nag_check_deriv (c05zbc)

1. Purpose

nag_check_deriv (c05zbc) checks that a user-supplied C function for evaluating a vector of functions and the matrix of their first derivatives produces derivative values which are consistent with the function values calculated.

2. Specification

```c
#include <nag.h>
#include <nagc05.h>

void nag_check_deriv(Integer n, double x[], double fvec[], double fjac[],
    Integer tdfjac,
    void (*f)(Integer n, double x[], double fvec[],
    double fjac[], Integer tdfjac, Integer *userflag),
    NagError *fail)
```

3. Description

nag_check_deriv checks the derivatives calculated by user-supplied C functions, e.g. functions of the form required for nag_zero_nonlin_eqns_deriv (c05pbc). As well as the C function to be checked f, the user must supply a point \( x = (x_1, x_2, \ldots, x_n)^T \) at which the check will be made.

nag_check_deriv first calls f to evaluate both the \( f_i(x) \) and their first derivatives, and uses these to calculate the sum of squares

\[
F(x) = \sum_{i=1}^{n} |f_i(x)|^2,
\]

and its first derivatives

\[
g_j = \frac{\partial F}{\partial x_j} \bigg|_x, \quad \text{for } j = 1, 2, \ldots, n.
\]

The components of \( g \) along two orthogonal directions (defined by unit vectors \( p_1 \) and \( p_2 \), say) are then calculated; these will be \( g^T p_1 \) and \( g^T p_2 \) respectively. The same components are also estimated by finite differences, giving quantities

\[
v_k = \frac{F(x + hp_k) - F(x)}{h}, \quad k = 1, 2
\]

where \( h \) is a small positive scalar. If the relative difference between \( v_1 \) and \( g^T p_1 \) or between \( v_2 \) and \( g^T p_2 \) is judged too large, an error indicator is set.

4. Parameters

- **n**
  Input: the number \( n \) of variables, \( x_j \), for use with nag_zero_nonlin_eqns_deriv (c05pbc).
  Constraint: \( n > 0 \).

- **x[n]**
  Input: \( x[j-1] \), for \( j = 1, 2, \ldots, n \) must be set to the co-ordinates of a suitable point at which to check the derivatives calculated by \( f \). ‘Obvious’ settings, such as 0 or 1, should not be used since, at such particular points, incorrect terms may take correct values (particularly zero), so that errors can go undetected. For a similar reason, it is preferable that no two elements of \( x \) should have the same value.

- **fvec[n]**
  Output: unless userflag is set negative when evaluating \( f_i \) at the point given in \( x \), \( fvec[i-1] \) contains the value of \( f_i \) at the point given by the user in \( x \), for \( i = 1, 2, \ldots, n \).
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Output: unless userflag is set negative when evaluating the Jacobian at the point given in x, \( f_jac[i-1][j-1] \) contains the value of the first derivative \( \partial f_i / \partial x_j \) at the point given in x, as calculated by f, for \( i = 1, 2, \ldots, n; j = 1, 2, \ldots, n \).

**tdfjac**

Input: the last dimension of array fjac as declared in the function from which nag_check_deriv is called.

Constraint: \( tdfjac \geq n \).

f

f must calculate the values of the functions at a point x or return the Jacobian at x. nag_zero_nonlin_eqns_deriv (c05pbc) gives the user the option of resetting a parameter to terminate immediately. nag_check_deriv (c05zbc) will also terminate immediately, without finishing the checking process, if the parameter in question is reset.

The specification of f is:

```c
void f(Integer n, double x[], double fvec[], double fjac[],
        Integer tdfjac, Integer *userflag)
```

- **n**
  - Input: the number of equations, n

- **x[]**
  - Input: the components of the point x at which the functions or the Jacobian must be evaluated.

- **fvec[]**
  - Output: if userflag = 1 on entry, fvec must contain the function values \( f_i(x) \) (unless userflag is set to a negative value by f).
    - If userflag = 2 on entry, fvec must not be changed.

- **fjac[]**
  - Output: if userflag = 2 on entry, \( f_jac[(i-1)*tdfjac+j-1] \) must contain the value of \( \partial f_i / \partial x_j \) at the point x, for \( i = 1, 2, \ldots, n; j = 1, 2, \ldots, n \) (unless userflag is set to a negative value by f).
    - If userflag = 1 on entry, fjac must not be changed.

