Estimating Absolute-Phase Maps Using ESPIRiT and Virtual Conjugate Coils

Martin Uecker and Michael Lustig

Department of Electrical Engineering and Computer Sciences,
University of California, Berkeley, California.

July 16, 2015

Running head: Estimating Absolute Phase with ESPIRiT

Address correspondence to:
Martin Uecker
545D Cory Hall
University of California, Berkeley
Berkeley, CA 94720
uecker@eecs.berkeley.edu

This work was supported by NIH R01EB009690 and GE Healthcare.

Approximate word count: 189 (Abstract) 2300 (body)

Part of this work has been presented at the ISMRM Annual Conference 2014.

Abstract

Purpose: To develop an ESPIRiT-based method to estimate coil sensitivities with image phase. **Theory and Methods:** ESPIRiT is a new framework for calibration of the coil sensitivities and reconstruction in parallel Magnetic Resonance Imaging (MRI). Applying ESPIRiT to a combined set of physical and virtual conjugate coils yields sensitivities with unknown joint phase but known relative phase, which allows the determination of the absolute phase up to a sign ambiguity. The accuracy of the computed maps with phase is experimentally evaluated using a test based on projection onto fully sampled coil images. The use of multiple ESPIRiT maps in a phase-constrained parallel imaging reconstruction is compared to the use of a single map. **Results:** The proposed method can estimate accurate sensitivities which include low-resolution image phase. In case of high-frequency phase variations ESPIRiT yields additional set of maps which capture the high-frequency phase component. Taking this additional set of maps into account can improve the robustness of phase-constrained parallel imaging. **Conclusion:** The extended VCC-ESPIRiT is a new promising framework for estimation of phase information and phase-constrained imaging.

Key words: parallel imaging, partial Fourier, ESPIRiT, virtual coil

1 Introduction

The primary quantity measured in MRI, the spin density, can be described by a real and positive function. In principle, this prior knowledge can reduce the amount of k-space data necessary to reconstruct an image to one half [1]. In practice, various phase effects from B1, flow, off-resonance, and others cause phase variations in the image. By using a low-resolution phase map, homodyne reconstruction [2] or SENSE-based parallel imaging with a phase constraint can sometimes be applied [3, 4], but this is problematic for data affected by high-frequency phase variations. Recently, ESPIRiT has been described - a new method to obtain highly accurate estimations of the coil sensitivities from a fully-sampled calibration region in the k-space center [5]. The estimates of the maps are defined up to multiplication with an unknown complex-valued function, *i.e.* only relative coil sensitivities are obtained. Here, we demonstrate that ESPIRiT can be extended to produce coil sensitivities which include absolute image phase. This is accomplished by applying ESPIRiT to virtual conjugate coils (VCC-ESPIRiT), which have been introduced previously to improve GRAPPA reconstruction [6, 7]. When using ESPIRiT maps with image phase in a SENSE reconstruction, the resulting image is real up to noise and a real-value constraint can be directly enforced in the reconstruction. If phase effects can not be described by a single set of smooth maps VCC-ESPIRiT will automatically produce a second set of maps in the affected areas of the image. Part of this work has been presented at the 22nd ISMRM Annual Conference [8].

2 Theory

Although the spin density is a positive and real quantity the magnetization image $\rho(\vec{x})$ measured in MRI usually has phase. Formally, the image $\rho(\vec{x})$ can be assumed to be real-valued in the signal equations for parallel imaging if the image phase $\psi(\vec{x})$ is fully absorbed into the coil sensitivities:

$$\begin{aligned} y_j(\vec{k}) &= \int \mathrm{d}\vec{x}\, \underbrace{|\rho(\vec{x})|e^{i\psi(\vec{x})}}_{\rho(\vec{x})} c_j(\vec{x}) e^{-i2\pi\vec{k}\cdot\vec{x}} \\ &= \int \mathrm{d}\vec{x}\, |\rho(\vec{x})| \underbrace{e^{i\psi(\vec{x})} c_j(\vec{x})}_{\hat{c}_j(\vec{x})} e^{-i2\pi\vec{k}\cdot\vec{x}} \end{aligned}$$

For example, such coil sensitivities $\hat{c}_j$ which include image phase can be estimated directly from the k-space center [9], using a recently proposed extension of Walsh's method [10, 11], or by non-linear inversion with real-value constraint [12]. All of these methods assume a smooth phase and may lead to inaccurate estimates for high-frequency phase variations. In the following, an ESPIRiT-based method is presented, which - while still based on the assumption of smooth phase - offers robustness even if the image phase is not smooth.

