Center for
European,
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Discussion Papers

Number 405 – October 2020

# ASYMMETRIC GENERAL OLIGOPOLISTIC EQUILIBRIUM

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**ISSN:** 1439-2305

# Asymmetric General Oligopolistic Equilibrium

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October 2020

## Abstract

We develop an asymmetric general oligopolistic equilibrium (AGOLE) model, which extends the range of possible applications in general oligopolistic equilibrium modelling. The AGOLE allows to incorporate endogenous and asymmetric marginal utilities of income across countries. As a first exemplary application, we analyze the effects of asymmetric labor market policies. When one country increases its labor supply per capita, it is optimal for its firms to supply a part of the additional production to the other country at reduced prices to artificially inflate domestic prices. This results in a spillover effect letting consumption increase abroad due to a change in the terms of trade. In AGOLE, oligopolistic competition can induce asymmetric price reactions that shift real income and demand between the two countries. We argue that incorporating this cross-country demand channel is crucial for analyzing asymmetric countries or policies in presence of firms with market power.

**JEL-Codes:** F12, D51, L13, F16, D33

**Keywords:** General oligopolistic equilibrium, strategic trade, international trade and labor market interactions, factor income distribution.

[*]We thank Udo Kreickemeier and Robert Schwager for valuable comments. All remaining errors are our own. Department of Economics, University of Göttingen, Germany. E-mail: aquint@uni-goettingen.de; jonas.rudsinske@uni-goettingen.de

# 1 Introduction

In the last four decades we have observed a large increase in the share of pure profits of gross value added, while the labor share has declined strongly. Industry data suggests that this could be driven by an increase in industry concentration.[1] Accordingly, modeling strategic interactions among large firms is of growing importance. In a series of papers, J. Peter Neary proposed a trade model that allows to analyze oligopoly in general equilibrium (GOLE).[2] His key insight is that firms need to be modeled as ”large in the small“ sector they supply to, but ”small in the large“ economy. This approach avoids issues resulting from firms taking their influence on aggregates of the economy into account and ensuing problems with the choice of a numéraire. The GOLE literature allows to analyze the highly relevant effects of market power in a global setting. These models focus on fundamentally symmetric countries.[3] We extend this framework to incorporate fundamental asymmetries between countries. This allows to analyze welfare and distribution effects of asymmetric policies and country characteristics. Additionally, we integrate segmented markets, which allows to distinguish supply decisions of individual firms across countries. This asymmetric general oligopolistic equilibrium (AGOLE) model opens a wide range of new applications without compromising the advantages of the original GOLE concept.

As a first exemplary implementation, we analyze the effects of asymmetric labor market policies in such a context. Liberalizing the labor market, i.e. increasing the labor supply without changing the population size, in one of the countries increases consumption in both countries and the total traded quantity. Additionally, this causes a decline in domestic real wages. Whenever that country's labor market has been more liberal already, i.e. the per capita labor supply has been larger, real profits rise and the labor share declines in the other country.

[1]See for example Barkai (2020), De Loecker et al. (2020), Autor et al. (2020) and Shepotylo and Vakhitov (2020).

[2]See Neary (2007), Eckel and Neary (2010), Neary (2003b), Neary (2016), Neary (2003a) and Neary (2010).

[3]The marginal utilitiy of income has to be identical in both countries, which is only the case, when countries are fundamentally symmetric. The only asymmetries allowed are those which keep the countries' marginal utility of income equal, e.g. a mirror-inverted sectoral productivity distribution that keeps the average productivity symmetric.

These effects stem from the interaction of oligopolistic competition with asymmetries in countries' real income per capita, which affect their representative consumers' marginal utility of income. As oligopolists maximize profits while treating both markets as segmented, they take the asymmetric demand aggregates in form of the marginal utilities of income into account, but perceive them as exogenous. In our application, one of the countries starts to produce more thereby increasing its real income per capita. However, firms will now find it optimal to supply a part of the additional production to the other country at reduced prices in order to artificially inflate domestic prices until marginal revenues equal marginal costs again in both markets. This mechanism causes a spillover effect, where consumption increases in the other country, that now benefits from the labor market liberalization abroad due to a change in the terms of trade. Oligopolistic competition thereby induces asymmetric price reactions in the two countries that raise real income and demand abroad.

It is tremendously important when analyzing asymmetric countries or even asymmetric policies between symmetric countries in presence of firms with market power to consider these strategic incentives. The mechanism shapes the welfare and distribution effects of policies and globalization. Thus, we consider it as vital for many theoretical tasks to take such a cross-country demand channel into account. The AGOLE framework incorporates this channel. We apply it to asymmetric taxation and tax-motivated transfer pricing in Quint and Rudsinske (2020), while in Rudsinske (2020) asymmetric import tariffs are analyzed.

