1 Purpose

nag_1d_cheb_intg (e02ajc) determines the coefficients in the Chebyshev-series representation of the indefinite integral of a polynomial given in Chebyshev-series form.

2 Specification

void nag_1d_cheb_intg (Integer n, double xmin, double xmax, const double a[],
                        Integer i1, double qatm1, double aint[], Integer iaint1, NagError *fail)

3 Description

nag_1d_cheb_intg (e02ajc) forms the polynomial which is the indefinite integral of a given polynomial. Both the original polynomial and its integral are represented in Chebyshev-series form. If supplied with the coefficients $a_i$, for $i = 0, 1, \ldots, n$, of a polynomial $p(x)$ of degree $n$, where

$$p(x) = \frac{1}{2}a_0 + a_1 T_1(x) + \cdots + a_n T_n(x),$$

the function returns the coefficients $a'_i$, for $i = 0, 1, \ldots, n + 1$, of the polynomial $q(x)$ of degree $n + 1$, where

$$q(x) = \frac{1}{2}a'_0 + a'_1 T_1(x) + \cdots + a'_{n+1} T_{n+1}(x),$$

and

$$q(x) = \int p(x) \, dx.$$

Here $T_j(x)$ denotes the Chebyshev polynomial of the first kind of degree $j$ with argument $x$. It is assumed that the normalised variable $\tilde{x}$ in the interval $[-1, +1]$ was obtained from the user’s original variable $x$ in the interval $[x_{\min}, x_{\max}]$ by the linear transformation

$$\tilde{x} = \frac{2x - (x_{\max} + x_{\min})}{x_{\max} - x_{\min}},$$

and that the user requires the integral to be with respect to the variable $x$. If the integral with respect to $\tilde{x}$ is required, set $x_{\max} = 1$ and $x_{\min} = -1$.

Values of the integral can subsequently be computed, from the coefficients obtained, by using nag_1d_cheb_eval2 (e02akc).

The method employed is that of Chebyshev-series (see Chapter 8 of Modern Computing Methods (1961)), modified for integrating with respect to $x$. Initially taking $a_{n+1} = a_{n+2} = 0$, the function forms successively

$$a'_i = \frac{a_{i-1} - a_{i+1}}{2i} \times \frac{x_{\max} - x_{\min}}{2}, \quad i = n + 1, n, \ldots, 1.$$

The constant coefficient $a'_0$ is chosen so that $q(x)$ is equal to a specified value, $qatm1$, at the lower endpoint of the interval on which it is defined, i.e., $\tilde{x} = -1$, which corresponds to $x = x_{\min}$.

4 References

Modern Computing Methods (1961) Chebyshev-series

NPL Notes on Applied Science 16 (2nd Edition)

HMSO
5 Parameters

1:  
   \n – Integer  
   \n On entry: n, the degree of the given polynomial p(x).
   
 Constraint: n ≥ 0.

2:  
   xmin – double  
   \n On entry: the lower and upper end-points respectively of the interval [xmin, xmax]. The Chebyshev-series representation is in terms of the normalised variable \( \bar{x} \), where
   
   \[ \bar{x} = \frac{2x - (x_{\text{max}} + x_{\text{min}})}{x_{\text{max}} - x_{\text{min}}} \].
   
 Constraint: xmax > xmin.

3:  
   xmax – double  
   \n On entry: the lower and upper end-points respectively of the interval [xmin, xmax]. The Chebyshev-series representation is in terms of the normalised variable \( \bar{x} \), where
   
   \[ \bar{x} = \frac{2x - (x_{\text{max}} + x_{\text{min}})}{x_{\text{max}} - x_{\text{min}}} \].
   
 Constraint: xmax > xmin.

4:  
   a[\text{dim}] – const double  
   \n Note: the dimension, \text{dim}, of the array a must be at least 1 + n × ia1.
   
 On entry: the Chebyshev coefficients of the polynomial p(x). Specifically, element \( i \times \text{ia1} \) of a must contain the coefficient \( a_i \), for \( i = 0, 1, \ldots, n \). Only these \( n + 1 \) elements will be accessed.

5:  
   ia1 – Integer  
   \n On entry: the index increment of a. Most frequently the Chebyshev coefficients are stored in adjacent elements of a, and \text{ia1} must be set to 1. However, if for example, they are stored in \( a[0], a[3], a[6], \ldots \), then the value of \text{ia1} must be 3. See also Section 8.
   
 Constraint: \text{ia1} ≥ 1.