ESPIRiT is a new method for auto-calibrating parallel imaging. It determines the signal space spanned by local patches in k-space using singular value decomposition of a calibration matrix constructed from auto-calibration data. Because there are local correlations in k-space due to field-of-view (FOV) limitations and correlations induced by the receive coils, this space is a small subspace of the space of all possible patches. ESPIRiT recovers the sensitivity maps of the receive coils as eigenvectors to the eigenvalue one of a a single reconstruction operator, which is derived from the requirement that all k-space patches lie in the signal subspace. In general, subspace-based methods are highly robust to many types of errors, because the estimated subspace automatically adapts to inconsistencies in the data. In ESPIRiT, multiple sets of sensitivity maps may appear as eigenvectors to the eigenvalue one in case of such inconsistencies. These maps can be taken into account in an extended SENSE-like reconstruction which is then as robust as traditional k-space methods. This has been demonstrated for aliasing in the case of a small FOV, motion corruption, and chemical shift [5].

Because the sensitivities are computed as the point-wise eigenvectors of a reconstruction operator, the point-wise joint phase is undefined. Usually, one channel is selected as a reference and for each pixel the phase of all channels is rotated so that the reference has zero phase. With an extension of ESPIRiT sensitivities $\hat{c}_j$ can be estimated, which ideally would result in a real image, when noise and other errors are ignored. This is done by exploiting the property of ESPIRiT that relative phase between different channels is preserved. The proposed procedure is described in the following and illustrated in Figure 1. By flipping and conjugating k-space virtual conjugate channels

$$y_{j\star}(\vec{k}) := y_j^\star(-\vec{k}) = \int \mathrm{d}\vec{x}\,|\rho(\vec{x})|\hat{c}_j^\star(\vec{x})e^{-i2\pi\vec{k}\cdot\vec{x}}$$

are constructed which are then included as additional virtual channels, doubling the total number of channels. ESPIRiT calibration is then applied to the extended data set

$$[y_1(\vec{k}), \cdots, y_N(\vec{k}), y_{1\star}(\vec{k}), \cdots y_{N\star}(\vec{k})]$$

to estimate the coil sensitivities for all channels. This yields a vector of sensitivity maps for all physical and virtual sensitivities up to an unknown pixel-wise phase $\phi(\vec{x})$:

$$[\tilde{c}_1(\vec{x}), \cdots, \tilde{c}_N(\vec{x}), \tilde{c}_{1\star}(\vec{x}), \cdots \tilde{c}_{N\star}(\vec{x})] = e^{i\phi(\vec{x})}[\hat{c}_1(\vec{x}), \cdots, \hat{c}_N(\vec{x}), \hat{c}_{1\star}(\vec{x}), \cdots \hat{c}_{N\star}(\vec{x})]$$

Note that $\phi(\vec{x})$ is an arbitrary phase difference between the estimated sensitivities $\tilde{c}_j$ and the sensitivities with image phase $\hat{c}_j(\vec{x}) = e^{i\psi(\vec{x})}c_j(\vec{x})$, *i.e.* $\tilde{c}_j(\vec{x}) = e^{i(\phi(\vec{x})+\psi(\vec{x}))}c_j(\vec{x})$. Because conjugate channels $y_{j\star}$ have conjugate sensitivities, *i.e.* $\hat{c}_{j\star}(\vec{x}) = \hat{c}_j^\star(\vec{x})$, the unknown phase ϕ can be determined up to π:

$$\frac{1}{2}\operatorname{Im}\log\sum_j \tilde{c}_j(\vec{x})\tilde{c}_{j\star}(\vec{x}) = \frac{1}{2}\operatorname{Im}\log e^{i2\phi(\vec{x})}\sum_j |c_j(\vec{x})|^2 = \phi(\vec{x}) + l\pi \qquad l \in \mathbf{Z}$$

