n(\kappa)\) is the modified Bessel function. This series expansion can be represented as a nested expression of terms involving the modified Bessel function ratio \(R_n\),

\[
R_n(\kappa) = \frac{I_n(\kappa)}{I_{n-1}(\kappa)}, \quad n = 1, 2, 3, \ldots,
\]

which is calculated using backwards recursion.

For large values of \(\kappa\) (see Section 7) an asymptotic Normal approximation is used. The angle \(\Theta\) is transformed to the nearly Normally distributed variate \(Z\),

\[
Z = b(\kappa) \sin \frac{\Theta}{2},
\]

where

\[
b(\kappa) = \frac{\sqrt{2/\kappa}}{I_0(\kappa)}
\]

and \(b(\kappa)\) is computed from a continued fraction approximation. An approximation to order \(\kappa^{-4}\) of the asymptotic normalizing series for \(z\) is then used. Finally the Normal probability integral is evaluated.

For a more detailed analysis of the methods used see Hill (1977).

4 References


5 Parameters

1: \( t \) – double \( \quad \text{Input} \)
   \( On \ entry: \) the observed von Mises statistic, \( \theta \), measured in radians.

2: \( v_k \) – double \( \quad \text{Input} \)
   \( On \ entry: \) the concentration parameter \( \kappa \), of the von Mises distribution.
   \( Constraint: \ v_k \geq 0. \)

3: \( \text{fail} \) – NagError * \( \quad \text{Input/Output} \)
   The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

\textbf{NE_REAL}

On entry, \( v_k = \langle \text{value} \rangle \).
Constraint: \( v_k \geq 0.0 \).

\textbf{NE_BAD_PARAM}

On entry, parameter \( \langle \text{value} \rangle \) had an illegal value.

\textbf{NE_INTERNAL_ERROR}

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

\textit{nag_prob_von_mises (g01erc)} uses one of two sets of constants depending on the value of machine precision. One set gives an accuracy of six digits and uses the Normal approximation when \( v_k \geq 6.5 \), the other gives an accuracy of 12 digits and uses the Normal approximation when \( v_k \geq 50 \).

8 Further Comments

Using the series expansion for small \( \kappa \) the time taken by \textit{nag_prob_von_mises (g01erc)} increases linearly with \( \kappa \); for larger \( \kappa \), for which the asymptotic Normal approximation is used, the time taken is much less.

If angles outside the region \( -\pi \leq \theta < \pi \) are used care has to be taken in evaluating the probability of being in a region \( \theta_1 \leq \theta \leq \theta_2 \) if the region contains an odd multiple of \( \pi \), \( (2n + 1)\pi \). The value of \( F(\theta_2; \kappa) - F(\theta_1; \kappa) \) will be negative and the correct probability should then be obtained by adding one to the value.

9 Example

Four values from the von Mises distribution along with the values of the parameter \( \kappa \) are input and the probabilities computed and printed.
9.1 Program Text

/* nag_prob_von_mises (g01erc) Example Program. */
/* Copyright 2001 Numerical Algorithms Group. */
/* Mark 7, 2001. */
#include <stdio.h>
#include <nag.h>
#include <nagg01.h>
#include <nag_stdlib.h>

int main(void)
{
    /* Scalars */
    double p, t, vk;
    Integer exit_status, i__, n;
    NagError fail;

    INIT_FAIL(fail);
    exit_status = 0;

    Vprintf("%s
\n", "g01erc Example Program Results");

    /* Skip heading in data file */
    Vscanf("%*[^
\] ");
    Vscanf("%ld%*[^
\] ", &n);

    for (i__ = 1; i__ <= n; ++i__)
    {
        Vscanf("%lf%lf%*[^
\] ", &t, &vk);
        p = g01erc(t, vk, &fail);
        if (fail.code == NE_NOERROR)
        {
            Vprintf(" p = %10.4f\n", p);
        }
        else
        {
            Vprintf("Error from g01erc.\n\n", fail.message);
            exit_status = 1;
            goto END;
        }
    }

    END:
    return exit_status;
}

9.2 Program Data

g01erc Example Program Data

4
7.0 0.0
2.8 2.4
1.0 1.0
-1.4 1.3

9.3 Program Results

g01erc Example Program Results

<table>
<thead>
<tr>
<th>p</th>
<th>0.6141</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>0.9983</td>
</tr>
<tr>
<td>p</td>
<td>0.7944</td>
</tr>
<tr>
<td>p</td>
<td>0.1016</td>
</tr>
</tbody>
</table>