6:  
   qatm1 – double  
   \n On entry: the value that the integrated polynomial is required to have at the lower end-point of its interval of definition, i.e., at \( \bar{x} = -1 \) which corresponds to \( x = x_{\text{min}} \). Thus, qatm1 is a constant of integration and will normally be set to zero by the user.

7:  
   aint[\text{dim}] – double  
   \n Note: the dimension, \text{dim}, of the array aint must be at least 1 + (n + 1) × ia1.
   
 On exit: the Chebyshev coefficients of the integral \( q(x) \). (The integration is with respect to the variable \( x \), and the constant coefficient is chosen so that \( q(x_{\text{min}}) \) equals qatm1). Specifically, element \( i \times \text{iaint1} \) of aint contains the coefficient \( a'_i \), for \( i = 0, 1, \ldots, n + 1 \).

8:  
   ia1int – Integer  
   \n On entry: the index increment of aint. Most frequently the Chebyshev coefficients are required in adjacent elements of aint, and \text{ia1int} must be set to 1. However, if, for example, they are to be stored in aint[0], aint[3], aint[6], \ldots, then the value of \text{ia1int} must be 3. See also Section 8.
   
 Constraint: \text{ia1int} ≥ 1.

9:  
   fail – NagError *  
   \n The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT
   
 On entry, n = \langle\text{value}\rangle.
 Constraint: n ≥ 0.
On entry, \( \text{ia1} = \langle \text{value} \rangle \).
Constraint: \( \text{ia1} \geq 1 \).

On entry, \( \text{iaint1} = \langle \text{value} \rangle \).
Constraint: \( \text{iaint1} \geq 1 \).

\text{NE_REAL}_2
On entry, \( \text{xmax} \leq \text{xmin} \): \( \text{xmax} = \langle \text{value} \rangle \), \( \text{xmin} = \langle \text{value} \rangle \).

\text{NE_BAD_PARAM}
On entry, parameter \( \langle \text{value} \rangle \) had an illegal value.

\text{NE_INTERNAL_ERROR}
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy
In general there is a gain in precision in numerical integration, in this case associated with the division by \( 2^i \) in the formula quoted in Section 3.

8 Further Comments
The time taken is approximately proportional to \( n + 1 \).

The increments \( \text{ia1}, \text{iaint1} \) are included as parameters to give a degree of flexibility which, for example, allows a polynomial in two variables to be integrated with respect to either variable without rearranging the coefficients.

9 Example
Suppose a polynomial has been computed in Chebyshev-series form to fit data over the interval \([-0.5,2.5]\]. The following program evaluates the integral of the polynomial from 0.0 to 2.0. (For the purpose of this example, \( \text{xmin}, \text{xmax} \) and the Chebyshev coefficients are simply supplied. Normally a program would read in or generate data and compute the fitted polynomial).

9.1 Program Text
/* nag_1d_cheb_intg (e02ajc) Example Program. */
/* Copyright 2001 Numerical Algorithms Group. */
/* Mark 7, 2001. */
/
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nage02.h>

int main(void)
{
    /* Initialized data */
    const double xmin = -0.5;
    const double xmax = 2.5;
    const double a[7] = { 2.53213,1.13032,0.2715,0.04434,0.00547,5.4e-4,4e-5 };

    /* Scalars */
    double ra, rb, result, xa, xb, zero;
    Integer exit_status, n, one;
    NagError fail;
/* Arrays */
double *aint = 0;

INIT_FAIL(fail);
exit_status = 0;
Vprintf("e02ajc Example Program Results\n");
n = 6;
zero = 0.0;
one = 1;

/* Allocate memory */
if ( !(aint = NAG_ALLOC(n + 2, double)) )
{
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
}
e02ajc(n, xmin, xmax, a, one, zero, aint, one, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from e02ajc.\n", fail.message);
    exit_status = 1;
    goto END;
}
xa = 0.0;
xb = 2.0;
e02akc(n+1, xmin, xmax, aint, one, xa, &ra, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from e02akc.\n", fail.message);
    exit_status = 1;
    goto END;
}
e02akc(n+1, xmin, xmax, aint, one, xb, &rb, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from e02akc.\n", fail.message);
    exit_status = 1;
    goto END;
}
result = rb - ra;
Vprintf("\n");
Vprintf("Value of definite integral is %10.4f\n", result);
END:
if (aint) NAG_FREE(aint);
return exit_status;

9.2 Program Data
None.

9.3 Program Results
e02ajc Example Program Results
Value of definite integral is 2.1515