There is a growing literature using the GOLE model.[4] Its fundamental structure is described in Neary (2016). Neary (2007) shows that trade liberalization can lead to international mergers thereby increasing specialization and trade. Bastos and Kreickemeier (2009) use it to analyze the effects of unionization in a symmetric country setting. Egger and Etzel (2012) examine the impact of trade in such a unionization context on welfare and the income distribution. Egger and Etzel (2014) implement an asymmetric degree of labor union centralization in such a general oligopolistic equilibrium model, where countries are

[4]For a detailed survey of the GOLE literature see Colacicco (2015).

still fundamentally symmetric. Eckel and Neary (2010) introduce multi-product firms in a GOLE model. Fujiwara and Kamei (2016) focus on the effects of a symmetric tariff reduction in a GOLE model with an explicit division of labor. Beladi and Chakrabarti (2019) analyze the effects of divisionalization on the extensive margin of trade. In all these models, countries are fundamentally symmetric and markets are not segmented, so a single world market is solved to derive the equilibrium. This allows to consider only the world average marginal utility of income, which improves tractability, but comes at the cost of not being able to determine total traded quantities. Therefore, we extend the model to the case of segmented markets. As a result, we derive equilibrium marginal utilities of income for each country separately, which allows them to be fundamentally asymmetric. To keep things as simple as possible, we abstract from introducing other elements of the literature like market entry, multi-product firms or divisionalization.

Segmented markets allow to explain the empirical observation of pricing-to-market (see e.g. Fitzgerald and Haller (2014)), although in our Cournot-framework firms optimize their supplied quantities separately for different markets, which only then results in differing prices in the two countries. Head and Spencer (2017) provide anecdotal justification for segmented markets by noting that contracts often include conditions that restrict sales to other countries, such that arbitrage is not possible. Ben-Zvi and Helpman (1988) show how trade costs cause market segmentation in partial oligopoly. While their model collapses to a single integrated world market in the absence of trade costs, our model collapses to an integrated world market when countries are completely symmetric, but country asymmetries give rise to market segmentation even in the absence of trade costs. Markusen (2013) finds that in trade models with imperfect competition non-homothetic preferences can explain higher price levels in richer countries, which nicely fits to the empirical observation of a positive per-capita-income coefficient in gravity equations. Our model with quadratic preferences in oligopolistic competition exhibits this mechanism.

Brander (1981) was the first to stress that strategic interactions can give rise to two-way trade in identical commodities. After removing a Ricardian technological comparative advantage in our model for simplicity, trade functions similarly to the famous reciprocal dumping model in Brander and Krugman (1983), where the rivalry of oligopolistic firms is the single cause of international trade.

In chapter two, we point out the basic model structure including the Cournot equilibrium. We turn to the solution strategy for the general equilibrium in chapter three. An exemplary application is provided in chapter four for the case of asymmetric labor market policies. The last chapter concludes.

# 2 The Model Setup

In this section, we will describe the underlying elements of the model. We assume oligopolistic competition within each sector of the two-country economy and bring the sectors together in a general equilibrium approach. To this end, we adapt the model of international trade in general oligopolistic equilibrium developed by Neary (2016). However, we do not incorporate a world market for each sector, but distinct national markets. We will usually only present the expressions for the Home country. Expressions for Foreign are analogous.

## 2.1 Demand

Each country is inhabited by a representative consumer, whose preferences over the consumed goods are additively separable. Following Neary (2016) we use continuum-quadratic preferences:

$$U[\{y(z)\}] = \int_0^1 u[y(z)]dz \qquad \text{with} \quad \frac{\partial U}{\partial y(z)} > 0 \text{ and } \frac{\partial^2 U}{\partial y(z)^2} < 0$$
$$\text{where} \quad u[y(z)] = a\ y(z) - {}^1\!/\!_2\ b\ y(z)^2.$$

The consumption of a homogeneous good produced in sector $z \in [0,1]$ is denoted by $y(z)$ and $a, b > 0$ are exogenous parameters. The consumer is indifferent between domestically produced goods and imports in each sector $z$.

The representative consumer inelastically supplies $L$ units of labor to a perfectly competitive labor market. The yet to be determined wage rate will result in a wage income of $wL$. Additionally, aggregate profits ($\Pi$) of all firms producing in Home are disbursed to the representative consumers. Therefore, his income is given by

$$I = wL + \Pi. \tag{1}$$

With price $p(z)$ per unit of the good in sector $z$, the budget constraint is

$$\int_0^1 p(z)y(z)dz \leq I. \tag{2}$$

Utility function and budget constraint lead to the utility maximization problem represented by the Lagrangian:

$$\max_{y(z),\forall z} \mathcal{L} = \int_0^1 \left(ay(z) - {}^1\!/\!{}_2\, by(z)^2\right) dz + \lambda \left(I - \int_0^1 p(z)y(z)dz\right).$$

The first order condition then gives

$$0 = a - by(z) - \lambda p(z) \qquad \forall z$$

with $\lambda$ being the Lagrange-parameter and therefore the marginal utility of income. The inverse Frisch demand follows straightforwardly and is given by

$$p(z) = \lambda^{-1}\frac{\partial u[y(z)]}{\partial y(z)} = {}^1\!/\!{}_\lambda[a - by(z)]. \tag{3}$$

Frisch demands specify a relation between the price, the quantity demanded and the marginal utility of income. The inverse demand functions (3) depend on the marginal utility of income negatively. The marginal utility of income $\lambda$ acts as a demand aggregator where a higher value indicates a lower demand for goods in every sector. The inverse formulation ($\lambda^{-1}$) can be interpreted as the marginal costs or the price of utility (Browning et al. (1985)). The value of $\lambda$ will be determined in general equilibrium.

## 2.2 Supply

The producers aim to maximize their profits. They perceive the demand they face in the two seperate markets as well as the wage rates as given. Analogously to Neary (2016), firms are assumed to have market power in their respective markets. However, they do not have direct influence on aggregate economic factors, as many sectors $z$ exist and only jointly determine these aggregates. This especially includes the demand aggregator $\lambda$, but also the wage rate $w$, which all firms take as given.

