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STATISTICAL IDENTFICATION IN SVARS - MONTE CARLO EXPERIMENTS AND A COMPARATIVE ASSESSMENT OF THE ROLE OF ECONOMIC UNCERTAIN-TIES FOR THE US BUSINESS CYCLE

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Statistical identification in SVARs - Monte Carlo experiments and a comparative assessment of the role of economic uncertainties for the US business cycle

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Abstract

Structural vector autoregressive analysis aims to trace the contemporaneous linkages among (macroeconomic) variables back to underlying orthogonal structural shocks. In homoskedastic Gaussian models the identification of these linkages deserves external and typically not-data-based information. Statistical data characteristics (e.g, heteroskedasticity or non-Gaussian independent components) allow for unique identification. Studying distinct covariance changes and distributional frameworks, we compare alternative data-driven identification procedures and identification by means of sign restrictions. The application of sign restrictions results in estimation biases as a reflection of censored sampling from a space of covariance decompositions. Statistical identification schemes are robust under distinct data structures to some extent. The detection of independent components appears most flexible unless the underlying shocks are (close to) Gaussianity. For analyzing linkages among the US business cycle and distinct sources of uncertainty we benefit from simulation-based evidence to point at two most suitable identification schemes. We detect a unidirectional effect of financial uncertainty on real economic activity and mutual causality between macroeconomic uncertainty and business cycles.

Keywords: Independent components, heteroskedasticity, model selection, non-Gaussianity, structural shocks.

JEL Classification: C32, E00, E32, E44, G01.

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1 Introduction

In the past decades, various techniques for the identification of structural vector autoregressive (SVAR) models have been suggested with reference to distinct assumptions on the underlying economic principles and the data structures. Identification based on economic theory has been widely applied in macroeconometrics. Respective a-priori assumptions have been formalized, for instance, by means of zero restrictions on the impact effects or the long-run effects of structural shocks (Sims, 1980; Blanchard and Quah, 1989). Assuming a recursive causation scheme for simplicity (i.e., lower triangularity of the structural matrix) might be too restrictive in many applications. In contrast, sign restrictions (Faust, 1998) facilitate the derivation of a structural model and implied impulse response functions (IRFs) by reducing the set of all instantaneous effect patterns which are in line with reduced form model features to those that accord with a-priori economic reasoning. Nevertheless, the most reasonable structural model cannot be recovered if its sign pattern does not coincide with the one assumed (Fry and Pagan, 2007).

Alternatively, data-driven identification procedures can be particularly appealing if no well-founded economic restrictions on the effects of structural shocks are available a-priori. Additional assumptions on the covariance structure or on the distribution of the structural error terms can be seen as external information to solve the identification problem. Statistical identification schemes can be classified into two major categories. Firstly, using heteroskedasticity for unique identification of SVAR models has become a frequently applied approach. In this context, informative assumptions on the covariance structure of the reduced form model residuals include the presence of an unconditional exogenous covariance shift (Rigobon, 2003; Lanne and Lütkepohl, 2008), Markov switching mechanisms (Lanne et al., 2010), smooth transitions between the covariance states (Lütkepohl and Netšunajev, 2017b), or patterns of conditional heteroskedasticity (Normadin and Phaneuf, 2004). Secondly, more recent statistical identification procedures build upon non-Gaussianity of the structural shocks within frameworks of independent component analysis (ICA) (Moneta et al., 2013; Lanne et al., 2017; Gouriéroux et al., 2017). Under a non-Gaussian distribution, independent components can be uniquely identified (Comon, 1994).[1]

Although statistical identification schemes promise point identification of 'structural' relations, their application is far from straightforward for at least two reasons. Firstly, it is by no means guar-

[1]Notions of stochastic dependence and correlation coincide under Gaussianity. Furthermore, the assumption of non-Gaussianity might be reasonable for economic data allowing, for example, leptokurtic distributions (see e.g., Chib and Ramamurthy, 2014; Cúrdia et al., 2014, for dynamic stochastic general equilibrium models with t-distributed shocks). It is also worth highlighting that the concepts independence and second order heterogeneity might complement each other in empirical data.

anteed that uniquely identified shocks have economically meaningful properties. In this respect, the issue of so-called 'shock labelling' is an important modelling step in the context of statistical identification. Herwartz and Lütkepohl (2014) and Herwartz and Plödt (2016a) show how statistical identification approaches can be combined with external information garnered from economic theory. Secondly, yet having a viable variety of alternative statistical approaches to the modelling of - lastly - latent structural relationships at hand, the uniqueness of shocks is somehow traded against the multitude of identification schemes. Put differently, method selection becomes an important step of statistical identification and the application of any specific identification scheme comes with risks of inefficient or even biased assessments of the structural model parameters. The literature on the comparative performance of statistical identification schemes is still scant. Lütkepohl and Netšunajev (2017a) review alternative heteroskedasticity-based models by pointing out their advantages and drawbacks, while Lütkepohl and Schlaak (2018) provide guidelines on how to choose between these models conditional on distinct forms of underlying covariance changes. Herwartz and Plödt (2016a) compare stylized sign restrictions and identification by means of covariance shifts. Yet, however, little is known about how independence-based methods perform under covariance changes compared with sign restrictions and heteroskedasticity-based identification schemes. Moreover, it is still unclear how the latter approaches perform under non-Gaussian distributional frameworks.

The prime purpose of this paper is to compare representatives of two families of alternative statistical identification schemes (heteroskedasticity-based identification vs. independent component analysis). Seeing that theory-based sign restrictions have become a common strategy to resolve the imposition of ad-hoc triangular model structures, we complement the comparison of statistical identification schemes with a stylized implementation of sign restrictions as a second direction of our Monte Carlo analysis. Identification through heteroskedasticity builds upon unconditional covariance shifts (Rigobon, 2003; Lanne and Lütkepohl, 2008) or conditional patterns of heteroskedasticity (Normadin and Phaneuf, 2004). The considered variants of detecting independent components comprise parametric maximum likelihood (ML) estimation (Lanne et al., 2017) and two nonparametric procedures that build upon dependence diagnostics (Herwartz, 2018; Matteson and Tsay, 2017).

In the first part of this study and similar to Herwartz and Plödt (2016a), we assess the relative merits of alternative identification schemes (theory-based sign restrictions and statistical techniques) using the log linearized counterpart of a stylized dynamic stochastic general equilibrium (DSGE) model for data generation. Our simulation set-up highlights numerous aspects of performance characteristics conditional on the underlying data generating process (DGP). In the second part, we provide a comparative analysis of alternative identification methods to examine the interdependence between economic uncertainty and business cycle fluctuations in the US economy. While the related literature (e.g., Bloom, 2009; Jurado et al., 2015) focuses on the role of uncertainty in recessions, our analysis complements recent results of Ludvigson et al. (2018) who argue powerfully for a need to differentiate major categories of uncertainty, namely macroeconomic and financial uncertainty. Specifically, to provide a structural view at the relationship between the US business cycle and alternative sources of uncertainty, we unravel if and in how far alternative statistical identification designs obtain structural shocks which are (best) in line with the event and correlation constraints of Ludvigson et al. (2018). Our results highlight distinct effects of macroeconomic and financial uncertainty on real economic activity. Additionally, we detect strong support for the narrative restrictions of Ludvigson et al. (2018) in the estimated independent components. Using external information (beyond impact effects of the shocks on the variables) supports the economic labeling of statistically identified shocks. Furthermore, the plausibility of the theory based identification in Ludvigson et al. (2018) benefits from the results of the statistical models.

In the next section, we introduce the stylized SVAR model and describe six alternative identification schemes. Section 3 provides the simulation setting and the corresponding results. Section 4 is explicit on the relation between uncertainty and business cycles. Section 5 concludes. Appendix A depicts the dependence diagnostics and Appendix B provides details on the DSGE model adopted for the simulation study. Appendix C documents detailed simulation results and Appendix D shows additional results on the link among uncertainty and real economic activity. Throughout, computations have been pursued by means of the R package `svars` (Lange et al., 2018).[2]

2 Identification procedures for structural VAR analysis

This section provides a brief outline of the identification problem in SVAR models and subsequently sketches alternative identification schemes in more detail. Specifically, we consider a stylized variant of identification by means of sign restrictions, and two (three) representatives of identification schemes which exploit the informational content of covariance changes (independent components).

[2]The R package `svars` comprises a large variety of statistical identification schemes and diverse diagnostic tools which are popular in the SVAR literature. It is available on CRAN at https://cran.r-project.org/package=svars.

