NAG C Library Function Document

nag_zpbstf (f08utc)

1 Purpose

nag_zpbstf (f08utc) computes a split Cholesky factorization of a complex Hermitian positive-definite band matrix.

2 Specification

```c
void nag_zpbstf (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer kb,
                Complex bb[], Integer pdbb, NagError *fail)
```

3 Description

nag_zpbstf (f08utc) computes a split Cholesky factorization of a complex Hermitian positive-definite band matrix $B$. It is designed to be used in conjunction with nag_zhbgst (f08usc).

The factorization has the form $B = S^H S$, where $S$ is a band matrix of the same bandwidth as $B$ and the following structure: $S$ is upper triangular in the first $(n + k)/2$ rows, and transposed hence, lower triangular in the remaining rows. For example, if $n = 9$ and $k = 2$, then

$$
S = \begin{pmatrix}
  s_{11} & s_{12} & s_{13} & \cdots & s_{1k} \\
  s_{22} & s_{23} & s_{24} & \cdots & s_{2k} \\
  s_{33} & s_{34} & s_{35} & \cdots & s_{3k} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  s_{64} & s_{65} & s_{66} & \cdots & s_{6k} \\
  s_{75} & s_{76} & s_{77} & \cdots & s_{7k} \\
  s_{85} & s_{86} & s_{87} & \cdots & s_{8k} \\
  s_{98} & s_{99} & & & 
\end{pmatrix}.
$$

4 References

None.

5 Parameters

1: order – Nag_OrderType

*Input*

On entry: the order parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by order = Nag_RowMajor. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: order = Nag_RowMajor or Nag_ColMajor.

2: uplo – Nag_UploType

*Input*

On entry: indicates whether the upper or lower triangular part of $B$ is stored as follows:

- if uplo = Nag_Upper, the upper triangular part of $B$ is stored;
- if uplo = Nag_Lower, the lower triangular part of $B$ is stored.

Constraint: uplo = Nag_Upper or Nag_Lower.
3: n – Integer

On entry: n, the order of the matrix B.

Constraint: n ≥ 0.

4: kb – Integer

On entry: k, the number of super-diagonals of the matrix B if uplo = Nag_Upper, or the number of sub-diagonals if uplo = Nag_Lower.

Constraint: kb ≥ 0.

5: bb[dim] – Complex

Input/Output

Note: the dimension, dim, of the array bb must be at least max(1, pdbb × n).

On entry: the n by n Hermitian band matrix B. This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements b[i,j] depends on the order and uplo parameters as follows:

- If order = Nag_ColMajor and uplo = Nag_Upper,
  b[i,j] is stored in bb[k + i - j + (j - 1) × pdbb], for i = 1, ..., n and
  j = i, ..., min(n, i + k);

- If order = Nag_ColMajor and uplo = Nag_Lower,
  b[i,j] is stored in bb[i - j + (j - 1) × pdbb], for i = 1, ..., n and
  j = max(1, i - k), ..., i;

- If order = Nag_RowMajor and uplo = Nag_Upper,
  b[i,j] is stored in bb[j - i + (i - 1) × pdbb], for i = 1, ..., n and
  j = i, ..., min(n, i + k);

- If order = Nag_RowMajor and uplo = Nag_Lower,
  b[i,j] is stored in bb[k + j - i + (i - 1) × pdbb], for i = 1, ..., n and
  j = max(1, i - k), ..., i.

On exit: B is overwritten by the elements of its split Cholesky factor S.

6: pdbb – Integer

Input

On entry: the stride separating row or column elements (depending on the value of order) of the matrix B in the array bb.

Constraint: pdbb ≥ kb + 1.

7: fail – NagError *

Output

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, n = (value).

Constraint: n ≥ 0.

On entry, kb = (value).

Constraint: kb ≥ 0.

On entry, pdbb = (value).

Constraint: pdbb > 0.

NE_INT_2

On entry, pdbb = (value), kb = (value).

Constraint: pdbb ≥ kb + 1.
The factorization could not be completed, because updated element \( b(h_{\text{value}}; h_{\text{value}}) \) would be the square root of a negative number. Hence \( B \) is not positive definite. This may indicate an error in forming the matrix \( B \).

Memory allocation failed.

On entry, parameter \( h_{\text{value}} \) had an illegal value.

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

The computed factor \( S \) is the exact factor of a perturbed matrix \( B + E \), where

\[
|E| \leq c(k + 1)\varepsilon |S^H| |S|,
\]

\( c(k + 1) \) is a modest linear function of \( k + 1 \), and \( \varepsilon \) is the machine precision. It follows that

\[
|e_{ij}| \leq c(k + 1)\varepsilon \sqrt{(b_{ii}b_{jj})}.
\]

The total number of floating-point operations is approximately \( 4n(k + 1)^2 \), assuming \( n \gg k \).

A call to this function may be followed by a call to nag_zhbgst (f08usc) to solve the generalized eigenproblem \( Az = \lambda Bz \), where \( A \) and \( B \) are banded and \( B \) is positive-definite.

The real analogue of this function is nag_dpbstf (f08ufc).

See Section 9 of the document for nag_zhbgst (f08usc).