As firms do not affect economy-wide variables, it is natural to assume that they maximize profits in their specific sector. There is no alternative objective such as the overall welfare of their owner – the representative consumer – as firms are not able to take their economy-wide influence into account and each sector's individual influence on the overall welfare is negligible. Gabszewicz and Vial (1972) argue that firms are myopic in such a context. Firms can collectively influence relative prices, but cannot affect them individually because they are small in the large.

We assume that $n$ firms exist in Home in each sector $z$ and that there are neither fixed costs of production nor transport costs. The firms play a static one-stage game where they compete à la Cournot over supply in the Home and the Foreign market. Irrespective of the functional form of $\lambda$ they perceive the inverse demand as linear.

Production occurs with constant returns to scale and common technology in each sector, such that cost $c(z)$ in sector $z$ is linear in output. Labor is the only factor of production and moves freely across sectors within a country, but not across national borders. The wage rate is determined at the country level by combining the inelastically supplied labor and the demand for labor that results from domestic companies' production.

The unit-cost function for sector $z$ is then given by $c(z) = w\gamma(z)$, where $\gamma(z)$ is the sector-specific unit-labor requirement. For simplicity, we only consider the case of identical technology across sectors and countries $\gamma(z) = \gamma^*(z) = 1 \; \forall z$ so we can drop $z$ throughout. Thus, the model does not capture a Ricardian-style technological comparative advantage anymore as it did in Neary (2016). The reasons for trade in our setting are the strategic considerations among firms in Cournot competition. However, we retain the assumption of a multitude of sectors even though they will be symmetrical. As companies remain small in the large, they still do not take their effect on wages into account when maximizing their profits. Therefore, marginal costs are constant independent of the sales' destination.

As already mentioned, we assume that there are distinct markets in the two countries. Companies supply their output to both countries, where the underlying demand may differ. Demand differences can result from differing labor endowments or the countries' industrial structures.[5] The profit of a firm in one sector is given by

$$\pi \quad = \quad (p-w)y_h + (p^* - w)y_f, \tag{4}$$

where $p^{(*)}$ are the inverse Frisch demands in equation (3) in the Home and the Foreign market respectively. The quantities a Home firm supplies in Home is $y_h$, while $y_f$ gives the supply to Foreign by Home firms.[6]

## 2.3 Cournot Equilibrium

The firms will maximize their profits by choosing the amount of goods to produce and sell given the demand and the other companies' supply.

$$\begin{aligned} \max_{y_h, y_f} \pi &= [p-w]\, y_h + [p^* - w]\, y_f \\ & \quad \text{with } p = {}^1\!/\!_\lambda[a - by] \text{ and } p^* = {}^1\!/\!_{\lambda^*}[a - by^*], \end{aligned}$$

where $y = ny_h + n^* y_h^*$ describes the total supply of the good in Home and analogously $y^*$ in Foreign.

The first order conditions for the firms' profit maximization are

$$\frac{\partial \pi}{\partial y_h} \quad = \quad \frac{1}{\lambda}\left(a - 2\ b\ y_h - b((n-1)\tilde{y}_h + n^* y_h^*)\right) - w = 0 \tag{5}$$

[5]Additionally, other policies, implemented by governments, may also alter the countries' demands.
[6]Supplies by Foreign firms are marked with an asterisk: $y_i^*$ with $i = h, f$.

$$\frac{\partial \pi}{\partial y_f} = \frac{1}{\lambda^*}\Big(a - 2\,b\,y_f - b((n-1)\tilde{y}_f + n^* y_f^*)\Big) - w = 0 \tag{6}$$

where $\tilde{y}_i$ is the quantity that each of the other Home companies supplies in the respective market. As companies producing in the same country are symmetric, they will supply the same quantity, $\tilde{y}_i = y_i$, in market $i$. The first order conditions result in the well-known relation that marginal costs have to equal marginal revenues in an oligopolistic equilibrium. The first order conditions can be transformed into reaction functions depending on the supply of Foreign companies.

$$y_h = \frac{a - \lambda w - b\,n^*\,y_h^*}{b(n+1)} \tag{7}$$

$$y_f = \frac{a - \lambda^* w - b\,n^*\,y_f^*}{b(n+1)} \tag{8}$$

For firms from Foreign, these reaction functions are defined analogously. We obtain the Cournot-equilibrium supply by each firm in the two countries by combining the reaction functions. The equilibrium supply of each individual company to the Home market is

$$y_h = \frac{\lambda}{b(n+n^*+1)}\left\{\frac{a}{\lambda} - w + n^*(w^* - w)\right\} \tag{9}$$

$$y_h^* = \frac{\lambda}{b(n+n^*+1)}\left\{\frac{a}{\lambda} - w^* + n(w - w^*)\right\}. \tag{10}$$

The supplied quantities in Foreign are analogous:

$$y_f = \frac{\lambda^*}{b(n+n^*+1)}\left\{\frac{a}{\lambda^*} - w + n^*(w^* - w)\right\} \tag{11}$$

$$y_f^* = \frac{\lambda^*}{b(n+n^*+1)}\left\{\frac{a}{\lambda^*} - w^* + n(w - w^*)\right\}. \tag{12}$$

The number of firms in either country has the usual competitive effects on the supplied quantities. A new entrant will lead to reduced marginal revenues resulting in reduced supply by incumbent firms. Changes in wage rates have two effects on the Cournot equilibrium. Higher wages will reduce the supply by companies, which bear the rising production costs. In reaction, firms from the other country will expand their supply as they benefit from a cost-based advantage.