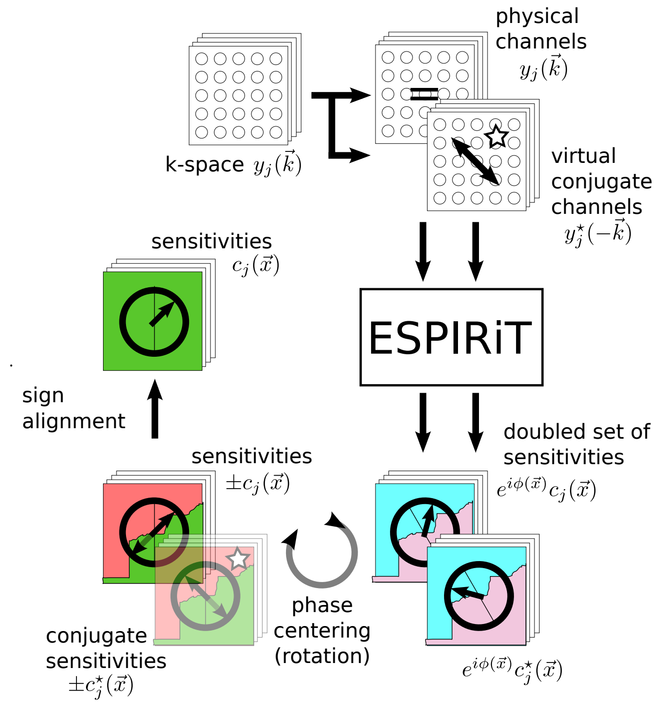

Figure 1: Processing steps in the proposed methods (clock-wise): Extension of k-space with additional virtual conjugate coils, ESPIRiT calibration, phase centering, sign unwrapping. After phase centering the virtual channels are redundant and can be discarded.

This ambiguity in phase corresponds to unknown sign in the estimated image. When enforcing a real-value constraint in SENSE or in similar reconstructions, the remaining sign ambiguity of the estimated sensitivities can be ignored. Nevertheless, the sign should be chosen correctly in model-based reconstructions or when using a positivity constraint. It also introduces artificial variations which do not fit well with sparsity or similar constraints. In general, the sign can be determined by algorithms adapted from phase unwrapping. While phase unwrapping is considered a hard problem in general, in this case only the sign of a smooth vector-valued function needs to be determined. In this work, the sign has simply been aligned to a low-resolution reference which seems to produce reasonable results in most cases. After the estimation of the phase, the virtual conjugate coils are not needed anymore and can be discarded.

A more complicated situation arises if the phase of the image is not smooth, *i.e.* it cannot be represented by a single smooth sensitivity map. In case of such inconsistencies VCC-ESPIRiT automatically produces a second set of sensitivity maps [8, 7]. As shown later, taking this second map into account will essentially relax the phase constraint in affected areas of the image, which can prevent artifacts in phase-constrained reconstructions.

3 Methods

Fully-sampled data from a human brain was acquired with 3D FLASH at 3T (TR/TE = 11/4.9 ms) using 32-channels and was retrospectively under-sampled with an acceleration factor of R = 3 in in the first phase-encoding dimension and a partial Fourier factor of PF = 5/8 in the second phase-encoding direction. Experiments with a calibration region consisting of 24x24 auto-calibration signal (ACS) lines and with 40x40 ACS lines were performed.
The Berkeley Advanced Reconstruction Toolbox (BART) was used for calibration and image reconstruction [13][1]. In the interest of reproducible research, code and data to reproduce the experiments are made available on Github [2].
Sensitivity maps were computed using ESPIRiT and the proposed VCC-ESPIRiT method. For ESPIRiT, default parameters were used: The kernel size was five and the null-space threshold in the first step of the ESPIRiT calibration was 0.001 of the maximum singular-value [5]. Based on preliminary experiments, a larger kernel size of eight was used for VCC-ESPIRiT to better capture the image phase which has more variations than the coil sensitivities. For image reconstruction, maps were weighted with a smooth S-curve transition between 1 and 0.75 of the local eigenvalue [14].
The quality of the estimated maps was directly evaluated with a projection test in a similar way as described before [5]: Coil images m_j obtained with discrete Fourier transform from fully sampled data were projected onto the span of normalized sensitivity maps by summing the images after multiplication with conjugate sensitivities and then multiplying with the sensitivities again.

$$(Pm)_s(\vec{x}) = \frac{\hat{c}_s(\vec{x}) \sum_{t=1}^{N} \hat{c}_t^\star(\vec{x}) m_t(\vec{x})}{\sum_j |\hat{c}_j(\vec{x})|^2}$$