- **tdfjac**
  - Input: the last dimension of array fjac as declared in the function from which nag_check_deriv is called.

- **userflag**
  - Input: userflag = 1 or 2.
    - If userflag = 1, fvec is to be updated.
    - If userflag = 2, fjac is to be updated.
  - Output: in general, userflag should not be reset by f. If, however, the user wishes to terminate execution (perhaps because some illegal point x has been reached), then userflag should be set to a negative integer. This value will be returned through fail.errnum.

**fail**

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. **Error Indications and Warnings**

**NE_INT_ARG_LE**

On entry, n must not be less or equal to 0: n = ⟨value⟩.

**NE_2_INT_ARG_LT**

On entry tdfjac = ⟨value⟩ while n = ⟨value⟩. These parameters must satisfy tdfjac ≥ n.

**NE_ALLOC_FAIL**

Memory allocation failed.
NE_DERIV_ERRORS
Large errors were found in the derivatives of the objective function.

The user should check carefully the derivation and programming of expressions for the $\frac{\partial f_i}{\partial x_j}$, because it is very unlikely that $f$ is calculating them correctly.

NE_USER_STOP
User requested termination, user flag value = ⟨value⟩.

6. Further Comments
Before using nag_check_deriv (c05zbc) to check the calculation of the first derivatives, the user should be confident that $f$ is evaluating the functions correctly.

6.1. Accuracy
fail.code is set to NE_DERIV_ERRORS if

$$(v_k - g^T p_k)^2 \geq h \times ((g^T p_k)^2 + 1)$$

for $k = 1$ or $2$. (See Section 3 for definitions of the quantities involved.) The scalar $h$ is set equal to $\sqrt{\varepsilon}$, where $\varepsilon$ is the machine precision.

7. See Also
nag_zero_nonlin_eqns_deriv (c05pbc)

8. Example
This example checks the Jacobian matrix for the problem solved in the example program for nag_zero_nonlin_eqns_deriv (c05pbc).

8.1. Program Text
/* nag_check_deriv(c05zbc) Example Program */
* * Mark 2, 1991.
*/
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagc05.h>

#ifdef NAG_PROTO
static void f(Integer n, double xc[], double fvecc[], double fjacc[], Integer tdj, Integer *userflag);
#else
static void f();
#endif

main()
{
#define NMAX 5

double fjac[NMAX][NMAX], fvecc[NMAX], x[NMAX];
Integer i, j, n, tdfjac;
static NagError fail;
fail.print = TRUE;
Vprintf("c05zbc Example Program Results\n");
n = 3;
tdfjac = NMAX;
/* Set up an arbitrary point at which to check the 1st derivatives */
x[0] = 9.2e-01;
x[1] = 1.3e-01;
x[2] = 5.4e-01;
Vprintf("The test point is ");
for (j=0; j<n; ++j)
  Vprintf("%13.3e", x[j]);
Vprintf("\n\n");
c05zbc(n, x, fvec, (double *)fjac, tdfjac, f, &fail);
if (fail.code != NE_NOERROR) exit(EXIT_FAILURE);
Vprintf("1st derivatives are consistent with residual values.\n\n");
Vprintf("At the test point, f() gives\n\n");
for (i=0; i<n; ++i)
  
  Vprintf("%13.3e", fvec[i]);
  for (j=0; j<n; ++j)
    Vprintf("%13.3e", fjac[i][j]);
  Vprintf("\n");
exit(EXIT_SUCCESS);
}

#define FJAC(I,J) fjac[((I))*tdfjac+(J)]

8.2. Program Data

None.
8.3. Program Results

c05zbc Example Program Results
The test point is \( 9.200e-01 \quad 1.300e-01 \quad 5.400e-01 \)

1st derivatives are consistent with residual values.

At the test point, \( f() \) gives

\[
\begin{array}{cccc}
\text{Residuals} & 1.807e+00 & -6.800e-01 & -2.000e+00 & 0.000e+00 \\
-6.438e-01 & -1.000e+00 & 2.480e+00 & -2.000e+00 \\
1.907e+00 & 0.000e+00 & -1.000e+00 & 8.400e-01 \\
\end{array}
\]