2.1 The structural model representation

Consider a K-dimensional vector autoregressive model of order p

$$\begin{aligned} y_t &= \nu_t + A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t, \\ &= \nu_t + A_1 y_{t-1} + \ldots + A_p y_{t-p} + \mathsf{B}\varepsilon_t, \\ \Leftrightarrow \quad A(L) y_t &= \nu_t + \mathsf{B}\varepsilon_t, \qquad t = 1, \ldots, T, \end{aligned} \tag{1}$$

with vector valued deterministic terms ν_t and $A(L) = I_K - A_1 L - \ldots - A_p L^p$, where I_K denotes the $K \times K$ identity matrix. Furthermore, the model is causal by assumption, i.e., $\det(A(z)) \neq 0$ for all $|z| \leq 1$.[3] The stochastic model components are commonly characterized from two perspectives: Firstly, zero mean *reduced-form residuals* u_t, $E(u_t) = 0$, are subject to cross equation correlation with covariance matrix $\Sigma_u = \mathsf{B}\mathsf{B}'$. Secondly, *structural shocks* $\varepsilon_t = \mathsf{B}^{-1} u_t$ are uncorrelated across equations with $E(\varepsilon_t) = 0$ and $\Sigma_\varepsilon = I_K$. While reduced form residuals can be estimated consistently by means of least squares (LS) or ML techniques, it is more challenging to obtain the structural shocks since the decomposition of the covariance matrix $\Sigma_u = \mathsf{B}\mathsf{B}'$ is not unique. For instance, alternative covariance decompositions obtain as

$$\Sigma_u = DD' = DQQ'D = DQ(DQ)', \tag{2}$$

with Q denoting any rotation matrix ($Q \neq I_K, QQ' = I_K$) and D, e.g., the lower triangular Choleski factor of Σ_u. Accordingly, the representation B=DQ highlights B as a specific member from a space of covariance factors which are all in line with the reduced form model ($\Sigma_u = \mathsf{B}\mathsf{B}'$). In parametric form, the matrix $Q(\theta)$ could be specified as a product of Givens rotation matrices defined through the associated $(K(K-1)/2) \times 1$ dimensional vector of rotation angles $\theta = (\theta_1, \ldots, \theta_{K(K-1)/2})$. Noticing that $u_t = \mathsf{B}\varepsilon_t$, the structural matrix B formalizes the instantaneous impacts of the structural shocks on the variables of the system. Hence, it carries informational content for causal relationships within a dynamic system. Therefore, a particular goal in structural analysis is to identify the matrix B properly. The literature on SVAR models yet covers several approaches to solve this identification problem assuming either economic or statistical properties of the structural shocks.

In the following, we briefly describe identification by means of sign restrictions which grounds on a-priori economic assumptions. Subsequent to this prominent approach of set identification, we

[3]While approaches exist to identify noncausal SVARs, standard methods have been developed under the stability condition. The consideration of noncausal SVARs model is of particular relevance if the underlying economic model includes forward looking behavioral relations (e.g. fiscal/Ricardian foresight).

outline some identification schemes which build upon statistical properties of the data. These approaches allow the recovery of a unique (up to column permutation and scaling) structural matrix. Broadly speaking, the statistical properties which have been suggested as informative for identification fall in two not necessarily disjoint categories, heteroskedasticity and non-Gaussian distributed independent components. For a broad overview and textbook treatment of identification in SVARs we refer the reader to Kilian and Lütkepohl (2017).

2.2 Identification based on sign restrictions (SR)

A typical element of B, b_{ij}, quantifies the direction and magnitude of the contemporaneous effect of a (positive) structural unit shock ε_{jt} on the i-th variable in the system. For both characteristics - direction and (relative) magnitude - economic theory might offer plausible restrictions. Throughout the SVAR literature, several variants of identification by means of sign restrictions build upon restricting the structural parameter matrix B to have an economically reasonable sign pattern (see, for instance, Faust, 1998; Fry and Pagan, 2007).

In this study, we consider a stylized sign restriction approach which formalizes directional effects throughout. After obtaining LS estimates of the reduced-form covariance matrix, $\widehat{\Sigma}_u$, identification by means of sign restrictions consists of generating a large set of $Q(\theta)$ rotation matrices as formalized in equation (2). Drawing the rotation angles θ_i, $i = 1, \ldots, K(K-1)/2$, uniformly from the interval $[0, \pi]$ ensures to cover the entire space of covariance decompositions (for more technical details, see, for instance, Canova and Nicolo, 2002). A particular draw is admissible for identification if the associated decomposition $\mathsf{B}(\theta) = D\,Q(\theta)$ fulfills the a-priori specified sign restrictions. The sampling proceeds until a prespecified number of successful draws (e.g., 10000) has been obtained. After this sampling exercise the collection of admissible matrices provides a set identification of B. For purposes of point estimation, for instance, the median of this set of admissible matrices could be considered as matrix estimate. Henceforth we denote this point estimate as $\widehat{\mathsf{B}}_{\mathrm{SR}}$.

2.3 Identification through heteroskedasticity

Time-varying variances characterize many (macroeconomic) time series and, likewise, may be used to identify underlying structural shocks (see, e.g., Sentana and Fiorentini, 2001; Rigobon, 2003, for discussions of this theoretical result). In the following, we consider two specific variants which either build upon the assumption of external covariance changes (Rigobon, 2003; Lanne and Lütkepohl, 2008) or on patterns of generalized autoregressive conditional heteroskedasticity (GARCH, Normadin and Phaneuf, 2004; Bouakez and Normadin, 2010).

2.3.1 Unconditional heteroskedasticity (UH)

Rigobon (2003) suggests to exploit covariance shifts at pre-specified time points for the identification of structural shocks. The covariance structure reads accordingly as

$$E(u_t u_t') = \Sigma_u(m), \tag{3}$$

where $m = 1, \ldots, M$ indicates variance regimes. In the simplest framework of two covariance states ($M = 2$) and a structural break occurring at time $T_{sb} \in \{1, \ldots, T\}$, the covariance matrices are

$$E(u_t u_t') = \begin{cases} \Sigma_1 \text{ for } t = 1, ..., T_{sb} - 1 \\ \Sigma_2 \text{ for } t = T_{sb}, ..., T. \end{cases} \tag{4}$$

Thus, the first regime is characterized by the covariance matrix Σ_1 and the second regime by Σ_2, where $\Sigma_1 \neq \Sigma_2$. Since the structural parameter matrix B is time invariant, the covariance matrices can be rewritten as $\Sigma_1 = \mathsf{B}\mathsf{B}'$ and $\Sigma_2 = \mathsf{B}\Lambda\mathsf{B}'$ such that Λ is a diagonal matrix with diagonal elements $\lambda_{ii} > 0, i = 1, 2, \ldots, K$. By construction, the structural shocks have unit variance in the first regime and variance λ_{ii} in the second regime. The structural matrix can be uniquely identified if the diagonal elements in Λ are distinct.

Conditional on the break point T_{sb}, the estimation of B and Λ might follow ML principles. Under the assumption of Gaussian residuals u_t and $M = 2$ variance regimes, the log-likelihood function is (without constant)

$$l = -\frac{T_{sb} - 1}{2}\left(\log\det(\Sigma_1) + \text{tr}(\widehat{\Sigma}_1(\Sigma_1)^{-1})\right) - \frac{T - T_{sb} + 1}{2}\left(\log\det(\Sigma_2) + \text{tr}(\widehat{\Sigma}_2(\Sigma_2)^{-1})\right). \tag{5}$$

The numerical maximization of the log-likelihood is conditional on T_{sb}. For the selection of a suitable break point the analyst might use external information. Alternatively one might fit the model conditional on a range of break point candidates (for instance, within the interval $[0.15T,\ 0.85T]$) and, subsequently, select the model with the highest log-likelihood as in Lütkepohl and Schlaak (2018). In this study we use the latter approach to obtain estimates denoted $\widehat{\mathsf{B}}_{\text{UH}}$.

2.3.2 Conditional heteroskedasticity (CH)

The identification of structural shocks through patterns of conditional heteroskedasticity has been proposed by Normadin and Phaneuf (2004), Lanne and Saikkonen (2007) and Bouakez and Normadin (2010), amongst others. For the formal exposition let $\mathcal{F}_t$ denote a filtration that summarizes systemic information which is available up to time t. Then, time varying covariances allow for a representation as

$$E(u_t u_t' | \mathcal{F}_{t-1}) = \Sigma_{t|t-1} = \mathsf{B}\Lambda_{t|t-1}\mathsf{B}', \tag{6}$$

where

$$\Lambda_{t|t-1} = \text{diag}(\sigma^2_{1,t|t-1}, ..., \sigma^2_{K,t|t-1}) \tag{7}$$

is a $(K \times K)$ matrix with GARCH-type variances on the main diagonal. Assuming a low order GARCH(1,1) specification, for instance, the individual variances exhibit a dynamic structure as

$$\sigma^2_{k,t|t-1} = (1 - \gamma_k - g_k) + \gamma_k \varepsilon^2_{k,t-1} + g_k \sigma^2_{k,t-1|t-2}, \qquad k = 1, .., K. \tag{8}$$

Under suitable distributional and parametric restrictions, $\gamma_k > 0$, $g_k \geq 0$ and $\gamma_k + g_k < 1$, the marginal GARCH processes $\varepsilon_{k,t}$ are covariance stationary (Milunovich and Yang, 2013). Sentana and Fiorentini (2001) show that if there are at least $K-1$ GARCH-type variances present in $\Lambda_{t|t-1}$ the matrix B is unique (up to the permutation of columns and signs). The Gaussian log-likelihood function is

$$l = -\frac{1}{2}\sum_{t=1}^{T} \log\det(\Sigma_{t|t-1}) - \frac{1}{2}\sum_{t=1}^{T} u_t' \Sigma_{t|t-1} u_t. \tag{9}$$

For practical implementation of identification through conditional heteroskedasticity, we follow the approach of Lütkepohl and Milunovich (2016) and rely on two-step ML estimation as suggested in Lanne and Saikkonen (2007). The structural estimator is denoted as $\widehat{\mathsf{B}}_{\text{CH}}$.

2.4 Independence based identification

Instead of focusing on changing covariance structures, an alternative approach is to impose a restriction on the distribution of the structural shocks (i.e., non-Gaussianity). A fundamental result of Comon (1994) implies that a vector of independent components ε_t allows the unique recovery of B from reduced form residuals if at most one independent component exhibits a Gaussian distribution.[4]

Building on the uniqueness of independent components, Lanne et al. (2017) have suggested a fully parametric ML approach for targeting at independent (standardized) Student-t distributed structural shocks. Weakening the distributional assumptions in comparison with ML approaches, two further identification strategies allow an interpretation of the structural model as Hodges Lehman estimation (HL estimation, Hodges and Lehmann, 2006).[5] Principles of HL estimation motivate the detection of least dependent structural shocks by the minimization of two alternative nonparametric dependence criteria, namely the so-called distance covariance (dCov) of Székely et al. (2007) and the Cramér-von Mises (CvM) distance of Genest and Rémillard (2004). While the former has already been employed in the context of independent component analysis (Matteson and Tsay, 2017), the latter has been successfully employed for macroeconometric modelling in Herwartz and Plödt (2016b) and Herwartz (2018).

