NAG C Library Function Document

nag_1d_quad_inf_1 (d01smc)

1 Purpose

nag_1d_quad_inf_1 (d01smc) calculates an approximation to the integral of a function \( f(x) \) over an infinite or semi-infinite interval \([a, b]\):

\[
I = \int_a^b f(x) \, dx.
\]

2 Specification

#include <nag.h>
#include <nagd01.h>

void nag_1d_quad_inf_1 (double (*f)(double x, Nag_User *comm),
                   Nag_BoundInterval boundinf, double bound, double epsabs,
                   double epsrel, Integer max_num_subint, double *result,
                   double *abserr, NAG_QuadProgress *qp, NAG_User *comm, NagError *fail)

3 Description

This function is based on the QUADPACK routine QAGI (Piessens et al. (1983)). The entire infinite integration range is first transformed to \([0, 1]\) using one of the identities

\[
\int_{-\infty}^a f(x) \, dx = \int_0^1 f \left( a - \frac{1 - t}{t} \right) \frac{1}{t^2} \, dt
\]

\[
\int_a^{\infty} f(x) \, dx = \int_0^1 f \left( a + \frac{1 - t}{t} \right) \frac{1}{t^2} \, dt
\]

\[
\int_{-\infty}^{\infty} f(x) \, dx = \int_0^\infty \left( f(x) + f(-x) \right) \, dx = \int_0^1 \left[ f \left( \frac{1 - t}{t} \right) + f \left( -1 + \frac{1}{t} \right) \right] \frac{1}{t^2} \, dt
\]

where \( a \) represents a finite integration limit. An adaptive procedure, based on the Gauss 7-point and Kronrod 15-point rules, is then employed on the transformed integral. The algorithm, described by De Doncker (1978), incorporates a global acceptance criterion (as defined by Malcolm and Simpson (1976)) together with the \( c \)-algorithm (Wynn (1956)) to perform extrapolation. The local error estimation is described by Piessens et al. (1983).

4 Parameters

1: \( f \) – function supplied by user

The function \( f \), supplied by the user, must return the value of the integrand \( f \) at a given point.

The specification of \( f \) is:

\[
\text{double } f(\text{double } x, \text{Nag_User *comm})
\]

1: \( x \) – double

On entry: the point at which the integrand \( f \) must be evaluated.

[NP3491/6]
2: \textbf{comm} – Nag_User *

\textit{On entry/on exit:} pointer to a structure of type \textbf{Nag_User} with the following member:

\textbf{p} – Pointer \hspace{1cm} \textit{Input/Output}

\textit{On entry/on exit:} the pointer \textbf{comm}→\textbf{p} should be cast to the required type, e.g.,
\texttt{struct user *s = (struct user *)comm→p}, to obtain the original object’s address with appropriate type. (See the argument \textbf{comm} below.)

2: \textbf{boundinf} – Nag_BoundInterval \hspace{1cm} \textit{Input}

\textit{On entry:} indicates the kind of integration interval:

\hspace{1cm} if \textbf{boundinf} = \textbf{Nag_UpperSemiInfinite}, the interval is \([\text{bound}, +\infty)\);

\hspace{1cm} if \textbf{boundinf} = \textbf{Nag_LowerSemiInfinite}, the interval is \((-\infty, \text{bound})\);

\hspace{1cm} if \textbf{boundinf} = \textbf{Nag_Infinite}, the interval is \((-\infty, +\infty)\).

\textit{Constraint:} \textbf{boundinf} = \textbf{Nag_UpperSemiInfinite}, \textbf{Nag_LowerSemiInfinite}, or \textbf{Nag_Infinite}.

3: \textbf{bound} – double \hspace{1cm} \textit{Input}

\textit{On entry:} the finite limit of the integration interval (if present). \textbf{bound} is not used if \textbf{boundinf} = \textbf{Nag_Infinite}.

4: \textbf{epsabs} – double \hspace{1cm} \textit{Input}

\textit{On entry:} the absolute accuracy required. If \textbf{epsabs} is negative, the absolute value is used. See Section 6.1.

5: \textbf{epsrel} – double \hspace{1cm} \textit{Input}

\textit{On entry:} the relative accuracy required. If \textbf{epsrel} is negative, the absolute value is used. See Section 6.1.

6: \textbf{max_num_subint} – Integer \hspace{1cm} \textit{Input}

\textit{On entry:} the upper bound on the number of sub-intervals into which the interval of integration may be divided by the function. The more difficult the integrand, the larger \textbf{max_num_subint} should be.

\textit{Suggested values:} a value in the range 200 to 500 is adequate for most problems.