This operation is also one of the projections repeatedly applied in POCSENSE [15]. The result of the projection $(Pm)_j$ is then subtracted from the original coil images to obtain an error map $E(\vec{x}) = m_t(\vec{x}) - (Pm)_t(\vec{x})$ where any remaining signal different from noise indicates that the maps do not describe the space the data lives in correctly. This procedure can be extended to include a real-value constraint by setting the imaginary part of the image to zero after combination of all channels and before multiplication with the sensitivities (similar to PF-POCSENSE [15]), and to project onto multiple set of ESPIRiT maps by summing the projection onto each individual set.
Iterative reconstruction using a single set of maps and without phase constraint was performed and compared to reconstructions using a real-value constraint and using either one or two sets of maps. These maps are computed using

[1] `https://mikgroup.github.io/bart/`
[2] `https://github.com/mikgroup/vcc-espirit`

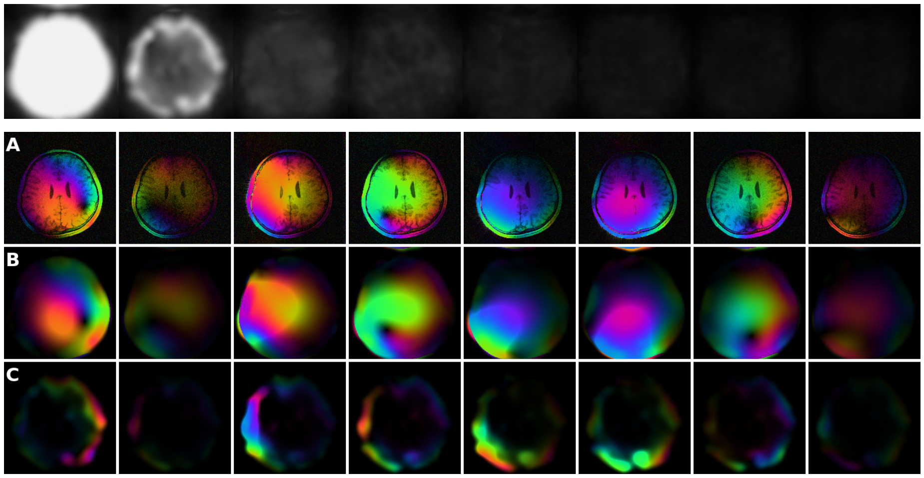

Figure 2: **Top:** Eigenvalue maps obtained from VCC-ESPIRiT calibration (only the first eight from 64 maps are shown). In some regions a second eigenvalue close to one appears which defines a second set of eigenvector maps. **Bottom:** Coil images (A), first set of maps (B), and second set of maps (C) as estimated with VCC-ESPIRiT for a human brain data set (only the eight out of 32 physical channels are shown). The color encodes the phase. The combined low-frequency phase of the image and coils is accurately captured in the first set of maps. High-frequency phase variation appear in regions with fat or blood vessels and are captured in a second set of maps.

VCC-ESPIRiT calibration and defined in all areas of the image where the corresponding eigenvalue map close to one. To avoid truncation artifacts, a smooth transition function based on the eigenvalue has been used as mentioned above.

4 Results

Figure 2 shows individual coil images and the first two sets of maps computed with VCC-ESPIRiT for the first eight channels of the brain data set. The primary set of maps represents the coil sensitivities with image phase which visually matches the phase of the coil images except for high-frequency phase components. A second set of maps appears in image regions affected by this high-frequency phase, which - in this example - is caused by off-resonance from fat and blood vessels.

Figure 3 shows the results of the projection test for the maps. The projection of the coil images onto the real part of the space spanned by the coil sensitivities shows that the image and coil phase are accurately captured in most parts of the image, but that residual signal occurs in areas with high-frequency phase from fat and blood vessels which cannot be modelled with a single set of smooth maps