2.4.1 Identification through non-Gaussian ML estimation (nGML)

Lanne et al. (2017) suggest to determine the structural matrix B by means of maximizing the joint density of independently and not-normally distributed variables. Let f_k denote the densities of independent components $\varepsilon_{t,k}$, $k = 1, \ldots, K$. The corresponding distributional parameters are collected in λ_k.[6] Furthermore, the matrix of structural parameters is column-wise normalized by the associated standard deviation σ_k. The resulting matrix with unit values on the diagonal is denoted as $\check{\mathsf{B}}$ and β collects the vectorized off-diagonal elements of $\check{\mathsf{B}}$.

ML estimation of B proceeds in two steps.[7] In the first step, reduced form residuals $\hat{u}_t$ are extracted from the VAR model by means of LS. Conditional on $\hat{u}_t$, the log-likelihood is

$$l(\beta, \sigma, \lambda) = T^{-1} \sum_{t=1}^{T} l_t(\beta, \sigma, \lambda), \tag{10}$$

[4] In the case of multiple independent Gaussian components the system lacks full identification, however, partial identification of the non-Gaussian components is possible (Maxand, 2019).

[5] Conditional on a particular nuisance free test statistic, the HL estimator of a parameter of interest is the specific parameter value obtaining the largest p-value when subjected to testing.

[6] Note that the component densities f_k each depend on (possibly distinct) parameter values λ_k which can, for instance, correspond to the family of t-distributions with λ_k degrees of freedom (Lanne et al., 2017).

[7] We apply the two-step ML estimation procedure rather than a simultaneous estimation of the reduced form VAR and the structural parameters, since the latter is computationally demanding even for medium dimensions K and time series of moderate length (Lanne et al., 2017).

where time specific contributions are

$$l_t(\beta, \sigma, \lambda) = \sum_{k=1}^{K} \log f_k(\sigma_k^{-1} \iota_k' \check{\mathsf{B}}^{-1} \hat{u}_t; \lambda_k) - \log \det(\check{\mathsf{B}}) - \sum_{k=1}^{K} \log \sigma_k, \tag{11}$$

with ι_k denoting the k-th unit vector. In the second step l, is maximized with respect to the parameter vector $(\beta', \sigma', \lambda')'$. The vector comprises standard deviations $\sigma = (\sigma_1, \ldots, \sigma_K)'$, the parameter vector β and the component specific distribution parameters $\lambda = (\lambda_1, \ldots, \lambda_K)'$. With ML estimates $\hat{\sigma}$ and $\hat{\beta}$ the matrix B then obtains as $\widehat{\mathsf{B}}_{\mathrm{nGML}} = \check{\mathsf{B}}\ \mathrm{diag}(\hat{\sigma})$. Gouriéroux et al. (2017) propose pseudo ML estimation as a less restrictive generalization of the fully parametric ML model.

2.4.2 Nonparametric identification techniques and HL estimation

ML estimation (Lanne et al., 2017) proceeds under the assumption of a well and fully specified distributional framework, however, the exact distribution of the underlying data is mostly unknown, or might not be fully described by any existing parametric distribution. In contrast, nonparametric dependence measures offer alternative approaches for identification without specifying the distribution of the elements in ε_t explicitly.

The two subsequent algorithms share an interpretation of HL estimation in the sense that they provide matrix estimates $\widehat{\mathsf{B}}$ such that the corresponding structural shocks $\hat{\varepsilon}_t = \widehat{\mathsf{B}}^{-1} \hat{u}_t$ minimize a dependence criterion. Hence, the structural shocks are least dependent according to a particular test statistic. Similar to the identification by means of sign restrictions, the detection of shocks with minimal dependence departs from the space of covariance decompositions formalized in (2). Instead of random sampling sets of rotation angles, HL estimation targets at specific choice of $Q(\theta)$ to minimize the contemporaneous dependence among implied shocks $\hat{\varepsilon}_t(\theta) = \mathsf{B}(\theta)^{-1}\hat{u}_t$. At the implementation side we use two alternative nonparametric dependence diagnostics, i.e. the distance covariance (Székely et al., 2007) or the Cramér-von Mises (CvM) statistic (Genest and Rémillard, 2004).[8]

Distance covariance (dCov) Matteson and Tsay (2017) suggest the so-called distance covariance of Székely et al. (2007), denoted $\mathcal{U}_T$, for the implementation of ICA. In the sense of HL

[8] Apart from computational merits, targeting at structural shocks with weakest dependence in terms of the CvM diagnostic holds the advantage that $\mathcal{B}$ is consistent against any form of dependence (Genest and Rémillard, 2004). Additionally, the two dependence criteria applied here outperform alternative dependence measures in terms of power against a wide range of dependence structures (Herwartz and Maxand, 2019). Matteson and Tsay (2017) show by means of a simulation study that the ICA algorithm employed in Capasso and Moneta (2016) (*FastICA*) shows larger mean errors compared with the algorithm implemented in *steadyICA*.

estimation, the distance covariance $\mathcal{U}_T$ is minimized by $\hat{\theta} = \text{argmin}_\theta \, \mathcal{U}_T(\hat{\varepsilon}_t(\theta))$ which consequently determines the estimated matrix $\widehat{\mathsf{B}}_{\text{dCov}} = \mathsf{B}(\hat{\theta})$. For details on the exact minimization procedure and the empirical definition of the dependence measure we refer to Appendix A and Matteson and Tsay (2017). In this study, we apply the function *steadyICA* from the `R` package `steadyICA` (Risk et al., 2015) to determine $\widehat{\mathsf{B}}_{\text{dCov}}$.

Cramér-von Mises statistic (CvM) As an alternative nonparametric dependence criterion Genest and Rémillard (2004) have introduced the CvM distance between the empirical copula and the independence copula of the components of the random sample, which we denote as $\mathcal{C}_T$ (the exact definition of $\mathcal{C}_T$ is given in Appendix A). Then, the HL estimate of the optimal specification of the rotation matrix in (2) is $\hat{\theta} = \text{argmin}_\theta \, \mathcal{C}_T(\hat{\varepsilon}_t(\theta))$ which consequently determines the estimated matrix $\widehat{\mathsf{B}}_{\text{CvM}} = \mathsf{B}(\hat{\theta})$. We use the implementation of $\mathcal{C}_T$ in the `R` package `copula` (Hofert et al., 2015).

2.5 Summary of identification procedures

Wrapping up the descriptions of this section, Table 1 provides a summary of the distributional assumptions on the error term ε_t in relation to the different identification techniques. Stylized sign restrictions build on economic assumptions on the relationship between the variables in the system and base on standard assumptions in terms of the error distribution, i.e., homoskedasticity and Gaussianity. In contrast, the statistical methods require some informational content from either changing covariance patterns or independence in non-Gaussian distributional frameworks. Moreover, obtaining the underlying shocks via likelihood based models needs an explicit parametrization of the distribution of the error terms, whereas estimating the structural parameter through nonparametric methods is possible under much weaker assumptions on the distribution.

3 Simulation study

The simulation study documented in this Section sheds light on the performance of the six identification schemes described above. More specifically, we compare the estimated structural parameter matrices $\widehat{\mathsf{B}}_\bullet$, $\bullet \in \{\text{SR, UH, CH, nGML, dCov, CvM}\}$, under five distributional scenarios and three alternative characteristics of the covariance structure of the DGP. In the following, we discuss the DGP, stochastic characteristics of the structural shocks and the criteria that we use for performance evaluation. Subsequently, we report the simulation results.

Model	Assumptions on the variance of ε_t			Assumptions on the distribution of ε_t		
	Homoskedasticity	Heteroskedasticity		Gaussian	Non-Gaussian	
		Unconditional	Conditional		Arbitrary	t-distribution
Economic-theory based identification						
SR	✓			✓		
Statistical identification						
Heteroskedasticity based identification						
UH		✓		✓		
CH			✓	✓		
Independence based identification						
nGML	✓					✓
dCov	✓				✓	
CvM	✓				✓	

Table 1: Summary of the identification techniques and the respective underlying assumptions on the errors ε_t.

3.1 Data generation

3.1.1 Autoregressive dynamics

VAR processes y_t are generated by means of an economically reasonable simulation framework as described in Herwartz and Plödt (2016a). The employed DGP resembles a log-linearized version of a stylized three-equation DSGE model comprising the output gap (x_t), inflation (π_t) and nominal interest rates (r_t) (Gertler et al., 1999; Carlstrom et al., 2009; Castelnuovo, 2013, 2012, 2016). First order autoregressive innovations characterize demand, supply and monetary policy shocks. The resulting three-dimensional SVAR reads as

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \mathsf{B}\varepsilon_t, \quad t = 1, \ldots, T, \tag{12}$$

where $y_t = (x_t, \pi_t, r_t)'$. Based on typical calibrations of the underlying DSGE model, the associated autoregressive matrices A_1, A_2 and the structural parameter matrix B are

$$A_1 = \begin{pmatrix} 1.24 & -0.09 & -0.16 \\ 0.13 & 0.94 & -0.06 \\ 0.24 & 0.30 & 1.03 \end{pmatrix}, \; A_2 = \begin{pmatrix} -0.37 & 0.05 & 0.08 \\ -0.07 & -0.22 & 0.03 \\ -0.12 & -0.15 & -0.27 \end{pmatrix} \text{ and } \mathsf{B} = \begin{pmatrix} 2.32 & -0.48 & -0.41 \\ 0.72 & 2.32 & -0.22 \\ 0.98 & 1.57 & 0.76 \end{pmatrix}, \tag{13}$$

respectively. The matrix B shows a unique pattern of instantaneous effects of the structural shocks on the variables.[9] The first shock exerts an on-impact increase of all variables. Hence, it can be considered as a stylized demand shock. A supply shock raises the levels of prices and interest rates,

[9] Herwartz and Plödt (2016a) show that the response pattern implied by B is robust under a broad set of parameter calibrations of the underlying DSGE model. For further information see Appendix B.

but at the same time it causes a negative response of output. Moreover, a positive interest rate shock reduces inflation and dampens economic activity. The monetary policy shock is already identified by means of its counter directional impact on policy rates and prices. Conditional on the DGP in (12) and (13) we analyze the performance of alternative identification procedures under distinct (co)variance settings and various choices of the distribution of the structural shocks $\varepsilon_t = \mathsf{B}^{-1}u_t$.