# 3 General Equilibrium

In general equilibrium we need to determine the wage rates and the demand aggregators in both countries. We will first solve the labor market equilibrium before turning to our solution strategy for the demand aggregators.

## 3.1 Labor Market

With the Cournot-Nash-equilibrium supply derived above we now turn to the clearing of the labor market. As described, the representative consumer inelastically supplies $L^{(*)}$ units of labor. The labor demand depends on the equilibrium production in a country. Each company in one sector in Home will supply

$$\bar{y} \;=\; y_h + y_f = \frac{1}{b(n+n^*+1)}\left\{2a - \bar{\lambda}\; w + \bar{\lambda} n^*(w^* - w)\right\},$$

where $\bar{\lambda} \equiv \lambda + \lambda^*$. The total labor demand is given by $L^D = \int_0^1 n\; \bar{y}\; dz$. In equilibrium, demand has to equal supply.

$$L = \int_0^1 n\; \bar{y}\; dz = \frac{n}{b(n+n^*+1)}\left\{2a - \bar{\lambda}\; w + \bar{\lambda} n^*(w^* - w)\right\} \tag{13}$$

This relation defines the nominal wage rate in Home. In combination with the analogously defined equilibrium on the foreign labor market, wages in both countries are given by

$$w \;=\; \frac{1}{\bar{\lambda}}\left\{2a - b\left[\frac{n+1}{n}L + L^*\right]\right\} \tag{14}$$

$$w^* \;=\; \frac{1}{\bar{\lambda}}\left\{2a - b\left[\frac{n^*+1}{n^*}L^* + L\right]\right\}. \tag{15}$$

Here, we can already see that for constant $\bar{\lambda}$ an increase in labor supply reduces the wage rate in both countries.

## 3.2 Balance of Payments Equilibrium

The model is characterized by nine equations In equilibrium. The Cournot equilibrium quantities determine the supply of each multinational company to each country given the wages and the marginal utilities of income. The labor market clearing in each country determines the wage rates given the produced quantities in the respective country. Additionally, the prices are given by the representative consumers' inverse Frisch demand functions.

The last equation results from the budget constraint of the representative consumer in either country. We use the budget constraint to attain an implicit definition of the marginal utilities of income in equilibrium.[7] The budget constraint in Home is given by

$$p \cdot (n\ y_h + n^*\ y_h^*) \quad = \quad w\ L + n\ \pi$$

This can be rearranged using the definition of $\pi$ in equation (4) to obtain a straightforward relationship that has to hold in equilibrium:

$$p^*\ n\ y_f = p\ n^*\ y_h^* \tag{16}$$

To close the model, we need a balance of payments equilibrium. In our simple setting, the capital balance is zero. Thus, the respective values of trade have to be equal in general equilibrium. We have the value of exports from Home to Foreign on the left that have to be equal to the value of imports from Foreign to Home at the right. The equality of these values does not imply that the traded quantities of goods are equal. If prices or the number of firms differ, one country might import more units of a good than it exports.

To attain the general equilibrium values of the endogenous variables and to solve the system of equations, we need to determine the marginal utilities of income. The foundation of our model is given by Neary (2016), but in his case this step is less difficult. Most importantly, in his case the two countries are symmetric such that $\bar{\lambda} = 2\lambda$, which

[7] Neary (2016) uses the same relationship to determine the marginal utility of income, see his footnote 13.

significantly reduces the complexity of the equilibrium. In our case, we cannot reduce the result that way, as countries are not necessarily symmetric.

We normalize the aggregate marginal utility of income to unity, i.e. $\bar{\lambda} = 1$. Hence, the aggregate marginal utility of income is used as numéraire. This translates into the relationship between $\lambda$ and $\lambda^*$ that $\lambda^* = 1 - \lambda$, which allows us to substitute all $\lambda^*$. Because both marginal utilities of income have to be positive, it follows that both will lie between zero and one, which is useful for determining the signs of derivatives later on.

As Neary (2016) notes, the model is not sensitive to the choice of numéraire. Most importantly, firms do only have an influence in their own sector, but not on the factor market or national income. If this would be different, companies would exert their monopsonistic power in the labor market and would account for their influence on income. In that case profit maximization could be an inadequate objective for the companies, while there could also result a dependency on normalization rules (Neary (2003b)). In GOLE models, this issue is solved by modeling a continuum of sectors, such that firms do not exert this influence. This inability of individual firms to influence aggregates is comparable to models with monopolistic or perfect competition (Neary (2003c)).

We can now further simplify the system of equations by expressing all endogenous variables, such that they only depend on exogenous parameters and $\lambda$. These formulations can then be used in the balance of payments condition such that we only have one equation in one variable left. We can show that for the admissible range $\lambda \in (0, 1)$ there is only one solution to this equation.

**Proposition 1** (Existence and Uniqueness)**.** *There exists a unique solution to the condition of an even balance of payments in* $\lambda \in (0, 1)$.