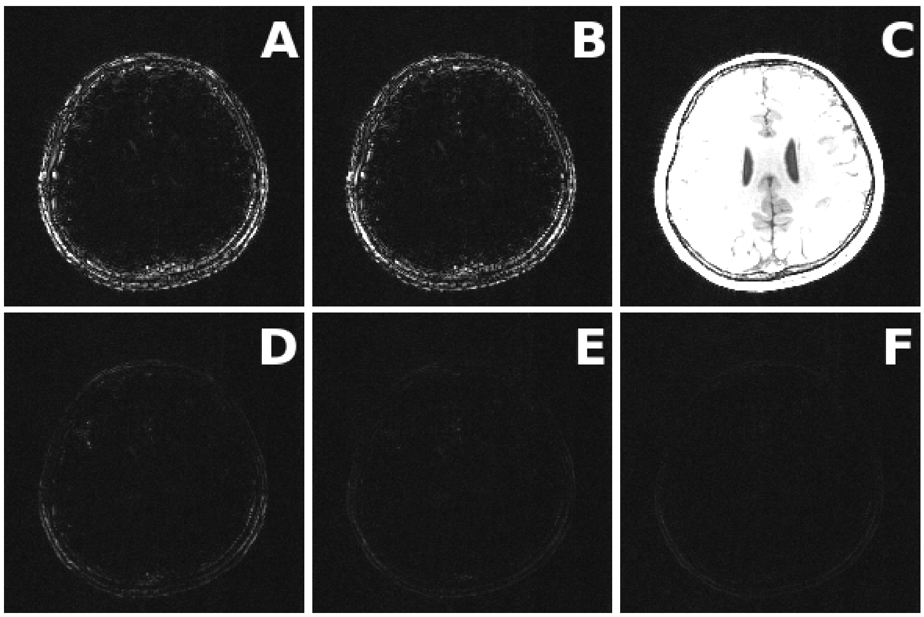

Figure 3: This figure shows the combined residual (unexplained) signal from all channels after projection of the coil images onto different spaces: real-projection onto the first set of VCC-ESPIRiT maps from 24x24 ACS lines (A), real-projection onto both sets of VCC-ESPIRiT maps from 24x24 ACS lines (B), full signal for comparison (C), real-projection onto the first set of VCC-ESPIRiT maps from 40x40 ACS lines (D), real-projection onto both sets of VCC-ESPIRiT maps from 40x40 ACS lines (E), complex-projection onto a single set of conventional ESPIRiT maps from 24x24 ACS lines (F). For each case, the residual signals for all channels have been combined into a single image using the root-sum-of-squares method. The images have been scaled up to aid visualization.

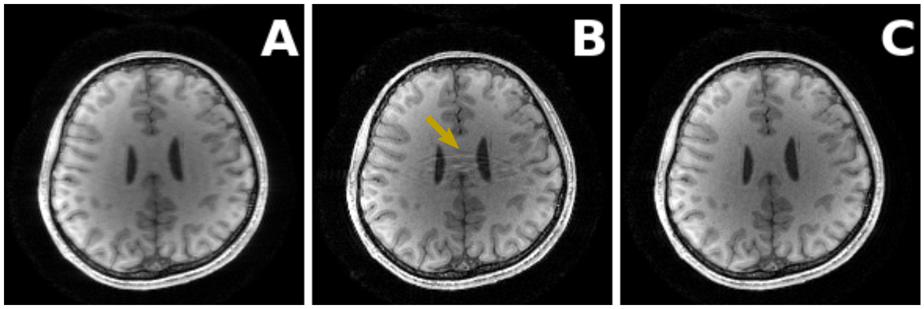

Figure 4: Iterative reconstruction from under-sampled partial Fourier data (R = 3, PF=5/8, 40x40 ACS lines) using VCC-ESPIRiT. (A) An iterative SENSE-type reconstruction without real-value constraint shows pronounced blurring due to the missing k-space information. (B) Using a real-value constraint yields a sharp image, but causes an artifact (arrow) because some regions of the image have high-frequency phase variations which are not accurately described with a single set of smooth maps. (C) The VCC-ESPIRiT reconstruction takes a second set of maps into account which relaxes the real-value constraint in affected regions. In this example, this yields an almost artifact-free reconstruction.

even with a larger calibration region. With a second set of maps this residual signal can be explained. The quality of the second map improves with a larger calibration region and then describes the data almost as good as a complex projection which allows arbitrary phase.

Figure 4 demonstrates reconstruction of data from an accelerated parallel-imaging partial-Fourier acquisition. While a conventional SENSE reconstruction is affected by blurring due to the missing information in one half of k-space, adding a real-value constraint allows recovery of a sharp image. Unfortunately, this method is often not robust if high-frequency phase errors occur as seen in this example. By taking the second map into account, *i.e.* by relaxing the phase constraint in affected images regions, an improved reconstruction can be obtained.