3.1.2 Covariance settings

The reduced form covariance matrix of the DGP in equation (12) and (13) is $\Sigma_u = \mathsf{B}\Lambda\mathsf{B}'$. While B is time invariant, the diagonal matrix Λ allows the formalization of distinct (co)variance scenarios. We consider three variants of Λ:

1. Homoskedasticity
   Setting $\Lambda = I_3$ establishes a constant (co)variance $\Sigma_u = \mathsf{B}\mathsf{B}'$, $\forall t$.

2. Unconditional heteroskedasticity
   An unconditional (co)variance change obtains by subjecting Λ to a fixed shift at time ($T_{sb} = 0.5T$). Specifically, Λ is the identity matrix up to observation T_{sb} and $\Lambda = \text{diag}\{9,4,1\}$ afterwards. The magnitude of the variance shift is empirically plausible and in line with comparable simulation studies (e.g., Cavaliere et al. (2010) and Herwartz and Plödt (2016a)).

3. Conditional heteroskedasticity
   Conditional variances follow the GARCH$(1,1)$ dynamics

$$\sigma^2_{k,t|t-1} = (1-\gamma_k - g_k) + \gamma_k \varepsilon^2_{k,t-1} + g_k \sigma^2_{k,t-1|t-2}, \qquad k = 1,..,3. \tag{14}$$

   Univariate variances enter the model as $\Lambda = \text{diag}(\sigma^2_{1,t|t-1},...,\sigma^2_{3,t|t-1})$. The parameter choices of the three variance equations, $\gamma = (0.15, 0.1, 0.17)$ and $g = (0.75, 0.7, 0.8)$, are based on the simulation experiments in Lütkepohl and Milunovich (2016).

3.1.3 Distributional frameworks

To mimic a variety of practically relevant distributional features (normality, leptokurtosis, skewness) isolated structural shocks ε_{kt} in (12) are drawn independently and identically from the following five alternative univariate distributions:

1. Gaussian;

2. Standardized Student-$t(df)$ with alternative degrees of freedom $df = 4, 8$;

3. Centered and standardized $\chi^2_{(3)}$;

4. Centered and standardized inverse-Gaussian IG$(1, 1)$.

This selection of distributions is representative for possible characteristics of the structural shocks. After standardization the Student's t-distribution is characterized by excess kurtosis and both the χ^2- and IG-distribution are skewed. With the vector of structural shocks ε_t and the matrices A_1, A_2 and B in (13), we generate samples $\{y_t\}_{t=-1000}^{T}$ of size $T = 100, 200, 500, 1000$ according to the dynamic model in (12).[10] Sample sizes of $T = 100, 200$ ($T = 500$) are representative for macroeconomic series at quarterly (monthly) frequency. Higher frequency data are rarely considered in macroeconomic applications, but are of interest in financial econometrics. From the generated processes, we estimate LS residuals $\hat{u}_t$ under the assumption that the true autoregressive order ($p = 2$) is known. Conditional on sample information $\{\hat{u}_t\}_{t=1}^{T}$, we estimate the structural matrix B by means of the alternative procedures described in Section 2 to obtain a set of estimators $\widehat{\mathsf{B}}_\bullet$, $\bullet \in \{$SR, UH, CH, nGML, dCov, CvM$\}$. Similar to Lütkepohl and Schlaak (2018), we fit all identification schemes to all series regardless of the underlying DGP. Hence, we mimic the simplified case of an analyst who has no further knowledge about the properties of the data. Each Monte Carlo experiment covers $L = 500$ replications.

3.2 Performance evaluation

We evaluate the performance of the alternative identification techniques in terms of two stylized criteria. Firstly, we record the relative mean squared error (MSE) of the estimated matrices with respect to the true structural matrix B in (13). Let l, $l = 1, 2, \ldots, L$, $L = 500$ denote an indexation of single Monte Carlo experiments. The relative MSE of $\widehat{\mathsf{B}}_\bullet$ is

$$\widehat{MSE}_\bullet = \frac{1}{L} \sum_{r=1}^{L} \inf_{P \in \mathcal{P}} \sqrt{\sum_{i=1}^{K} \sum_{j=1}^{K} \left(\frac{\mathsf{B}_{ij} - (\widehat{\mathsf{B}}_{\bullet,l} P)_{ij}}{\mathsf{B}_{ij}} \right)^2}, \tag{15}$$

where $\widehat{\mathsf{B}}_{\bullet,l}$ is the estimated structural matrix for each replication l and identification scheme $\bullet \in \{$SR, UH, CH, nGML, dCov, CvM$\}$. Since any identification outcome is 'unique' up to column signs and column ordering, the infimum in equation (15) is taken over all matrices $P \in \mathcal{P}$ of the

[10] After the generation step we drop the first 1000 observations to immunize simulation results against the effects of initial conditions.

subset of signed column permutation matrices of $\widehat{\mathsf{B}}_{\bullet,l}$ within the set of the nonsingular $K \times K$ matrices.[11] Accordingly, the definition in (15) accounts for the non-uniqueness of the estimated matrices with respect to signed permutations, as it evaluates the matrix P which fits best to the true matrix of structural parameters B. Secondly, complementing the relative MSE in (15),

Table 2: Shock labeling

	Shocks		
Variable	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$
x	+	−	−
π	+	+	−
r	+	+	+

we evaluate alternative identification outcomes in a more economic sense noticing that structural shocks are typically qualified in terms of their impact effects on macroeconomic aggregates. The data generating matrix B in (13) holds a characteristic sign pattern which is supported by economic theory and displayed in Table 2. To assess data-based identification outcomes we examine in every iteration if an estimated structural matrix $\widehat{\mathsf{B}}_\bullet$ holds this specific sign pattern or if single shocks can be identified in terms of a unique (and correct) pattern of effect signs. Aggregating over single iterations we report the frequency of fully or partially identified sign patterns of $\widehat{\mathsf{B}}_\bullet$, $\bullet \in$ {UH, CH, nGML, dCov, CvM}.[12]

3.3 Simulation results

3.3.1 Mean squared errors

Figure 1 shows estimates $\widehat{MSE}_\bullet$, $\bullet \in$ {SR, UH, CH, nGML, dCov, CvM}, with respect to alternative sample sizes, underlying distributions and patterns of heteroskedasticity. Detailed simulation results are documented in the Tables A1 to A4 in Appendix C.

Studying the three alternative (co)variance scenarios separately, MSE estimates of structural matrices identified by means of sign restrictions, $\widehat{MSE}_{\text{SR}}$, are invariant over sample sizes and alternative distributional models (see also the results in Appendix C). Noticing that the application of sign restrictions can be understood as a censored sampling from the set of possible covariance decomposition matrices $\mathsf{B}= DQ(\theta)$ (see equation (2)), an estimation bias arises by construction.[13] Unlike $\widehat{MSE}_{\text{SR}}$, estimates obtained from statistical identification, $\widehat{MSE}_\bullet$, $\bullet \in$ {UH, CH,

[11] With $b_{.k}$ denoting a typical column of B, it holds that $\mathsf{BB}' = \sum_{k=1}^{K} b_{.k} b'_{.k}$.

[12] Since the identified matrices $\widehat{\mathsf{B}}_{\text{SR}}$ hold the true sign pattern by assumption, we do not include identification by means of (correct) sign restrictions in this direction of performance assessment.