*Proof.* First, we reformulate the condition that the balance of payments is zero. In this paper's setting the balance of payment is equal to the balance of trade ($BoT$).

$$\begin{aligned} BoP \equiv BoT = 0 &= p^*(\lambda)\ n\ y_f(\lambda) - p(\lambda)\ n^*\ y_h^*(\lambda) \\ &= x_1 + x_2\lambda + x_3 \left( \frac{\lambda n}{1-\lambda} - \frac{n^*}{\lambda} \right), \end{aligned}$$

where

$$\begin{aligned}
x_1 &= \frac{a}{n+n^*+1}(L(3n+3n^*+1)+L^*(n+n^*-1)) \\
&\quad +\frac{a^2}{b}\frac{2n^*-n-2(n+n^*)^2}{(n+n^*+1)^2}-b(L+L^*)L \\
x_2 &= b(L+L^*)^2-4a\frac{n+n^*}{n+n^*+1}(L+L^*)+4\frac{a^2}{b}\frac{(n+n^*)^2}{(n+n^*+1)^2} \\
x_3 &= \frac{a^2}{b(n+n^*+1)^2}.
\end{aligned}$$

In the limiting cases of the admissible $\lambda$, we can show in the supplement that the $BoT$ is

$$\lim_{\lambda\to 0^+} BoT = -\infty$$
$$\lim_{\lambda\to 1^-} BoT = \infty.$$

Additionally, the $BoT$ is differentiable with respect to $\lambda$, which implies continuity of the $BoT$. Therefore, there has to be at least one solution of the above equation for $\lambda \in (0,1)$.

At the same time we can show with mathematical software that the balance of trade has a strictly positive derivative with respect to $\lambda$ in $\lambda \in (0,1)$.

$$\frac{\partial BoT}{\partial \lambda} = x_2 + x_3\left(\frac{n}{(1-\lambda)^2}+\frac{n^*}{\lambda^2}\right) > 0$$

Therefore, there exists a unique solution. □

However, it is difficult to derive the equilibrium marginal utility of income in closed form. It is defined as the root of a cubic polynomial with a positive discriminant. Therefore, in our simple setting it is possible to obtain the expression of $\hat{\lambda}$ in closed form, as all solutions to the polynomial are real valued, but only one is within $(0,1)$. Still, it is complex and the derivation is obtained with mathematical software and deferred to the supplement.

However, in many cases it is not necessary to determine the closed-form value of $\hat{\lambda}$. Firstly, the derivative of the marginal utility of income in general equilibrium with respect to a parameter of interest can be derived from the balance of payments condition using implicit differentiation. Secondly, we know due to the normalization that $\hat{\lambda} \in (0, 1)$, which often suffices to determine signs of endogenous variables' derivatives with respect to a parameter of interest, given the sign of the derivative of $\hat{\lambda}$ with respect to that parameter. Hence, qualitative results can often be shown even in extensions of the model that are too complex to derive $\hat{\lambda}$ in closed form.

A defining feature of the model are the supply decisions by the oligopolists. To give the reader some intuition about the mechanisms that determine the supply in general equilibrium, we can break the system down into two fundamental conditions. The consumption indifference condition (CI) states that for utility maximization the origin of the product is inconsequential. The representative consumer is indifferent between products in the same sector that are produced in Home and in Foreign. The market indifference condition (MI) states that in equilibrium all firms have to be indifferent between selling the marginal unit in Home or in Foreign. The MI line represents all Cournot-equilibria for all possible demands firms face. We can plot these two conditions in a box diagram in Figure 1.

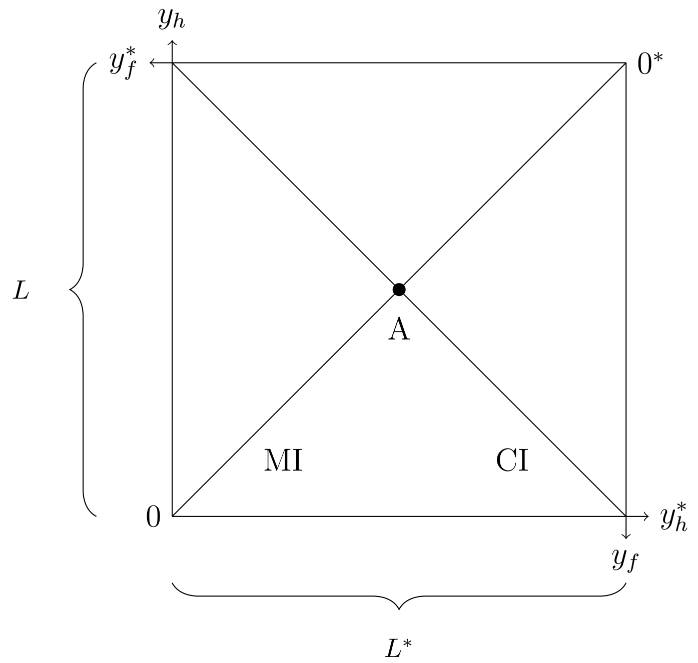


Figure 1: General Equilibrium with symmetric countries ($L = L^*$ and $n = n^*$)

In the box diagram, the supply decision of one firm from Home and of one firm from Foreign are depicted. As all firms producing in the same country are symmetric, this still provides useful insights not only into each firm's decision, but also the overall supply.

The CI line is easily derived from the budget constraint.

$$CI: \qquad y_h = \frac{I/p}{n} - \frac{n^*}{n} y_h^*$$

It gives us a function with perfect substitutability between the goods of a Home and a Foreign firm from the consumer's perspective, for whom real income $I/p$ is exogenous. The slope of the CI line is $-n^*/n$. If all Foreign firms increase their supply, Home firms need to reduce theirs such that the overall consumption is unchanged. An increase in real income in Home shifts the CI line towards the upper-right corner. In symmetric equilibrium ($L = L^*$), the intercept is exactly in the upper left corner of the graph. In this case both countries consume the same quantities. The same can analogously be done for the Foreign representative consumer giving us exactly the same line in the graph.