5 Discussion

In this work, a simple technique based for the computation of sensitivity maps which include image method has been described based on ESPIRiT with virtual conjugate channels (VCC-ESPIRiT) The computed sensitivities can then be used in a real-value constrained SENSE reconstruction to increase SNR or to exploit conjugate symmetry in image reconstruction for partial-Fourier acquisitions.

In cases where the image phase is not smooth, phase-constrained SENSE with low-resolution phase maps suffers from reconstruction artifacts. While phase-

constrained SENSE has been shown to be equivalent to SENSE with virtual conjugate coils (VCC-SENSE), which suffers from the same problem, methods using calibration in k-space such as GRAPPA or ESPIRiT with virtual conjugate coils are more robust [7]. Using the ESPIRiT formalism this observation can be explained: In regions of the image with high-frequency phase variations, a second eigenvector to the eigenvalue one in an eigendecomposition of the k-space reconstruction. That this second set of map is required to fit the data correctly has been shown directly in the present work by computing the residual after projection of full coil images onto the space spanned by the maps.
Instead of doing a reconstruction using virtual conjugate coils which is computationally very expensive, the post-processing technique described here allows the explicit computation of sensitivities with the right phase to make the image real. The reconstruction can then use an explicit real-value constraint similar to phase-constrained SENSE instead of including virtual conjugate coils as in VCC-GRAPPA. In contrast to phase-constrained SENSE, this is still robust due to the use of multiple maps typical for ESPIRiT.
As long as only high-frequency phase variations but no other kinds of corruption occur, the only essential information are the locations where a second eigenvalue close to one appears. This then indicates where the imaginary component of the signal cannot be neglected. Using the second map in a real-value constrained reconstruction relaxes the real-value constraint at the corresponding locations to allow arbitrary complex values. Of course, this could also be done in a simpler way by using only the location information from the eigenvalue maps to directly modify the constraint. Using multiple sensitivity maps has the advantage of being robust also against other types of errors.
In should be noted that with other types of errors the multiple sets of maps might appear as an arbitrary linear mixture of eigenvectors to eigenvalue one. If this is the case, the maps are usually not smooth due to mixing of different components. If smooth maps are desired, e.g. when sparsity constraints are applied to the coil-combined image, additional alignment steps similar to the ones proposed for ESPIRiT-based coil compression are needed [16].
Robust reconstruction with virtual coils or phase constraints usually requires somewhat more calibration information than a conventional calibration [7]. To avoid the increased scan time, another reconstruction method can be used to first recover a larger calibration area from partially undersampled k-space center, for example using iterative GRAPPA [7] or (robust) SAKE [17]. To exploit correlations between conjugate-symmetric parts of k-space at this stage, this could be applied to a k-space already extended with virtual conjugate coils, i.e. using VCC-SAKE. In fact, a very similar method called LORAKS has recently been proposed [18] for parallel imaging with (implicit) phase constraints.
Interestingly, information about the location of small objects with high-frequency phase such as blood vessels which appear in the second map can be recovered from a relatively small amount of data from the k-space center. This can be explained by the relationship of ESPIRiT to classical subspace-based frequency-estimation methods. From this point of view of sparse recovery, it seems more promising to use a sparsity penalty directly on the residual imaginary part as

proposed recently [19]. Although the information from the second map is then not required, such method still critically depends on accurate phase information and would benefit from the accurate estimation of the sensitivities with phase as described here.
In the present work, the technique has been used to estimate coil sensitivities which include the image phase. Although not demonstrated here, the same principle also works for phase correction of data from a single channel, e.g. a single physical (or virtual) channel or the synthetic Fourier transform of an existing complex-valued image. For example, the later might be useful for background phase removal as required for applications such as susceptibility weighted imaging.

6 Conclusion

An extension to ESPIRiT has been presented, which can be used to estimate coil sensitivities which include slowly-varying image phase. The sensitivities with phase can be directly used in a reconstruction with real-value constraint to improve SNR or to exploit conjugate-symmetry in k-space for partial Fourier acquisitions. In case of high-frequency phase variations, ESPIRiT yields a second set of maps, which can then be taken into account to make such phase-constrained reconstructions more robust.