[13] Reducing the set-identification to the median matrix allows to quantify the bias of $\widehat{\mathsf{B}}_{\text{SR}}$. Note that we consider

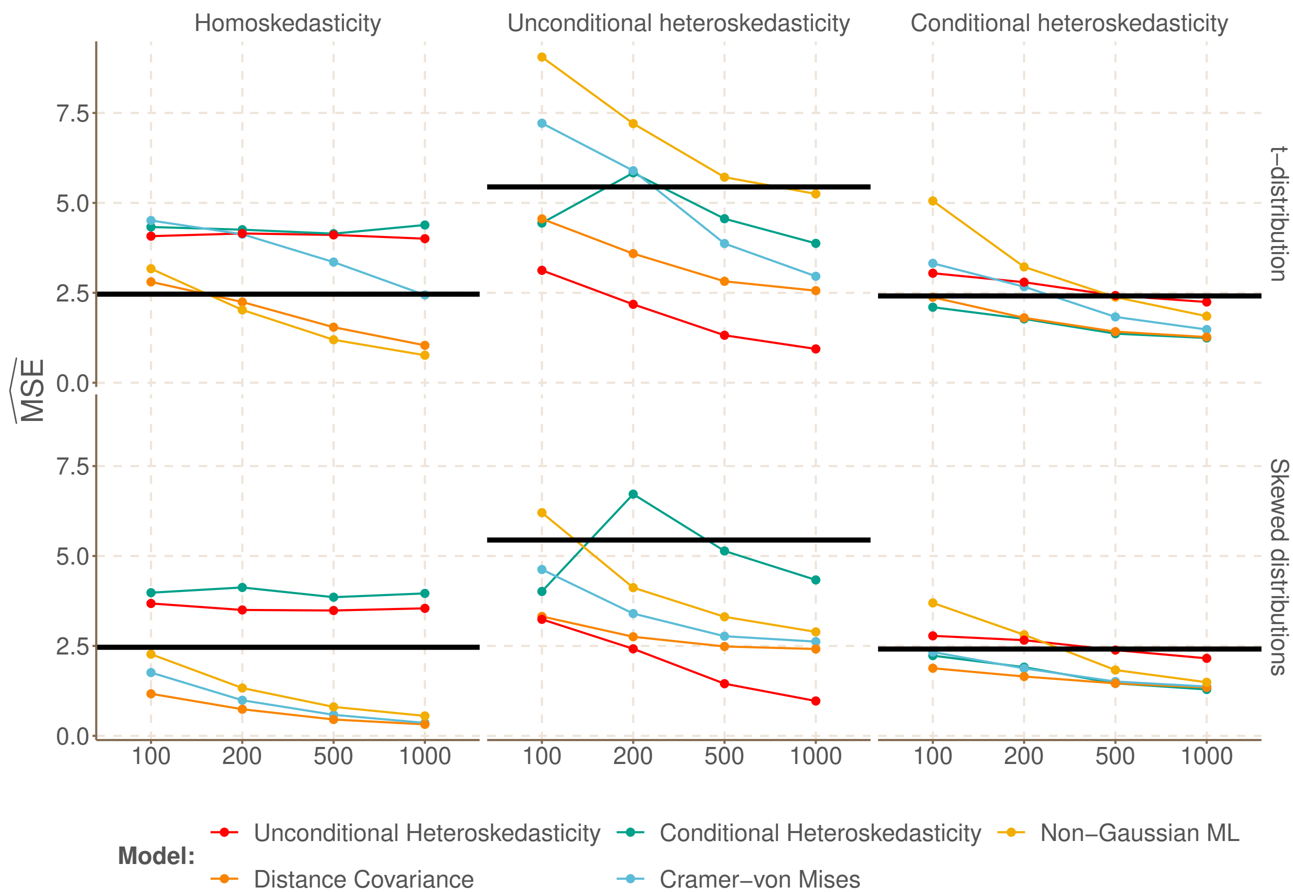

Figure 1: Summary of $\widehat{MSE}_{\bullet}, \bullet \in$ {SR, UH, CH, nGML, dCov, CvM} calculated as in equation (15) from $L = 500$ Monte Carlo experiments. The thick black lines represent the averaged $\widehat{MSE}_{SR}$. The first (second) row displays $\widehat{MSE}$ estimates for DGPs processing standardized Student-$t(4)$ and $t(8)$ (centered and standardized χ^2- and IG-) distributed structural shocks.

nGML, dCov, CvM}, vary with the sample size, the underlying distribution and the underlying (co)variance patterns. The estimated MSEs of the independence-based identification schemes (nGML, dCov and CvM) show a clear reduction conditional on larger sample sizes. Apparently, these identification techniques benefit from the consistency of the nonparametric independence diagnostics (dCov and CvM) against unspecified forms of dependence on the one hand. On the other hand, it is interesting to see that ML estimation benefits from consistency even if the (homoskedastic Student-t) likelihood is misspecified. As such, MSE outcomes illustrate the result of Gouriéroux et al. (2017) on the consistency of the Pseudo ML estimator under independent and identically distributed (iid) shocks. Unlike the detection of independent components, the perfor-

a stylized implementation of the sign restriction approach such that the model is fully restricted. The bias can be reduced by incorporating further characteristics of the data prior to estimation, for instance, in the framework of a more agnostic model (Arias et al., 2014).

mance of SVAR identification by means of heteroskedasticity depends on the correct diagnosis of the underlying form of heteroskedasticity (conditional vs. unconditional). These identification schemes fail to recover the true structural matrix under homoskedasticity or under a misspecified form of heteroskedasticity. However, $\widehat{MSE}_\bullet$ estimates, $\bullet \in \{\text{UH, CH}\}$, are almost unaffected by a misspecification of the likelihood, i.e., of the distribution. Regardless of the degrees of freedom of the underlying t-distribution, MSE estimates shrink to zero with increasing sample sizes as long as the underlying assumptions on the covariance structure are fulfilled. In contrast to this result, $\widehat{MSE}_\bullet$ estimates, $\bullet \in \{\text{nGML, dCov, CvM}\}$, change remarkably with distinct error distributions. The closer the distribution is to the Gaussian (e.g. comparing Student-$t(df)$ for $df = 8$ vs. $df = 4$) the larger is the estimated MSE. Moreover, the ML estimator $\widehat{\mathsf{B}}_{\text{nGML}}$ suffers from weak precision particularly in the presence of variance shifts (see also Lanne et al., 2017). Under changing (co)variances $\widehat{MSE}_\bullet$, $\bullet \in \{\text{dCov, CvM}\}$, increase on average, while their precision improves under conditional heteroskedasticity if structural shocks are Gaussian or close to Gaussian.

It is worth noticing that performance statistics assigned to $\widehat{\mathsf{B}}_{\text{dCov}}$ are more favorable than those of $\widehat{\mathsf{B}}_{\text{CH}}$ under conditional heteroskedasticity in small samples and skewed distributions. The estimator $\widehat{\mathsf{B}}_{\text{UH}}$ shows the smallest $\widehat{MSE}$ under unconditional covariance shifts and the $\widehat{\mathsf{B}}_{\text{nGML}}$ estimator outperforms rival approaches under homoskedastic Student-$t(4)$ distributed shocks and in samples larger than $T = 200$. Furthermore, set identification by means of sign restrictions obtains smallest $\widehat{MSE}$ statistics under homoskedastic Gaussian innovations and in small samples.

3.3.2 Frequencies of correct sign patterns

Figure 2 documents the frequency of the matrices $\widehat{\mathsf{B}}_\bullet$, $\bullet \in \{\text{UH, CH, nGML, dCov, CvM}\}$, which are fully in line with the theoretical sign pattern documented in Table 2. Conditional on homoskedastic structural shocks, these frequencies provide conclusions on the relative performance of data based identification schemes which are largely similar to those obtained from MSE estimates. Conditional on heteroskedastic shocks, however, the two criteria (MSE vs. sign pattern frequencies) do not necessarily agree in the assessment of an identification scheme under scrutiny. For instance, even though the estimator $\widehat{\mathsf{B}}_{\text{UH}}$ obtains the lowest $\widehat{MSE}$ under unconditional covariance shifts, it falls behind the estimators $\widehat{\mathsf{B}}_\bullet, \bullet \in \{\text{nGML}, \text{dCov}, \text{CvM}\}$ in recovering the true sign pattern. The inferior performance of the independence-based identification schemes in terms of MSE estimates reflects the fact that they do not account for the change in the covariance matrix of the structural shocks ($\mathsf{B}\Lambda\mathsf{B}'$) that applies in the second half of the sample. Rather, targeting at independent components