The MI line follows straightforwardly from the profit maximization in equations (5) and (6) as marginal revenues need to be equal across countries in equilibrium. The same relation needs to hold for Foreign firms.

$$\begin{aligned} \frac{1}{\lambda}\left(a - b((n+1)y_h + n^* y_h^*)\right) &= w = \frac{1}{\lambda^*}\left(a - b((n+1)y_f + n^* y_f^*)\right) \\ \frac{1}{\lambda}\left(a - b(y_h + (n^*+1)y_h^*)\right) &= w^* = \frac{1}{\lambda^*}\left(a - b(y_f + (n^*+1)y_f^*)\right) \end{aligned}$$

As both relations need to hold in the Cournot-equilibrium, they can be combined to attain

$$p - p^* = \frac{1}{\lambda}\left(a - b(ny_h + n^* y_h^*)\right) - \frac{1}{\lambda^*}\left(a - b(ny_f + n^* y_f^*)\right) = b\left(\frac{y_h}{\lambda} - \frac{y_f}{\lambda^*}\right) = b\left(\frac{y_h^*}{\lambda} - \frac{y_f^*}{\lambda^*}\right).$$

This gives us a direct link between the cross-country price difference and the firms' supply to the two markets – weighted with the respective marginal utilities of income. As $y_h^{(*)} + y_f^{(*)} = L^{(*)}/n^{(*)}$, we obtain a relation between the supply to Home by one Home firm and one Foreign firm, which has to hold in the Cournot-equilibrium.

$$MI: \qquad y_h = \lambda\left(\frac{L}{n} - \frac{L^*}{n^*}\right) + y_h^*. \tag{17}$$

This results in a line with the slope +1. The intercept consists of two parts. Firstly, within the brackets we have the Home production per firm minus the Foreign production per firm. If this is positive, Home firms have a larger quantity to supply than Foreign firms. Secondly, the marginal utility of income $\lambda$ gives the share of this excess production that is allocated to the Home market, whereas $1 - \lambda = \lambda^*$ is the share allocated to Foreign. Figure 1 shows how the equilibrium supplies are determined at the intersection of MI line and CI line.

# 4 Application: Asymmetric Labor Market Policy

The AGOLE model allows to analyze a wide variety of economic aspects, for example asymmetries in market concentration[8], trade policies or taxation.[9] We want to provide an exemplary application of the AGOLE model by analyzing changes in the labor market regime in one of the countries.

Changes in labor endowment $L$ in our setting may be interpreted as labor market policies that increase the average working hours in a country. Such changes in $L$ do not increase the population as given by the number of consumers, which are characterized by the representative consumer. This is because the structure of the utility function of a country's representative consumer does not change with $L$. An increase in the population would increase the labor endowment and the number of consumers equally, such that it should not affect consumption per capita and the marginal utility of consumption. In our application, only working hours per capita are rising, while the number of consumers is

[8]For $L = L^*$, it is easy to show the effects of changes in $n$. The domestic labor share falls with rising concentration in a country, while the other country is unaffected. With $L \neq L^*$ this becomes computationally more demanding.

[9]See Rudsinske (2020) for an analysis of asymmetric tariffs and Quint and Rudsinske (2020) for an analysis of asymmetric taxation and tax-motivated transfer pricing.

unaffected, such that consumption per capita rises and the marginal utility of consumption falls. We will focus on changes in $L$ in the Home country. For simplicity, we assume $n = n^* = 1$. Additionally, we set $b < \frac{a}{L+L^*}$ to ensure a positive marginal utility of consumption in any case. This also leads to positive wage rates in both countries.

Firms in a substantially smaller country may only supply to the larger country, but nothing to the country in which they produce, as demand and hence the marginal revenues are smaller there. To ensure an interior solution we assume that the countries' labor endowments are sufficiently similar to get positive supplies to all countries by all firms. The larger country may only have just under a 2.5 times larger endowment, i.e. $2/5 < L/L^* < 5/2$.[10] These restrictions reduce the possible values, $\hat{\lambda}$ may take. In full symmetry with $L = L^*$, the marginal utility of income will be one half in equilibrium in both countries. If $L > L^*$, we can show that $1/4 < \hat{\lambda} < 1/2$. For $L < L^*$, we have $1/2 < \hat{\lambda} < 3/4$.[11]

To analyze the changes in general equilibrium, we need the reaction of $\hat{\lambda}$ to changes in $L$. Therefore, we first derive the effect of an increase in $L$ on $\hat{\lambda}$.

**Lemma 1** (Effect of $L$ on $\hat{\lambda}$)**.** *An increase in $L$ will reduce the marginal utility of income in Home in general equilibrium ($\hat{\lambda}$).*

*Proof.* In the equilibrium condition in equation (16), $\hat{\lambda}$ is implicitly defined. The derivative of $\hat{\lambda}$ with respect to $L$ is then given by

$$\frac{\partial \hat{\lambda}}{\partial L} = -\frac{\partial BoT/\partial L}{\partial BoT/\partial \lambda}.$$

The derivative of the balance of trade with regard to $\lambda$ is positive as shown in the proof of uniqueness. Additionally, the derivative of the balance of trade with respect to $L$ is given by

$$\frac{\partial BoT}{\partial L} = \frac{a}{3}(7 - 8\lambda) + bL^*(2\lambda - 1) - 2bL(1 - \lambda).$$

In the supplement we show that this derivative is positive. It follows that the derivative

[10] We show in the supplement that all supplied quantities are positive under this condition.

