NAG C Library Function Document

nag_cubic_roots (c02akc)

1 Purpose

nag_cubic_roots (c02akc) determines the roots of a cubic equation with real coefficients.

2 Specification

void nag_cubic_roots (double u, double r, double s, double t, double zeror[], double zeroi[], double errrest[], NagError *fail)

3 Description

nag_cubic_roots attempts to find the roots of the cubic equation

\[ uz^3 + rz^2 + sz + t = 0, \]

where \( u, r, s \) and \( t \) are real coefficients with \( u \neq 0 \). The roots are located by finding the eigenvalues of the associated 3 by 3 (upper Hessenberg) companion matrix \( H \) given by

\[
H = \begin{pmatrix}
0 & 0 & -t/u \\
1 & 0 & -s/u \\
0 & 1 & -r/u
\end{pmatrix}.
\]

Further details can be found in Section 6.

To obtain the roots of a quadratic equation, nag_quartic_roots (c02alc) can be used.

4 Parameters

1: \( u \) – double \hspace{1cm} Input

On entry: \( u \), the coefficient of \( z^3 \).

Constraint: \( u \neq 0.0 \).

2: \( r \) – double \hspace{1cm} Input

On entry: \( r \), the coefficient of \( z^2 \).

3: \( s \) – double \hspace{1cm} Input

On entry: \( s \), the coefficient of \( z \).

4: \( t \) – double \hspace{1cm} Input

On entry: \( t \), the constant coefficient.

5: \( \text{zeror}[3] \) – double \hspace{1cm} Output

6: \( \text{zeroi}[3] \) – double \hspace{1cm} Output

On exit: \( \text{zeror}[i - 1] \) and \( \text{zeroi}[i - 1] \) contain the real and imaginary parts, respectively, of the \( i \)th root.

7: \( \text{errrest}[3] \) – double \hspace{1cm} Output

On exit: \( \text{errrest}[i - 1] \) contains an approximate error estimate for the \( i \)th root.
5 Error Indicators and Warnings

NE_REAL
On entry, u = 0.0.
Constraint: u ≠ 0.0.

NE_C02_OVERFLOW
The companion matrix H cannot be formed without overflow.

NE_C02_NOT_CONV
The iterative procedure used to determine the eigenvalues has failed to converge.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments
The method used by the routine consists of the following steps, which are performed by routines from LAPACK.
(a) Form H.
(b) Apply a diagonal similarity transformation to H to give H'.
(c) Calculate the eigenvalues and Schur factorization of H'.
(d) Calculate the left and right eigenvectors of H'.
(e) Estimate reciprocal condition numbers for all the eigenvalues of H'.
(f) Calculate approximate error estimates for all the eigenvalues of H' (using the 1-norm).

6.1 Accuracy
If fail.code = NE_NOERROR on exit, then the i'th computed root should have approximately \[|\log_{10}(\text{errest}[i - 1])|\] correct significant digits.

6.2 References

7 See Also
nag_quartic_roots (c02alc)

8 Example
To find the roots of the cubic equation
\[z^3 + 3z^2 + 9z - 13 = 0.\]
8.1 Program Text

/* nag_cubic_roots (c02akc) Example Program. */
/* Copyright 2000 Numerical Algorithms Group. */
/* NAG C Library */
/* Mark 6, 2000. */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagc02.h>

int main(void)
{
  double *errest=0, *zeroi=0, *zeror=0;
  double r, s, t, u;
  Integer i;
  Integer exit_status=0;
  NagError fail;

  INIT_FAIL(fail);
  Vprintf("c02akc Example Program Results\n\n");

  if
  {
    ! (errest = NAG_ALLOC(3, double)) ||
    ! (zeroi = NAG_ALLOC(3, double)) ||
    ! (zeror = NAG_ALLOC(3, double))
  }
  {
    Vprintf("Allocation failure\n");
    exit_status=-1;
    goto END;
  }

  /* Skip heading in data file */
  Vscanf("%*[\n] ");
  Vscanf("%lf %lf %lf %lf", &u, &r, &s, &t);

  c02akc (u, r, s, t, zeror, zeroi, errest, &fail);
  if (fail.code == NE_NOERROR)
  {
    Vprintf("\n Roots of cubic equation Error estimates\n");
    Vprintf(" (machine-dependent)\n\n");
    for (i = 0; i <= 2; ++i)
    {
      Vprintf(" z = %10.5f %10.5f%s %g\n",
        zeror[i], zeroi[i], "*i", errest[i]);
    }
  }
  else
  {
    Vprintf("Error from c02akc.\n\n", fail.message);
    exit_status = 1;
    goto END;
  }

  return
}
8.2 Program Data

c02akc Example Program Data
1.0 3.0 9.0 -13.0 : Values of u, r, s and t

8.3 Program Results

Roots of cubic equation     Error estimates
                          (machine-dependent)

<table>
<thead>
<tr>
<th>z</th>
<th>1.00000</th>
<th>0.000000*i</th>
<th>2.37689e-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>z</td>
<td>-2.00000</td>
<td>3.000000*i</td>
<td>3.08488e-15</td>
</tr>
<tr>
<td>z</td>
<td>-2.00000</td>
<td>-3.000000*i</td>
<td>3.08488e-15</td>
</tr>
</tbody>
</table>