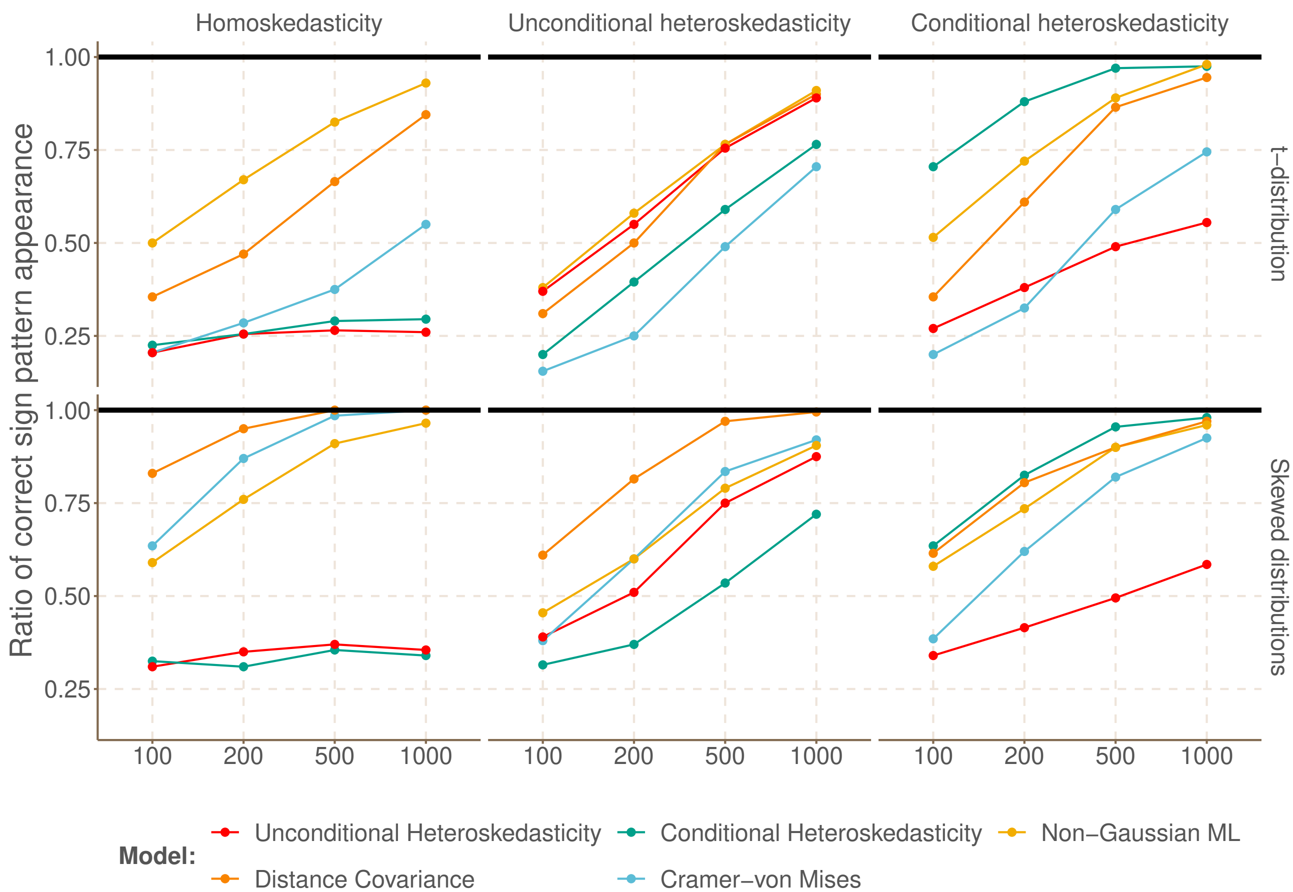

Figure 2: Frequency of matrices $\widehat{\mathsf{B}}_\bullet$, $\bullet \in \{\text{UH, CH, nGML, dCov, CvM}\}$ holding the corresponding sign pattern as in Table 2 based on $L = 500$ Monte Carlo experiments. For further notes see Figure 1.

results in a covariance decomposition $\widehat{\mathsf{B}}_\bullet \widehat{\mathsf{B}}_\bullet'$, $\bullet \in \{\text{nGML, dCov, CvM}\}$ to hold for the entire sample. While this effect implies an estimation bias, the signs of the elements of the structural matrix could remain unaffected.

Under conditional heteroskedasticity the $\widehat{\mathsf{B}}_{\text{CH}}$ estimator detects the true sign pattern most reliably and outperforms the identification of independent components irrespective of the underlying distribution. In addition, the detection of the correct sign pattern by means of $\widehat{\mathsf{B}}_\bullet, \bullet \in \{\text{dCov, CvM}\}$, is subject to the underlying distribution. Interestingly, in comparison with scenarios of homoskedasticity, both forms of heteroskedasticity might enable a more frequent detection of the true sign pattern by means of targeting at independent components. At least in large samples, independence based identification benefits from covariance changes, which apparently shift the distribution of the shocks away from normality.

3.3.3 Summary of the simulation results

The MSEs and sign patterns of the structural estimates $\widehat{\mathsf{B}}_\bullet$ highlight performance differentials among the identification schemes $\bullet \in \{$SR, UH, CH, nGML, dCov, CvM$\}$. Unlike the application of sign restrictions, the average performance of statistical identification schemes improves with increasing sample sizes. However, at least for the evaluation of scarce sample information, small frequencies of correctly identified effect directions (see Figure 2) indicate a notable risk of mislabelling shocks according to a specific statistically identified sign pattern.

Our results allow a general conclusion: If the distribution family for the non-Gaussian ML method is specified correctly (i.e., as standardized Student-t distribution) this estimator performs best (in large samples and under homoskedasticity). In other settings, nonparametric identification schemes show a superior performance. Identification based on unconditional heteroskedasticity is the preferred approach under covariance shifts and the identification by means of GARCH dynamics performs best under conditional heteroskedasticity (in larger samples). In terms of MSE and the frequency of correct sign patterns, identification based on the distance covariance outperforms the HL-estimator that builds upon the Cramér-von Mises statistic. Across all simulation scenarios (including normally distributed shocks) and sample sizes, the HL-estimator conditioning on the distance covariance obtains, on average, the smallest MSE estimates and highest frequencies of correctly detected sign patterns. As long as the shocks are non-Gaussian, it is worth considering the use of this identification technique.

4 Economic uncertainties and the business cycle

Similar to the analysis in Ludvigson et al. (2018), we consider a trivariate system to shed light on the ongoing debate if economic uncertainties are a cause or an effect of changes in real economic activity.[14] More precisely, we highlight the performance of the previously discussed statistical identification techniques to disentangle the causality between different types of uncertainty and fluctuations of the business cycle in the U.S. economy. As argued by Ludvigson et al. (2018), the relevant literature on the matter lacks a consensus on the signs of the economic relationships in such systems, thus, we refrain from using stylized sign restrictions for identification. Arguing against restrictive recursive model specifications, Ludvigson et al. (2018) suggest a combination of event and correlation constraints imposed on the structural shocks for model identification. As they are

[14] For further studies on this topic and insights from recursive identification schemes see, e.g., Bloom (2009), Bachmann et al. (2013) and Bekaert et al. (2013).

directly placed on the structural shocks, event constraints take the form of narrative restrictions in the spirit of Antolín-Díaz and Rubio-Ramírez (2018) on the one hand. Correlation constraints apply to the relation between structural shocks and first order autoregressive residuals from a market portfolio. Hence, on the other hand, using stock market information for instrumentation connects the approach of Ludvigson et al. (2018) to so-called proxy SVARs (Stock and Watson, 2012).[15] In the following, we evaluate if (i) the event and correlation constraints are generally in line with outcomes from statistical identification, and (ii) if a particular statistical identification approach provides a unique reflection of the external information and economic narratives suggested by Ludvigson et al. (2018). Thereby, our analysis is informative in how far external instruments or economic narratives are in line with heteroskedastic structural shocks and/or the independent component view at identification. In this case, the external or narrative information is obviously most helpful for the economic labelling of the statistically identified shocks. Similarly, the statistical information benefits the interpretation of the economic identification. On the one hand, the detection of heteroskedastic shocks is important for a realistic (i.e., likely state dependent) interpretation of the scale of 'unit' shocks and their effects which are of main interest in impulse response analysis. On the other hand, the detection of independent shocks is crucial for the stylized assumption that within impulse response analysis unit impulses hit a variable under scrutiny in isolation.

After a brief introduction of the data, we discuss statistical identification outcomes and evaluate the model implied structural shocks in terms of event and correlation constraints. We compare impulse responses to statistically identified shocks and their accordance with economic concepts in quantitative terms. Furthermore, we discuss potential limitations of recursive identification schemes and turn to an unrestricted analysis of the relationship between financial uncertainty, macroeconomic uncertainty and real economic activity.

4.1 Data

The three-dimensional VAR ($K = 3$) consists of the variables

- $U_t^{(M)}$ – one-month ahead macroeconomic uncertainty index (y_{1t}),
- q_t – linearly detrended log of U.S. real industrial production (y_{2t}),
- $U_t^{(F)}$ – one-month ahead financial uncertainty index (y_{3t}).

[15] Lütkepohl and Milunovich (2016) exploit conditional heteroskedasticity for model identification. Testing multiple restrictions on the structural parameters within an augmented model ($K = 4$), they cannot reject the null hypothesis of valid instrumentation.

The uncertainty indices have been constructed from a large set of macroeconomic and financial time series (see Ludvigson et al., 2018, for a detailed description). Extending the data set of Ludvigson et al. (2018) and Lütkepohl and Milunovich (2016) we consider monthly data from July 1960 to June 2018 which results in 696 observations.[16] The industrial production index has been downloaded from the Federal Reserve Bank of St. Louis database (FRED), and the uncertainty measures have been drawn from Sydney C. Ludvigson's website.[17] We set the lag order p in the estimated system to $p = 5$, as indicated by the Akaike information criterion (AIC). To evaluate the correlation constraints, we use first order autoregressive residuals ($\hat{w}_t$) extracted from monthly S&P500 returns. Ludvigson et al. (2018) use first order autoregressive residuals of value-weighted CRSP (Center for Research in Securities Prices) stock market returns as instrumental series. In sum, marginally different variable and instrument definitions, distinct lag orders for VAR estimation and a slightly extended sample period complicate a one-to-one comparison with benchmark results of Ludvigson et al. (2018).

As it is evident from the simulation study, non-normality is a crucial assumption for the uniqueness of identification outcomes achieved by targeting at independent components. To check for Gaussianity, we perform component-wise kurtosis and skewness tests as implemented in the `R` package `normtest` (Gavrilov and Pusev, 2014). Additionally, we test the number independent Gaussian components by means of fourth-order blind identification implemented in the `R` package `ICtest` (Nordhausen et al., 2018).[18] The results displayed in Table A5 indicate at least one skewed component and excess leptokurtosis in all three components. Moreover, we find no indication of Gaussianity. The extraction of independent components by means of minimizing nonparametric independence diagnostics bears the interpretation of HL-estimation. In the present case the maximization of the p-values of the CvM distance and of the distance covariance are 0.987 and 0.975, respectively. While we are aware that the supremum approach invalidates the standard interpretation of p-values, we assume that supremum p-values close to unity indicate the independence of shocks extracted by means of HL estimation. As argued by Lütkepohl and Milunovich (2016) the data are heteroskedastic and the model can be fully identified under the assumption of a GARCH structure.

[16] The sample sizes analyzed in Ludvigson et al. (2018) and Lütkepohl and Milunovich (2016) are 658 and 666, respectively.

[17] https://www.sydneyludvigson.com/

[18] Maxand (2019) finds satisfying power and size properties of univariate normality tests and tests based on fourth-order blind identification to evaluate non-Gaussianity of structural shocks.