[11] See the Supplement for the derivations.

${\partial\hat{\lambda}}/{\partial L}$ is negative. □

When average working hours $L$ increase, Home production rises. Home firms maximize their profits and thus set marginal revenues equal to marginal costs, which are identical for supplies to both markets. As a consequence, it is not optimal for Home firms to supply all newly produced goods to Home. They can increase their average mark-ups by supplying more to both markets. Thus, they will not only increase their domestic supply, but also increase their exports. Consequently, as long as prices do not react, Home will run a trade balance surplus. This is impossible in equilibrium. Therefore, the balance of trade needs to adjust. On the one hand, the price ratio ${p}/{p^*}$, which is also the Foreign terms of trade, increases. On the other hand, supply decisions adjust, but this does not reverse the initial impetus. Because both the traded quantity $y_f + y_h^*$ and the Foreign terms of trade increase, it follows that Foreign consumption rises as well.

**Proposition 2** (Consumption and Trade)**.** *Liberalizing the labor market, i.e. increasing the average working hours, in one of the countries increases consumption in both countries and raises the total traded quantity.*

*Proof.* Changes in consumption in Home and Foreign are given by

$$\begin{aligned}
\frac{\partial}{\partial L}(y_h + y_h^*) &= \lambda + \frac{\partial\hat{\lambda}}{\partial L}\left(L + L^* - \frac{4}{3}\frac{a}{b}\right) \\
\frac{\partial}{\partial L}(y_f + y_f^*) &= (1-\lambda) + \frac{\partial\hat{\lambda}}{\partial L}\left(\frac{4}{3}\frac{a}{b} - (L + L^*)\right)
\end{aligned}$$

The increase of consumption in Home is straightforward as $\frac{\partial\hat{\lambda}}{\partial L} < 0$ and $L + L^* < {4a}/{3b}$ under our restriction of $b < {a}/{(L+L^*)}$. For consumption in Foreign we show in the supplement that the derivative is positive in equilibrium.

The sum of exports will be affected by an increase in $L$ according to

$$\frac{\partial}{\partial L}(y_f + y_h^*) = 1 - \lambda + \frac{\partial\hat{\lambda}}{\partial L}(L^* - L).$$

For $L \geq L^*$ it follows immediately that the derivative is positive, but also for $L < L^*$ we show in the supplement that the derivative remains positive. □

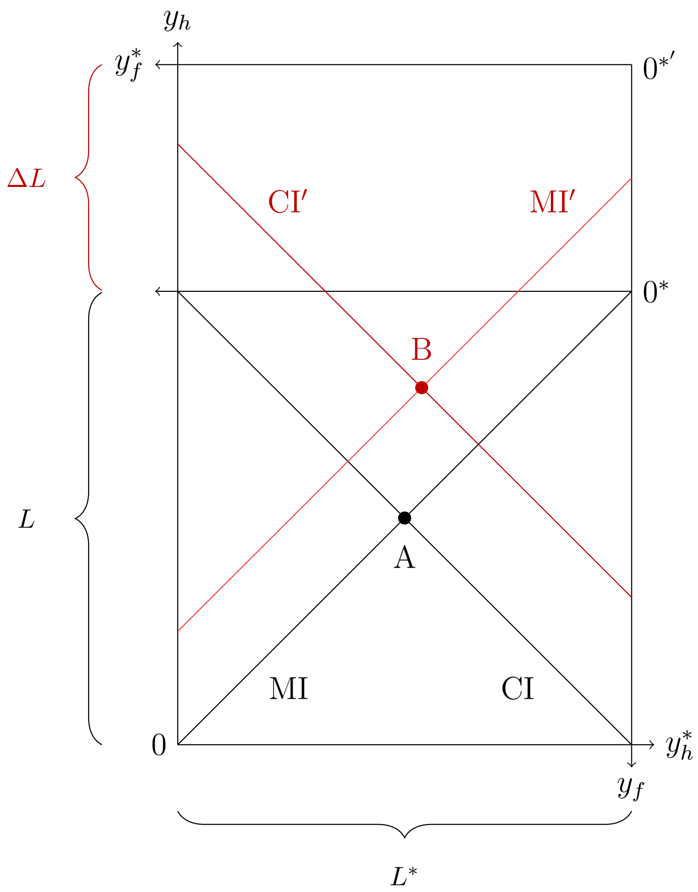


Figure 2: Home Labor Market Liberalization ($n = n^* = 1$)

Figure 2 graphically illustrates the effects. Both the CI and the MI lines shift upwards. The rise of consumption in Foreign can be described as a spillover effect. In consequence, the CI line shifts less than the initial rise in the labor endowment. This captures that less than 100 percent of the additional production is consumed in Home. Also the MI line shifts less than the rise in labor supply, because all firms have an incentive to increase exports. However, which curve shifts more is ambiguous. We depict the case where the CI shifts more, which is always true if $L^* \leq 2L$ and depends on the parameters $a$ and $b$ otherwise.

With the growing labor supply on a perfect labor market, nominal wages will fall in Home. Because of competitive interactions Foreign firms want to supply less, thereby reducing its labor demand, such that nominal wages in Foreign fall as well, though less than in Home. With declining prices, however, the real wage effect in Foreign is ambiguous.[12] By contrast, real wages unambiguously decrease in Home as prices increase there.