\textit{Constraint:} \textbf{max_num_subint} \geq 1.

7: \textbf{result} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} the approximation to the integral \(I\).

8: \textbf{abser} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} an estimate of the modulus of the absolute error, which should be an upper bound for \(|I - \text{result}|\).

9: \textbf{qp} – Nag_QuadProgress *

Pointer to structure of type \textbf{Nag_QuadProgress} with the following members:

\hspace{1cm} \textbf{num_subint} – Integer \hspace{1cm} \textit{Output}

\textit{On exit:} the actual number of sub-intervals used.

\hspace{1cm} \textbf{fun_count} – Integer \hspace{1cm} \textit{Output}

\textit{On exit:} the number of function evaluations performed by \texttt{nag_1d_quad_inf_1}.
sub_int_beg_pts – double *  Output
sub_int_end_pts – double *  Output
sub_int_result – double *  Output
sub_int_error – double *  Output

On exit: these pointers are allocated memory internally with max_num_subint elements. If an error exit other than NE_INT_ARG_LT, NE_BAD_PARAM or NE_ALLOC_FAIL occurs, these arrays will contain information which may be useful. For details, see Section 6.

Before a subsequent call to nag_1d_quad_inf_1 is made, or when the information contained in these arrays is no longer useful, the user should free the storage allocated by these pointers using the NAG macro NAG_FREE.

10:  comm – Nag_User *

On entry/on exit: pointer to a structure of type Nag_User with the following member:

p – Pointer  Input/Output

On entry/on exit: the pointer p, of type Pointer, allows the user to communicate information to and from the user-defined function f(). An object of the required type should be declared by the user, e.g., a structure, and its address assigned to the pointer p by means of a cast to Pointer in the calling program, e.g., comm.p = (Pointer)&s. The type Pointer is void *.

11:  fail – NagError *  Input/Output

The NAG error parameter (see the Essential Introduction).

Users are recommended to declare and initialise fail and set fail.print = TRUE for this function.

5  Error Indicators and Warnings

NE_INT_ARG_LT

On entry, max_num_subint must not be less than 1: max_num_subint = <value>.

NE_BAD_PARAM

On entry, parameter boundinf had an illegal value.

NE_ALLOC_FAIL

Memory allocation failed.

NE_QUAD_MAX_SUBDIV

The maximum number of subdivisions has been reached: max_num_subint = <value>. The maximum number of subdivisions has been reached without the accuracy requirements being achieved. Look at the integrand in order to determine the integration difficulties. If the position of a local difficulty within the interval can be determined (e.g., a singularity of the integrand or its derivative, a peak, a discontinuity, etc.) you will probably gain from splitting up the interval at this point and calling the integrator on the sub-intervals. If necessary, another integrator, which is designed for handling the type of difficulty involved, must be used. Alternatively, consider relaxing the accuracy requirements specified by epsabs and epsrel, or increasing the value of max_num_subint.

NE_QUAD_ROUNDOFF_TOL

Round-off error prevents the requested tolerance from being achieved: epsabs = <value>,
epsrel = <value>. The error may be underestimated. Consider relaxing the accuracy requirements specified by epsabs and epsrel.
NE_QUAD_BAD_SUBDIV

Extremely bad integrand behaviour occurs around the sub-interval \(<value>, <value>\).
The same advice applies as in the case of NE_QUAD_MAX_SUBDIV.

NE_QUAD_ROUNDOFF_EXTRAPL

Round-off error is detected during extrapolation.
The requested tolerance cannot be achieved, because the extrapolation does not increase the accuracy satisfactorily; the returned result is the best that can be obtained.
The same advice applies as in the case of NE_QUAD_MAX_SUBDIV.

NE_QUAD_NO_CONV

The integral is probably divergent or slowly convergent.
Please note that divergence can also occur with any error exit other than NE_INT_ARG_LT,
NE_BAD_PARAM or NE_ALLOC_FAIL.

NE_QUAD_BAD_SUBDIV_INTS

Extremely bad integrand behaviour occurs around one of the sub-intervals \(<value>, <value>\) or \(<value>, <value>\).
The same advice applies as in the case of NE_QUAD_MAX_SUBDIV.

6 Further Comments

The time taken by nag_1d_quad_inf_1 depends on the integrand and the accuracy required.
If the function fails with an error exit other than NE_INT_ARG_LT, NE_BAD_PARAM or NE_ALLOC_FAIL then the user may wish to examine the contents of the structure qp.
These contain the end-points of the sub-intervals used by nag_1d_quad_inf_1 along with the integral contributions and error estimates over the sub-intervals.