4.2 Economic narratives and instrumental information

4.2.1 Event and correlation constraints

Noticing that any identification outcome for the structural matrix B could be subjected to column reorganisation we assume in the following that the shocks in ε_t correspond to the variable ordering in $y_t = (U_t^{(M)}, q_t, U_t^{(F)})$. In terms of shock labelling, for instance, we call $\varepsilon_{U(M)}$, ε_q and $\varepsilon_{U(F)}$ the macroeconomic uncertainty shock, economic activity shock and financial uncertainty shock, respectively. Ludvigson et al. (2018) argue that the structural shocks related to the estimates $\widehat{\mathsf{B}}$ should align with the following three event constraints.

E1. Financial uncertainty is subject to a large positive shock ($\varepsilon_{U(F)} \geq 4$) in October 1987 (Black Monday).

E2. The financial uncertainty shocks show large positive value(s) ($\varepsilon_{U(F)} \geq 4$) for at least one month during the financial crisis (2007M12-2009M6).

E3. There are no large positive shocks to economic activity ($\varepsilon_q \leq 2$) during the financial crisis (2007M12-2009M6).

Moreover, theoretical arguments (e.g., Sharpe, 1964; Lintner, 1965) suggest that uncertainty shocks have an impact on stock market returns which is channeled through their influence on risk premia. Using this implication for model identification, Ludvigson et al. (2018) impose the following three inequality restrictions on the correlation between the structural shocks and first order autoregressive residuals ($\hat{w}_t$) extracted from a broad market portfolio (in their case the CRSP stock index).[19]

C1. Macroeconomic uncertainty shocks and financial uncertainty shocks are negatively correlated with $\hat{w}_t$ such that $-0.05 - \text{corr}(\hat{w}_t, \varepsilon_{U(M)}) \geq 0$ and $-0.05 - \text{corr}(\hat{w}_t, \varepsilon_{U(F)}) \geq 0$.

C2. Shocks to financial uncertainty are markedly stronger correlated with $\hat{w}_t$ than macroeconomic uncertainty shocks, i.e., $|\text{corr}(\hat{w}_t, \varepsilon_{U(F)})| - 2 \times |\text{corr}(\hat{w}_t, \varepsilon_{U(M)})| \geq 0$.

C3. The aggregated correlation between both uncertainty shocks and $\hat{w}_t$ exceeds 0.18, i.e., $\sqrt{\text{corr}(\hat{w}_t, \varepsilon_{U(M)})^2 + \text{corr}(\hat{w}_t, \varepsilon_{U(F)})^2} - 0.18 \geq 0$.

We evaluate if the shocks implied by the estimated structural matrices $\widehat{\mathsf{B}}_\bullet$, $\bullet \in \{$UH, CH, nGML, dCov, CvM$\}$ are in line with the event and correlation constraints. For this exercise it is worth

[19] For a detailed discussion on the choice of the external variable and the derivation of the correlation constraints see Ludvigson et al. (2018).

noticing that owing to presumed (co)variance changes the shocks $\widehat{\mathsf{B}}_{\bullet}^{-1}\hat{u}_t$, $\bullet \in \{\text{UH, CH}\}$, are heteroskedastic and their unconditional empirical covariance differs from the identity matrix by construction. To enable a comparison of model based structural shocks which is not subject to distinct unconditional scaling, the event constraints are not only compared with heteroskedastic shocks $\hat{\varepsilon}_{\bullet,t} = \widehat{\mathsf{B}}_{\bullet}^{-1}\hat{u}_t$, $\bullet \in \{\text{UH, CH}\}$, but also with their standardized counterparts, $\tilde{\varepsilon}_{\bullet,t} = \widehat{\Lambda}_{\bullet,t}^{-1/2}\hat{\varepsilon}_{\bullet,t}$, where $\widehat{\Lambda}_{\bullet,t}$ is the diagonal matrix comprising model specific variance estimates (i.e., $\bullet \in \{U, C\}$). Structural shocks with unit (unconditional) variance are displayed in Figure 3.

4.2.2 Qualitative evaluation of statistically identified shocks

With respect to the event constraints, we note that within the vectors of structural shocks $\hat{\varepsilon}_{\bullet}$, $\bullet \in \{\text{nGML, dCov, CvM}\}$ and the standardized shocks $\tilde{\varepsilon}_{\text{UH}}$ only the third series ($\varepsilon_{U(F)}$ in Figure 3 shows a large positive value (≥ 4) in October 1987 (Black Monday). Accordingly, the statistical identification confirms its labelling as the financial uncertainty shock. After standardization, the shocks retrieved from assuming conditional heteroskedasticity lack a large positive outcome on Black Monday. For the labelled financial uncertainty shocks, we find, moreover, several months during the financial crisis at which the shocks are beyond a rule-of-thumb threshold of two standard deviations. However, none of the shocks exceeds a value of four during this period, which would be necessary in order to fulfill the second event constraint E2 exactly. The third event constraint is clearly violated by the productivity shocks in $\tilde{\varepsilon}_{\text{UH}}$, while the productivity shocks from the other identification schemes hit the two standard deviation threshold only for some months in 2018. In summary, the shocks resulting from most statistical identification schemes are not exactly in line with all event restrictions, however, it is interesting to see that the statistical criteria mostly support the economic narratives in qualitative terms.

The left hand side of Table 3 displays correlation estimates between identified shocks and instrumental information ($\hat{w}_t$). Both uncertainty shocks are negatively correlated with the stock market residuals, and financial uncertainty shocks exhibit a stronger correlation with the instrument than macroeconomic uncertainty shocks. Overall, the correlation constraints are fully supported by the structural shocks obtained from the independence-based models and are partially supported by the (standardized) heteroskedastic shocks. Seeing that the event and correlation constraints are qualitatively in line with results from alternative statistical identification schemes, an immediate interest arises in unravelling which identified shocks allow a most precise approximation of the economically motivated constraints of Ludvigson et al. (2018). Next, we turn to a precise quantitative

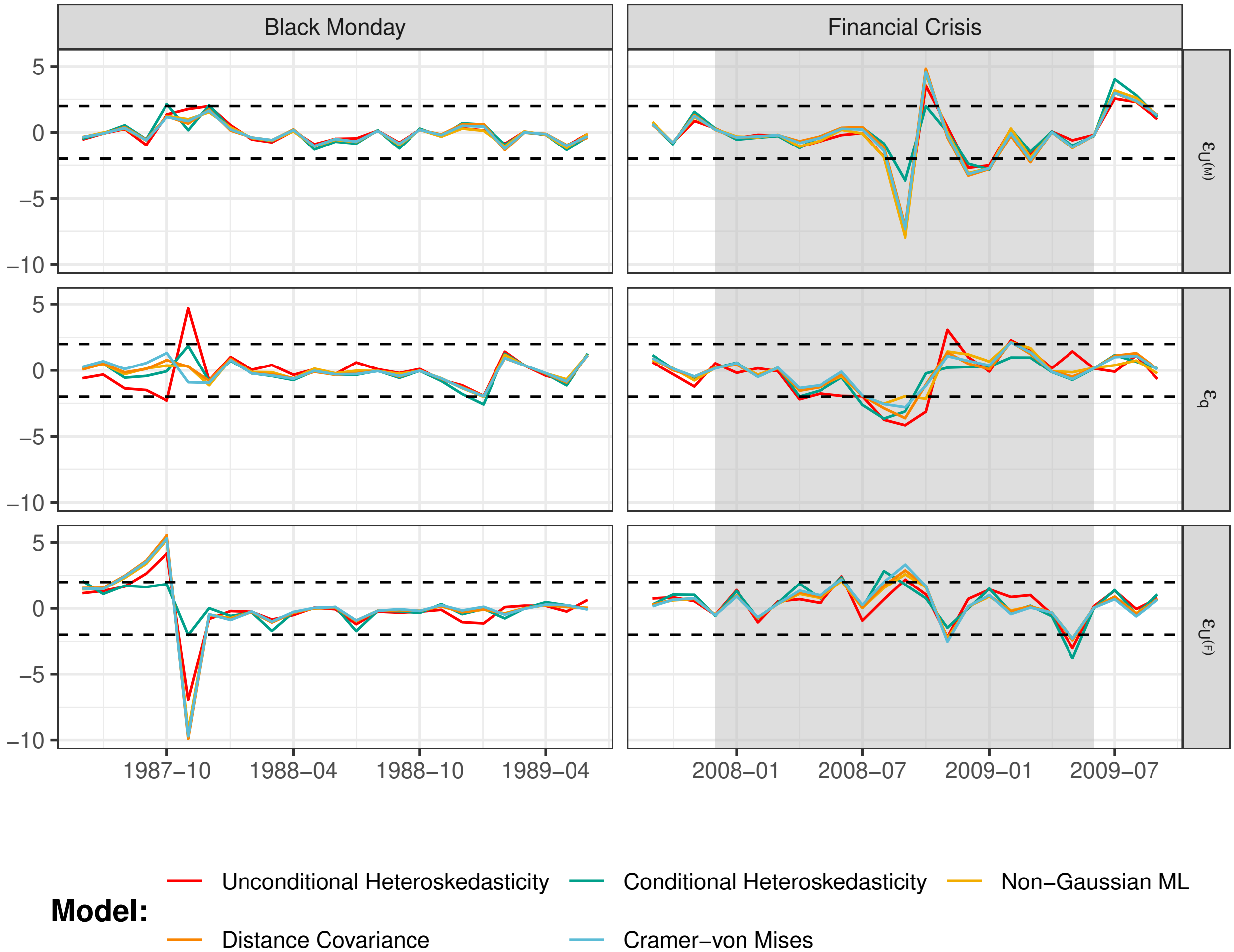

Figure 3: Magnitudes of the standardized structural shocks on black Monday and during the financial crisis (shaded area in the right hand side panel). The dashed lines correspond to ±2 standard deviations from the mean for all series and the shaded areas correspond to the period of the financial crisis 2007M12-2009M6. The three rows show the financial uncertainty shocks ($\varepsilon_{U(F)}$), productivity shocks (ε_q) and macroeconomic uncertainty shocks ($\varepsilon_{U(M)}$), respectively.

characterization of shocks obtained from alternative statistical identification schemes.

