NAG C Library Function Document

nag_dpbstf (f08ufc)

1 Purpose

nag_dpbstf (f08ufc) computes a split Cholesky factorization of a real symmetric positive-definite band matrix.

2 Specification

```c
void nag_dpbstf (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer kb, double bb[], Integer pdbb, NagError *fail)
```

3 Description

nag_dpbstf (f08ufc) computes a split Cholesky factorization of a real symmetric positive-definite band matrix $B$. It is designed to be used in conjunction with nag_dsbgst (f08uec).

The factorization has the form $B = S^T S$, where $S$ is a band matrix of the same bandwidth as $B$ and the following structure: $S$ is upper triangular in the first $(n + k)/2$ rows, and transposed hence, lower triangular in the remaining rows. For example, if $n = 9$ and $k = 2$, then

$$
S = \begin{pmatrix}
  s_{11} & s_{12} & s_{13} & s_{23} & s_{24} & s_{34} & s_{35} & s_{45} & s_{55} \\
  s_{22} & s_{23} & s_{24} & s_{34} & s_{35} & s_{45} & s_{55} \\
  s_{33} & s_{34} & s_{35} & s_{45} & s_{55} \\
  s_{44} & s_{45} & s_{55} \\
  s_{55} \\
  s_{64} & s_{65} & s_{66} & s_{76} & s_{77} \\
  s_{75} & s_{76} & s_{77} & s_{88} \\
  s_{86} & s_{87} & s_{88} & s_{99} \\
  s_{97} & s_{98} & s_{99}
\end{pmatrix}.
$$

4 References

None.

5 Parameters

1: order – Nag_OrderType

```
Input
```

On entry: the order parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by order = Nag_RowMajor. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: order = Nag_RowMajor or Nag_ColMajor.

2: uplo – Nag_UploType

```
Input
```

On entry: indicates whether the upper or lower triangular part of $B$ is stored as follows:

if uplo = Nag_Upper, the upper triangular part of $B$ is stored;
if uplo = Nag_Lower, the lower triangular part of $B$ is stored.

Constraint: uplo = Nag_Upper or Nag_Lower.
3: \( n \) – Integer 
   \( n \) \text{ is the order of the matrix } B.
   \text{Input}
   \text{On entry: } n, \text{ the order of the matrix } B.
   \text{Constraint: } n \geq 0.

4: \( kb \) – Integer 
   \text{Input}
   \text{On entry: } k, \text{ the number of super-diagonals of the matrix } B \text{ if } uplo = \text{Nag}\_\text{Upper}, \text{ or the number of sub-diagonals if } uplo = \text{Nag}\_\text{Lower}.
   \text{Constraint: } kb \geq 0.

5: \( bb[dim] \) – double 
   \text{Input/Output}
   \text{Note: the dimension, } dim, \text{ of the array } bb \text{ must be at least max}(1, pdbb \times n).
   \text{On entry: the } n \text{ by } n \text{ symmetric band matrix } B. \text{ This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements } b_{ij} \text{ depends on the order and uplo parameters as follows:}
   \text{if } order = \text{Nag}\_\text{ColMajor} \text{ and } uplo = \text{Nag}\_\text{Upper},
   b_{ij} \text{ is stored in } bb[k + i - j + (j - 1) \times pdbb], \text{ for } i = 1, \ldots, n \text{ and } j = i, \ldots, \min(n, i + k);
   \text{if } order = \text{Nag}\_\text{ColMajor} \text{ and } uplo = \text{Nag}\_\text{Lower},
   b_{ij} \text{ is stored in } bb[i - j + (j - 1) \times pdbb], \text{ for } i = 1, \ldots, n \text{ and } j = \max(1, i - k), \ldots, i;
   \text{if } order = \text{Nag}\_\text{RowMajor} \text{ and } uplo = \text{Nag}\_\text{Upper},
   b_{ij} \text{ is stored in } bb[j - i +(i - 1) \times pdbb], \text{ for } i = 1, \ldots, n \text{ and } j = i, \ldots, \min(n, i + k);
   \text{if } order = \text{Nag}\_\text{RowMajor} \text{ and } uplo = \text{Nag}\_\text{Lower},
   b_{ij} \text{ is stored in } bb[k + j - i +(i - 1) \times pdbb], \text{ for } i = 1, \ldots, n \text{ and } j = \max(1, i - k), \ldots, i.
   \text{On exit: } B \text{ is overwritten by the elements of its split Cholesky factor } S.

6: \( pdbb \) – Integer 
   \text{Input}
   \text{On entry: the stride separating row or column elements (depending on the value of order) of the matrix } B \text{ in the array } bb.
   \text{Constraint: } pdbb \geq kb + 1.

7: \( fail \) – NagError * 
   \text{Output}
   \text{The NAG error parameter (see the Essential Introduction).}

6 \text{ Error Indicators and Warnings}

\textbf{NE_INT}
   \text{On entry, } n = \langle value \rangle.
   \text{Constraint: } n \geq 0.
   \text{On entry, } kb = \langle value \rangle.
   \text{Constraint: } kb \geq 0.
   \text{On entry, } pdbb = \langle value \rangle.
   \text{Constraint: } pdbb > 0.

\textbf{NE_INT_2}
   \text{On entry, } pdbb = \langle value \rangle, \text{ kb = } \langle value \rangle.
   \text{Constraint: } pdbb \geq kb + 1.
NE_CONVERGENCE
The factorization could not be completed, because updated element $b_{{\langle value\rangle}, \langle value\rangle}$ would be the square root of a negative number. Hence $B$ is not positive definite. This may indicate an error in forming the matrix $B$.

NE_ALLOC_FAIL
Memory allocation failed.

NE_BAD_PARAM
On entry, parameter $\langle value\rangle$ had an illegal value.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy
The computed factor $S$ is the exact factor of a perturbed matrix $B + E$, where

$$|E| \leq c(k + 1)\varepsilon |S^T||S|,$$

$c(k + 1)$ is a modest linear function of $k + 1$, and $\varepsilon$ is the machine precision. It follows that $|e_{ij}| \leq c(k + 1)\varepsilon \sqrt{(b_{ii}b_{jj})}$.

8 Further Comments
The total number of floating-point operations is approximately $n(k + 1)^2$, assuming $n \gg k$.

A call to this function may be followed by a call to nag_dsbgst (f08uec) to solve the generalized eigenproblem $Az = \lambda Bz$, where $A$ and $B$ are banded and $B$ is positive-definite.

The complex analogue of this function is nag_zpbstf (f08utc).

9 Example
See Section 9 of the document for nag_dsbgst (f08uec).