Calibration for Parallel MRI Using Robust Low-Rank Matrix Completion

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Target Audience: The target audience for this document includes those who are interested in MRI acceleration and motion compensation.

Introduction: The goal of this work is to develop a practical calibration method for parallel MRI which is robust against both under-sampling and corruption of the calibration data. It was demonstrated in [1, 2] that low-rank matrix completion can reconstruct non-uniformly under-sampled k-space without specific auto-calibration data (ACS). Furthermore the work in [3] demonstrated correction of k-space data corrupted by sparse motion. Here, we show a generalized formulation for motion-robust auto-calibration and reconstruction from under-sampled data that is incorporated into ESPIRiT [4]. The method is general and can incorporate navigation information when available. The feasibility of the method was demonstrated in simulation and in-vivo experiment.

Methods:

Low-rank matrix completion: Multi-channel k-space data is correlated and can be reformatted to form a low-rank block-Hankel matrix [1, 3]. Sparse errors due to motion corruption and missing k-space can be thought of as outliers that violate the low-rank property [5]. It is possible to restore both the missing and corrupted k-space by an outlier-robust low-rank matrix completion approach [6] of minimizing:

$$\hat{y} = \arg\min_{y} \|Py - y\|_1, \text{ subject to: } \text{rank}(H^y) = k$$

Here, $\hat{y}$ is the estimated full k-space, $y$ is the acquired data, $P$ is the sampling operator, $H$ is an operator that constructs a so called calibration matrix from the multi-channel k-space in which rows are sliding windows of the multi-channel data [1], and $k$ is the apriori known rank of the calibration matrix. The $l_1$-norm in the objective is known to be robust to outliers. When navigator information is available, the robust method can be modified into a “weighted low-rank” algorithm [7, 8].

$$\hat{y} = \arg\min_w \|w(Py - y)\|_2^2, \text{ subject to: } \text{rank}(H^y) = k$$

Experimentally, $w$ can be calculated with a 3D navigator. These objective functions can be solved by iterative singular-value thresholding [2].

Combination with ESPIRiT: The central k-space, i.e. the calibration region for ESPIRiT, can be corrected by the proposed method. Then, this recovered auto-calibration region is used to compute eigenvector (sensitivity) maps. The full k-space can then be reconstructed using ESPIRiT. With navigator weights available, the weighted low-rank method is used to correct auto-calibration data and additionally, a weighted ESPIRiT is used to reduce corruptions in the final reconstructed image:

$$m = \arg\min_{m} \|\mathcal{F}(PmS - y)\|_2 + \|Lm\|_1$$

Here, $m$ is the reconstructed image, $\mathcal{F}$ is the Fourier transform, $S$ are ESPIRiT eigenvector maps, and $L$ is a sparsity regularization.

Simulation: An 8-channel T1-weighted brain data acquired from 3D RF-spoiled FLASH (TR/TE=12.2/5.2ms, TI = 450ms, FA=20°, matrix size: 256x180x230) was artificially corrupted by random phase shifts in k-space and then retrospectively under-sampled by variable-density Poisson-disc sampling. 20% of the phase encoding lines were corrupted, followed by under-sampling without ACS region at a reduction factor of 5. After that, the corrupted k-space was corrected by the proposed methods. Weights for weighted methods were computed from the simulated motion. For comparison, images were also reconstructed using “conventional” ESPIRiT without any k-space correction.

In-vivo Experiment: Abdominal data from fat-suppressed 3D spoiled gradient echo sequence (TR/TE=3/1.2ms, FA=15°, matrix size: 192x180x68, 32 channels) was acquired during free-breathing using variable-density view-ordering (VDRad) [8]. Weights were obtained with a Butterfly navigator [8]. From this data set, a temporal phase with undersampling factor 8.4 was extracted and reconstructed with the proposed method. The reconstruction was conducted slice by slice after a 1D iFFT along frequency encoding direction. In order to reduce noise amplification, $l_1$-wavelet constraints were used in the reconstructions [4].

Results and discussion: Fig. 1a compares the simulation results of robust low-rank and weighted low-rank methods. Fig 1b shows the eigenvector maps computed from the calibration region via ESPIRiT with and without correction. These results demonstrate the efficiency of low-rank matrix completion on fixing corrupted and under-sampled data. Fig. 2 illustrates the results of in-vivo experiment. Weighted ESPIRiT significantly reduced corruption compared to not weighted ESPIRiT.

Conclusion: The proposed combination of ESPIRiT with low-rank matrix completion is robust against both under-sampling and corruption of the calibration region. Simulation and in-vivo experiment indicated the efficiency of the proposed method.