

SHORT NOTE

Equations for potential annual direct incident radiation and heat load

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Abstract. Estimation of potential annual direct incident radiation has traditionally required numerical integration with simulation models. As an alternative, we present convenient equations for use in spreadsheet, GIS, and database applications. Input variables are latitude, slope, and aspect. The equations apply to 0-60° north latitude, slopes from 0-90°, and all aspects. By transforming aspect, the equations can also be applied as an index of heat load, symmetrical about a northeast to southwest axis.

Keywords: Aspect; Azimuth; Latitude; Light; Slope; Temperature; Topography.

Introduction

Aspect of a slope (the direction or azimuth that a slope faces) is commonly measured in field studies. It strongly influences potential direct incident radiation and temperature. Untransformed, aspect is a poor variable for quantitative analysis, since 1° is adjacent to 360° – the numbers are very different even though the aspect is about the same. So aspect needs to be transformed in one of several ways, depending on the precision with which it was measured and the environmental factor(s) the analyst would like it to represent.

Incident radiation

Useful tables of potential direct solar radiation for various slopes were presented by Buffo et al. (1972). They tabulated potential direct incident radiation (cloud cover not considered) with entry points by slope, latitude, and aspect. They assumed a transmissivity coefficient of 0.9, based on a typical value for a mountain top at 1500 m on a clear day. Interpolation is required and values must be sought manually.

Potential annual direct incident radiation can be esti-

mated by simulation modeling, numerically integrating equations based on the solar trajectory, typically with a one-hour time step (see e.g. Buffo et al. 1972; Flint & Childs 1987; Kaufmann & Weatherred 1982; Nikolov & Zeller 1992; Satterlund & Means 1978). SOLARRAD software is available on the internet and the Solar Analyst extension to ArcView. There is, however, no known general analytical solution to the equations over a one-year interval. This makes these models difficult or time consuming to apply to data sets from many sites, because each site requires a separate run of the simulation model.

Heat load

While potential direct incident radiation is symmetrical about the north-south axis, temperatures are not, because a slope with afternoon sun will have higher maximum temperatures than an equivalent slope with morning sun. A reasonable approximation to heat load is to make the scale symmetrical about the northeast-southwest line. The following equation rescales aspect to a scale of zero to one, with zero being the coolest slope (northeast) and one being the warmest slope (southwest):

$$\text{Heat load index} = \frac{1 - \cos(\theta - 45)}{2}$$

where θ = aspect in degrees east of north. A very similar equation but ranging from zero to two, was published by Beers et al. (1966). These equations have been applied in various ecological studies (e.g. Neitlich & McCune 1997). While this and related equations are useful, they do not consider the steepness of the slope. For example, a 1° south-facing slope would receive the same heat load index as a 30° south-facing slope, even though the latter will be considerably warmer.

Table 1. Definitions of variables. Note that because aspect ranges from 0-180°, it is essential to ‘fold over’ aspects about the north-south line (see text).

Variable	Symbol	Units	Range	Range, angles in degrees
Folded aspect	A	radians E of N	0 - π	0 - 180°
Slope	S	radians	0 - $\pi/2$	0 - 90°
Latitude	L	radians N	0 - $\pi/3$	0 - 60°
Potential direct incident radiation	Rad	MJ cm ⁻² yr ⁻¹	0.03 - 1.11	

A new approach

We developed equations that could be used in spreadsheets, database, and GIS applications to estimate direct incident radiation and an index of head load, based on slope, aspect, and latitude (Table 1). We fit equations to potential direct incident radiation as tabulated by Buffo et al. (1972), using stepwise linear regression (Anon. 1998). There was little danger of overfitting, since the tabulated values were produced by a deterministic process without a component of random error. Nevertheless, to make the equations easy to apply, we sought an equation with $R^2 > 0.95$ with as few independent variables as possible. We tried both log-transformed and untransformed radiation as dependent variables, selecting the best solution by plotting the residuals. The resulting equations (Table 2, Figs. 1 and 2) can be used in spreadsheet, database, and GIS applications to automate the calculation, replacing manual lookup and interpolation from Buffo’s tables.

The three equations (Table 2) have slightly different uses. Eq. 1 is broadest in application, covering all slopes $\leq 90^\circ$ in steepness at latitudes 0-60° N, but has the lowest R^2 (0.96). Eq. 2 increases the R^2 to 0.98 by excluding slopes steeper than 60°, an inconsequential omission for almost all data sets. Eq. 3 uses only three parameters to produce a slightly stronger model, but is applicable only to latitudes 30-60° N.

To apply the models in a spreadsheet, set up three columns with the input variables in radians. In Excel one can use the RADIANS(x) function on an angle in decimal degrees. Aspect must first be ‘folded’ about the north-south line, rescaling 0-360° to 0-180°, such that NE = NW, E = W, etc.:

$$\text{Folded aspect} = 180 - |\text{Aspect} - 180|$$

or

$$\text{Folded aspect} = \pi - |\text{Aspect} - \pi|$$

if aspect is in radians.