7 Acknowledgements

The authors thank Mariya Doneva for valuable comments.

References

[1] Margosian P, Schmitt F, Purdy D. Faster MR imaging: imaging with half the data. Health Care Instrum 1986; 1:195.

[2] Noll DC, Nishimura DG, Macovski A. Homodyne detection in magnetic resonance imaging. IEEE Trans Med Imaging 1991; 10:154–163.

[3] Pruessmann KP, Weiger M, Scheidegger MB, Boesiger P. SENSE: Sensitivity encoding for fast MRI. Magn Reson Med 1999; 42:952–962.

[4] Willig-Onwuachi JD, Yeh EN, Grant AK, Ohliger MA, McKenzie CA, Sodickson DK. Phase-constrained parallel MR image reconstruction. J Magn Reson 2005; 176:187–198.

[5] Uecker M, Lai P, Murphy MJ, Virtue P, Elad M, Pauly JM, Vasanawala SS, Lustig M. ESPIRiT – an eigenvalue approach to auto-calibrating parallel MRI: Where SENSE meets GRAPPA. Magn Reson Med 2014; 71:990–1001.

[6] Blaimer M, Gutberlet M, Kellman P, Breuer FA, Köstler H, Griswold MA. Virtual coil concept for improved parallel MRI employing conjugate symmetric signals. Magn Reson Med 2009; 61:93–102.

[7] Blaimer M, Heim M, Neumann D, Jakob PM, Kannengiesser S, Breuer FA. Comparison of phase-constrained parallel MRI approaches: Analogies and differences. Magn Reson Med 2015; . Epub, doi: 10.1002/mrm.25685.

[8] Uecker M, Lustig M. Robust partial Fourier parallel imaging using ESPIRiT and virtual conjugate coils. In: Proceecings of the 22nd Annual Meeting ISMRM, Milano, 2014. p. 1629.

[9] McKenzie C, Yeh E, Ohliger M, Price M, Sodickson D. Self-calibrating parallel imaging with automatic coil sensitivity extraction. Magn Reson Med 2002; 47:529–538.

[10] Walsh DO, Gmitro AF, Marcellin MW. Adaptive reconstruction of phased array MR imagery. Magn Reson Med 2000; 43:682–690.

[11] Inati SJ, Hansen MS, Kellman P. A solution to the phase problem in adaptive coil combination. In: Proceedings of the ISMRM 21th Annual Meeting, Salt Lake City, 2013. p. 2672.

[12] Uecker M, Karaus A, Frahm J. Inverse reconstruction method for segmented multishot diffusion-weighted MRI with multiple coils. Magn Reson Med 2009; 62:1342–1348.

[13] Uecker M, Ong F, Tamir JI, Bahri D, Virtue P, Cheng JY, Zhang T, Lustig M. Berkeley advanced reconstruction toolbox, version 0.2.07, 2015, DOI: 10.5281/zenodo.16040.

[14] Uecker M, Virtue P, Vasanawala SS, Lustig M. ESPIRiT reconstruction using soft SENSE. In: Proceedings of the 21st Annual Meeting ISMRM, Salt Lake City, 2013. p. 127.

[15] Samsonov AA, Kholmovski EG, Parker DL, Johnson CR. POCSENSE: POCS-based reconstruction for sensitivity encoded magnetic resonance imaging. Magn Reson Med 2004; 52:1397–1406.

[16] Bahri D, Uecker M, Lustig M. Three-nearest-neighbor alignment for smooth ESPIRiT maps. In: Proceedings of the 22nd Annual Meeting ISMRM, Milano, 2014. p. 4394.

[17] Zhu D, Uecker M, Cheng JY, Bi Z, Ying K, Lustig M. Calibration for parallel MRI using robust low-rank matrix completion. In: Proceecings of the 22nd Annual Meeting ISMRM, Milano, 2014. p. 741.

[18] Haldar JP. Low-rank modeling of local k-space neighborhoods (LORAKS) for constrained MRI. IEEE Trans Med Imaging 2014; 33:668–681.

[19] Li G, Hennig J, Raithel E, Büchert M, Paul D, Korvink JG, Zaitsev M. An L1-norm phase constraint for half-Fourier compressed sensing in 3D MR imaging. Magnetic Resonance Materials in Physics, Biology and Medicine 2015; . doi: 10.1007/s10334-015-0482-7.