In equilibrium, Home firms' costs, i.e. the nominal wages, decline more strongly than those of Foreign firms. Home firms can improve their nominal profits, while the real profit reaction is ambiguous.[13] The reaction of real profits in Foreign is more complex, but we show that, at least when the Home labor market has already been more liberal before Home further liberalizes it, Foreign real profits increase and the Foreign labor shares declines.

**Proposition 3** (Income and Distribution)**.** *Liberalizing the labor market, i.e. increasing the average working hours, in one country causes its real wages to fall. Whenever that country's labor market has been more liberal already, i.e. the average working hours have been larger, real profits rise and the labor share declines in the other country.*

*Proof.* The changes in nominal wages and prices in Home, where $L$ is increased, are given by

$$\begin{aligned} \frac{\partial w}{\partial L} &= -2b \\ \frac{\partial p}{\partial L} &= -\frac{1}{3}\frac{a}{\hat{\lambda}^2}\frac{\partial \hat{\lambda}}{\partial L} - b \end{aligned}$$

The change in real wages is accordingly

$$\frac{\partial}{\partial L}\left(\frac{w}{p}\right) = \frac{1}{p^2}\left(\frac{\partial w}{\partial L}p - w\frac{\partial p}{\partial L}\right),$$

where the expression in brackets determines the sign of the derivative. We show in the supplement that this expression is negative in general equilibrium.

If Foreign's labor endowment is smaller and $L$ increases, wage payments in Foreign $(w^* L^*)$ will decrease, as

$$\frac{\partial w^* L^*}{\partial L} = -bL^*.$$

[12] At least for $L = L^*$ Foreign real wages increase.

[13] At least for the initial increase at $L = L^*$ Home real profits increase.

The profits in Foreign will change according to

$$\frac{\partial \pi^*}{\partial L} = \frac{1}{9}\frac{a^2}{b}\frac{2\hat{\lambda}-1}{\hat{\lambda}^2(1-\hat{\lambda})^2}\frac{\partial \hat{\lambda}}{\partial L}.$$

As for $L > L^*$ we know that $\hat{\lambda} < 1/2$, this derivative is positive. Therefore the labor share will decrease.

The price in foreign changes according to

$$\frac{\partial p^*}{\partial L} = \frac{1}{3}\frac{a}{(1-\hat{\lambda})^2}\frac{\partial \hat{\lambda}}{\partial L} - b < 0$$

if $L$ increases. As this is negative, real profits will increase whenever nominal profits increase. □

# 5 Conclusion

We develop an asymmetric general oligopolistic equilibrium (AGOLE) model which opens a wide range of possible applications that so far have been outside the scope of general oligopolistic equilibrium modelling. As a first exemplary application, we analyze the effects of asymmetric labor market policies. When one country increases its labor supply per capita, it is optimal for its firms to supply a part of the additional production to the other country at reduced prices to artificially inflate domestic prices until marginal revenues equal marginal costs again in both markets, which results in a spillover effect letting consumption increase abroad due to a change in the terms of trade.

These effects stem from the interaction of oligopolistic competition with asymmetries in countries' real income per capita, which affect their representative consumers' marginal utilities of income. As oligopolists maximize their profits while treating both markets as segmented, they take the asymmetric demand aggregates in form of the marginal utilities of income into account, but percieve them as exogenous. We argue that incorporating this cross-country demand channel is important when analyzing asymmetric countries or policies in presence of firms with market power. The mechanism shapes the welfare and distribution effects of policies and globalization. Thus, we consider it as vital for many theoretical tasks to take such a cross-country demand channel into account.

There are many possible applications of AGOLE models and we hope to stimulate research in that area with the proposed methodology. Interesting future applications include, for example differentiated factor markets, government interventions or unequally developed countries.

# Appendix

## Endogenous variables depending on exogenous parameters and $\lambda$

$$\begin{aligned}
y_h &= \frac{\lambda}{b}\left\{b\frac{L}{n} + \frac{1-2\lambda}{n+n^*+1}\frac{a}{\lambda}\right\} \\
y_f &= \frac{1-\lambda}{b}\left\{b\frac{L}{n} + \frac{2\lambda-1}{n+n^*+1}\frac{a}{1-\lambda}\right\} \\
y_h^* &= \frac{\lambda}{b}\left\{b\frac{L^*}{n^*} + \frac{1-2\lambda}{n+n^*+1}\frac{a}{\lambda}\right\} \\
y_f^* &= \frac{1-\lambda}{b}\left\{b\frac{L^*}{n^*} + \frac{2\lambda-1}{n+n^*+1}\frac{a}{1-\lambda}\right\}
\end{aligned}$$

$$\begin{aligned}
p &= a\left(1+\frac{n+n^*+{}^{1-\lambda}/_{\lambda}}{n+n^*+1}\right) - b(L+L^*) \\
p^* &= a\left(1+\frac{n+n^*+{}^{\lambda}/_{1-\lambda}}{n+n^*+1}\right) - b(L+L^*)
\end{aligned}$$

$$\begin{aligned}
w &= 2a - b\left(\frac{n+1}{n}L + L^*\right) \\
w^* &= 2a - b\left(\frac{n^*+1}{n^*}L^* + L\right)
\end{aligned}$$

## Supplement

The supplement (Mathematica Notebook) is available from the authors upon reasonable request.

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