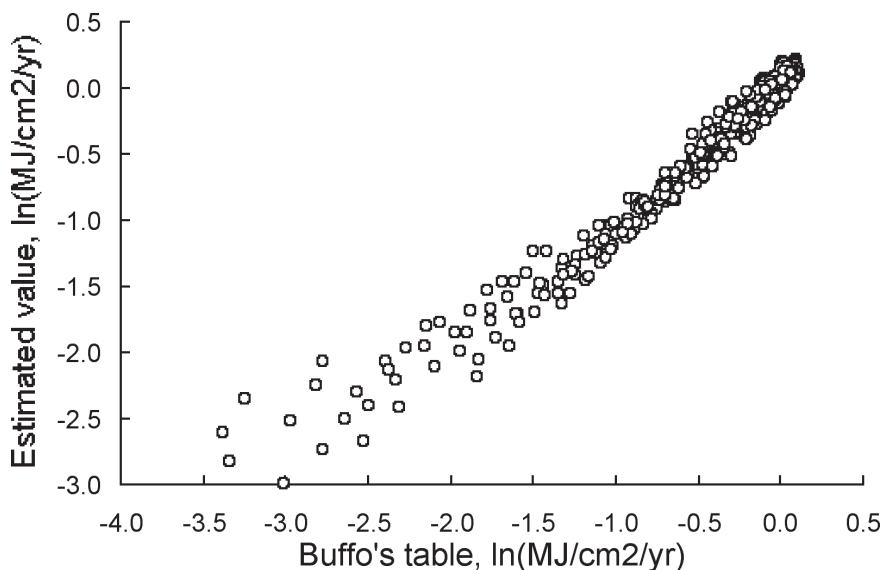


Fig. 1. Predicted potential annual direct incident radiation against values from Buffo et al. (1972), based on Eq. 1 (see Table 2). This includes 0-60° latitudes and 0-90° slopes.

Table 2. Characteristics and coefficients for the three models.

	Eq. 1	Eq. 2	Eq. 3
Number of parameters	4	4	3
Range in slope (degrees)	0-90	0-60	0-60
Range in latitude (degrees N)	0-90	0-90	30-60
Adjusted R ²	0.958	0.978	0.983
Dependent variable	ln(Rad)	ln(Rad)	Rad
Coefficients for independent variables, angles in radians			
constant	-1.467	-1.236	0.339
cos(L) × cos(S)	1.582	1.350	0.808
cos(A) × sin(S) × sin(L)	-1.500	-1.376	-
sin(L) × sin(S)	-0.262	-0.331	-0.196
sin(A) × sin(S)	0.607	0.375	-
cos(A) × sin(S)	-	-	-0.482
Example values			
40°N, 30° slope, N aspect	-0.939	-0.889	0.571
40°N, 30° slope, S aspect	-0.020	-0.005	1.053
40°N, flat	-0.255	-0.202	0.958

Assume that latitude, slope, and folded aspect are in columns G, H, and I, respectively, all in radians. In Microsoft Excel, the Eq. 1 for row 3 (for example) of the spreadsheet takes the following form:

$$=-1.467+1.582*\text{COS}(G3)*\text{COS}(H3)-1.*\text{COS}(I3)*\text{SIN}(H3)*\text{SIN}(G3)-0.262*\text{SIN}(G3)*\text{SIN}(H3)+0.607*\text{SIN}(I3)*\text{SIN}(H3)$$

This equation returns ln(Rad, MJ · cm⁻² · yr⁻¹). It can be returned to an arithmetic scale with the EXP(x) function. Test your equations against example values in Table 2.

To approximate heat load, the equation can be shifted from a maximum on south slopes to a maximum on southwest slopes and a minimum on northeast slopes. This is accomplished by ‘folding’ the aspect about the NE - SW line, such that NE becomes zero degrees and SW becomes 180°:

$$\text{Folded aspect} = |180 - \text{aspect} - 225|$$

or

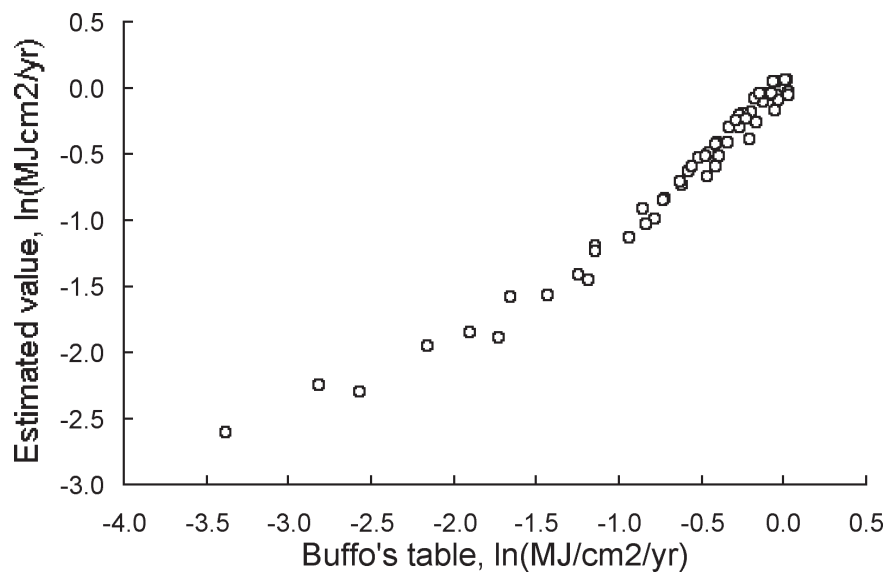
$$\text{Folded aspect} = |\pi - \text{aspect} - (5\pi/4)|$$

if aspect is in radians. One then applies the identical equation as for direct radiation. The result must, however, be considered a unitless index of heat load, since we have no basis for converting the result into a cumulative measure of temperature, such as degree days.

While these equations have proven useful, their limitations must be recognized. They do not give account to cloud cover, regional differences in the atmospheric coefficient, and shading by adjacent topography.

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Fig. 2. Predicted potential annual direct radiation against values from Buffo et al. (1972), showing only points at 40° latitude and 0 - 30° slopes, illustrating the range of values obtained in a typical hilly or mountainous landscape in a single region.



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