nag_deviates_normal_dist (g01cec)

1. Purpose
nag_deviates_normal_dist (g01cec) returns the deviate, \( x_p \), associated with the given lower tail probability, \( p \), of the standardised Normal distribution.

2. Specification
#include <nag.h>
#include <nagg01.h>
double nag_deviates_normal_dist(double p, NagError *fail)

3. Description
\( x_p \) is calculated for the given \( p \) such that
\[
p = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x_p} e^{-u^2/2} du \quad -\infty < x_p < \infty.
\]
The method used is an extension of that of Beasley and Springer (1977). \( p \) is first replaced by \( q = p - 0.5 \).

(a) If \(|q| \leq 0.3\), \( x_p \) is computed by a rational Chebyshev approximation
\[
x_p = s \frac{A(s^2)}{B(s^2)}
\]
where \( s = \sqrt{2\pi q} \) and \( A, B \) are polynomials of degree 7.

(b) If \( 0.3 < |q| \leq 0.42 \), \( x_p \) is computed by a rational Chebyshev approximation
\[
x_p = \text{sign } q \left( \frac{C(t)}{D(t)} \right)
\]
where \( t = |q| - 0.3 \) and \( C, D \) are polynomials of degree 5.

(c) If \(|q| > 0.42\), \( x_p \) is computed as
\[
x_p = \text{sign } q \left( \frac{E(u)}{F(u)} \right) + u
\]
where \( u = \sqrt{-2 \log(\min(p,1-p))} \) and \( E, F \) are polynomials of degree 6.

4. Parameters
\( p \)
Input: the probability, \( p \), from the standardised Normal distribution.
Constraint: \( 0.0 < p < 1.0 \).

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings
NE_REAL_ARG_LE
On entry, \( p \) must not be less than or equal to 0.0: \( p = \langle \text{value} \rangle \).

NE_REAL_ARG_GE
On entry, \( p \) must not be greater than or equal to 1.0: \( p = \langle \text{value} \rangle \).
6. Further Comments
If $X$ is a Normal random variable with mean $\mu$ and variance $\sigma^2$, the deviate corresponding to a lower tail probability of $p$ is $\mu + \sigma x_p$, where $x_p$ is the standardised Normal deviate returned by nag_deviates_normal_dist.

6.1. Accuracy
The function attempts to attain a relative precision of $5.0 \times 10^{-13}$.

6.2. References

7. See Also
nag_cumul_normal (s15abc)
nag_cumul_normal_complem (s15acc)

8. Example
The deviates corresponding to several lower tail probabilities from the standard Normal distribution are calculated and printed.

8.1. Program Text
/* nag_deviates_normal_dist(g01cec) Example Program */
* Copyright 1990 Numerical Algorithms Group.
* Mark 2 revised, 1992.
*/
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>
define NMAX 5

main()
{
    double x;
    static double p[] = {0.95, 0.5, 0.995, 0.75, 0.001};
    Integer i;

    Vprintf("g01cec Example Program Results\n");
    Vprintf(" Prob. Deviate\n");
    for (i=0; i<NMAX; i++)
    {
        x = g01cec(p[i], NAGERR_DEFAULT);
        Vprintf("%7.3f%11.4f\n", p[i], x);
    }
    exit(EXIT_SUCCESS);
}

8.2. Program Data
None.

8.3. Program Results

g01cec Example Program Results
    Prob.    Deviate
    0.950    1.6449
    0.500    0.0000
    0.995    2.5758
    0.750    0.6745
    0.001    -3.0902

3.g01cec.2