Specifically, for \(i = 1, 2, \ldots, n\), let \(r_i\) denote the approximation to the value of the integral over the sub-interval \([a_i, b_i]\) in the partition of \([a, b]\) and \(e_i\) be the corresponding absolute error estimate.

Then, \(\int_{a_i}^{b_i} f(x) \, dx \approx r_i\) and \(\text{result} = \sum_{i=1}^{n} r_i\) unless the function terminates while testing for divergence of the integral (see Section 3.4.3 of Piessens et al. (1983)).
In this case, result (and abserr) are taken to be the values returned from the extrapolation process. The value of \(n\) is returned in num_subint, and the values \(a_i, b_i, r_i\) and \(e_i\) are stored in the structure qp as

\[
\begin{align*}
  a_i &= \text{sub_int_beg_pts}[i-1], \\
  b_i &= \text{sub_int_end_pts}[i-1], \\
  r_i &= \text{sub_int_result}[i-1] \quad \text{and} \\
  e_i &= \text{sub_int_error}[i-1].
\end{align*}
\]

6.1 Accuracy

The function cannot guarantee, but in practice usually achieves, the following accuracy:

\[ |I - \text{result}| \leq tol \]

where

\[ tol = \max\{|\text{epsabs}|, |\text{epsrel}| \times |I|\} \]

and \text{epsabs} and \text{epsrel} are user-specified absolute and relative error tolerances. Moreover it returns the quantity \text{abserr} which, in normal circumstances, satisfies

\[ |I - \text{result}| \leq \text{abserr} \leq tol. \]
6.2 References
Wynn P (1956) On a device for computing the $e_{m}(S_{n})$ transformation Math. Tables Aids Comput. 10 91–96

7 See Also
nag_1d_quad_gen_1 (d01sjc)

8 Example
To compute
\[ \int_{0}^{\infty} \frac{1}{(x+1)\sqrt{x}} \, dx. \]

8.1 Program Text
/* nag_1d_quad_inf_1(d01smc) Example Program */
/* Copyright 1998 Numerical Algorithms Group. */
/* Mark 5, 1998. */
/* Mark 6 revised, 2000. */
*/
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <math.h>
#include <nagD01.h>

static double f(double x, Nag_User *comm);

main()
{
    double a;
    double epsabs, abserr, epsrel, result;
    Nag_QuadProgress qp;
    Integer max_num_subint;
    static NagError fail;
    Nag_User comm;

    Vprintf("d01smc Example Program Results\n"辏);
    epsabs = 0.0;
    epsrel = 0.0001;
    a = 0.0;
    max_num_subint = 200;

    d01smc(f, Nag_UpperSemiInfinite, a, epsabs, epsrel, max_num_subint,
&result, &abser, &qp, &comm, &fail);
Vprintf("a - lower limit of integration = %10.4fn", a);
Vprintf("b - upper limit of integration = infinity\n");
Vprintf("epsabs - absolute accuracy requested = %9.2en", epsabs);
Vprintf("epsrel - relative accuracy requested = %9.2e\n\n", epsrel);
if (fail.code != NE_NOERROR)
    Vprintf("%s\n", fail.message);
if (fail.code != NE_INT_ARG_LT && fail.code != NE_BAD_PARAM &&
    fail.code != NE_ALLOC_FAIL)
{
    Vprintf("result - approximation to the integral = %9.5fn", result);
    Vprintf("abser - estimate of the absolute error = %9.2en", abser);
    Vprintf("qp.fun_count - number of function evaluations = %4ld\n", 
        qp.fun_count);
    Vprintf("qp.num_subint - number of subintervals used = %4ld\n", 
        qp.num_subint);
    /* Free memory used by qp */
    NAG_Free(qp.sub_int_beg_pts);
    NAG_Free(qp.sub_int_end_pts);
    NAG_Free(qp.sub_int_result);
    NAG_Free(qp.sub_int_error);
    exit(EXIT_SUCCESS);
}
exit(EXIT_FAILURE);
}

static double f(double x, Nag_User *comm)
{
    return 1.0/((x+1.0)*sqrt(x));
}

8.2 Program Data

None.

8.3 Program Results

d01smc Example Program Results
a - lower limit of integration = 0.0000
b - upper limit of integration = infinity
epsabs - absolute accuracy requested = 0.00e+00
epsrel - relative accuracy requested = 1.00e-04
result - approximation to the integral = 3.14159
abser - estimate of the absolute error = 2.65e-05
qp.fun_count - number of function evaluations = 285
qp.num_subint - number of subintervals used = 10