4.2.3 Quantitative performance differentials of statistical identification schemes

For a comparative assessment of the alternative statistical identification techniques, we next evaluate which particular estimates $\widehat{\mathsf{B}}_{\bullet}$, $\bullet \in \{\text{UH, CH, nGML, dCov, CvM}\}$, obtain the most extreme shocks in the spirit of the three event constraints, and/or align most favorably with the correlation restrictions. Pursuing along these lines could support the selection of a 'most suitable' identification scheme, from the middle ground spanned by narrative and instrumental information. Figure 4 displays the (average) magnitudes of the shocks at time instances in question.

Table 3: Correlation between stock market returns AR(1) residuals ($\hat{w}_t$) and structural shocks. The correlation constraints are reported in the last three columns (positive values indicate that the correlation constraints are fulfilled). Largest positive values for the inequalities are marked in boldface and results for standardized shocks ($\tilde{\varepsilon}$) are indicated as $\widetilde{\text{UH}}$ and $\widetilde{\text{CH}}$.

Model	$\text{corr}(\hat{w}_t, \hat{\varepsilon}_{U(M)})$	$\text{corr}(\hat{w}_t, \hat{\varepsilon}_q)$	$\text{corr}(\hat{w}_t, \hat{\varepsilon}_{U(F)})$	C1		C2	C3
UH	-0.064	0.123	-0.133	0.014	0.083	0.202	-0.033
$\widetilde{UH}$	-0.037	0.136	-0.172	-0.013	0.122	**0.307**	-0.004
CH	-0.074	0.024	-0.175	0.024	**0.125**	0.276	0.010
$\widetilde{CH}$	-0.087	0.025	-0.096	**0.037**	0.046	0.105	-0.051
nGML	-0.070	0.039	-0.173	0.020	0.123	0.276	0.006
dCov	-0.082	0.017	-0.173	0.032	0.123	0.264	**0.011**
CvM	-0.077	0.002	-0.172	0.027	0.122	0.267	0.008

Firstly, looking for most extreme values for all event constrains, Figure 4 does not hint at one 'ideal' method. The structural shocks $\hat{\varepsilon}_\bullet$, $\bullet \in$ {dCov, CvM, } and $\tilde{\varepsilon}_{\text{CH}}$ each optimize one of the constraints. While the shocks $\hat{\varepsilon}_{\text{dCov}}$ comprise the most extreme values for two out of three event constraints among the independence-based approaches, we find the same relative performance outcome for the standardized shocks $\tilde{\varepsilon}_{\text{CH}}$ among the heteroskedasticity-based models. Secondly, while financial uncertainty shocks recovered from $\widehat{\mathsf{B}}_{CH}$ attains a large positive value on black Monday, their standardized counterparts are relatively small at this event. Hence, for the practical work with shocks that are extracted under assumptions of conditional (co)variance changes it is important to notice that structural outcomes might differ if one adjusts the heteroskedastic 'shocks' by scaling information which is already available before the shock occurs. Thirdly, the shocks derived from $\widehat{\mathsf{B}}_\bullet$, $\bullet \in$ {UH, nGML}, obtain intermediate diagnostics throughout.

As displayed in the right hand side of Table 3, the outcomes of the heteroskedasticity-based models with respect to the correlation constraints C1-C3 are ambiguous. On the one hand, C1 and C2 are maximized by the standardized shocks $\tilde{\varepsilon}_\bullet$, $\bullet \in$ {UH, CH}, on the other hand, assuming time-varying covariances for identification implies structural shocks, which violate at least one of the correlation constraints (with exception of $\hat{\varepsilon}_{\text{CH}}$). Overall, the constraints are jointly maximized by $\hat{\varepsilon}_{\text{CH}}$ followed by $\hat{\varepsilon}_{\text{dCov}}$.[20]

[20]Lütkepohl and Schlaak (2018) argue that the AIC is an appropriate criterion to choose between alternative heteroskedasticity-based SVARs. In our case we obtain for UH and CH the diagnostic outcomes

Model	$\log L$	AIC	Model	$\log L$	AIC
UH	-3121.66	6365.31	CH	-2929.31	5980.62

which support the results from Figure 4 and Table 3. Exploiting conditional heteroskedasticity for identification is superior to assuming unconditional covariance shifts.

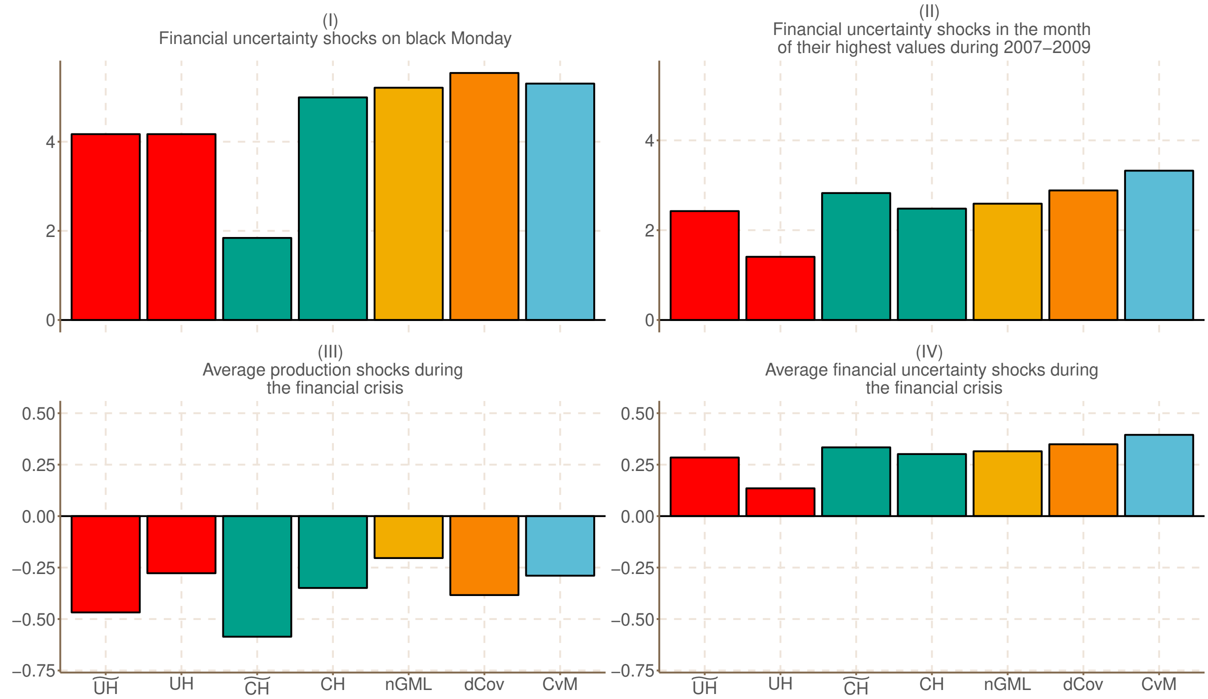

Figure 4: Comparison of the magnitudes of the structural shocks during event periods, obtained by $\widehat{\mathsf{B}}_{\bullet}, \in \{\text{UH}, \text{CH}, \text{nGML}, \text{dCov}, \text{CvM}\}$. Standardized shocks retrieved from heteroskedasticity-based identification schemes are indicated as $\widetilde{\text{UH}}$ and $\widetilde{\text{CH}}$. Panel (I) visualizes the first constraint (Black Monday), whereas the second constraint (financial crisis uncertainty) is associated with panel (II; maximum shock) and panel (IV; average). Panel (III) shows the averaged production shocks during the third event restriction (financial crisis economic activity).

4.3 Structural shocks under maximized constraints

Having seen that independent shocks are qualitatively and also largely quantitatively in line with event narratives and correlation constraints, it is tempting to check if - vice versa - structural shocks which conform best with economic narratives and instrumental information exhibit some higher order dependence beyond orthogonality. To address this issue, we follow two complementary approaches. Firstly, we adapt the identification scheme of Ludvigson et al. (2018) to our data. Secondly, we rescale the on impact impulse responses shown in Ludvigson et al. (2018) to provide a decomposition of the reduced form variance of our data.[21] From both exercises and for both dependence diagnostics, dCov and CvM, we obtain p-values below 1% which indicate significant violations of the null hypotheses of independent structural shocks obtained from maximum alignment with the event and correlation constraints of Ludvigson et al. (2018). As an implication of

[21]Let B_L denote a benchmark structural matrix which we obtain from linear interpolations of impact effects shown in Figure 4 of Ludvigson et al. (2018). Then the rescaled structural matrix adapted to our data is $\widetilde{\mathsf{B}}_L = \widehat{\Sigma}_u^{1/2}\left(\widehat{\mathsf{B}_L\mathsf{B}_L'}\right)^{-1/2}\mathsf{B}_L$, where the notation $G^{1/2}$ indicates the symmetric square root matrix of G.

higher order dependence, the tracing of isolated unit shocks by means of stylized IRFs might hide important actual transmission patterns among origins of uncertainty and business cycles.

4.4 Comparative analysis of IRFs

So far, we have investigated how statistically identified shocks align with economic narratives (event constraints) and external information (correlation constraints). Furthermore, we compare the insights obtained from statistical identification in terms of implied IRFs, which are depicted in Figure 5. The IRFs computed from $\widehat{\mathsf{B}}_{\mathrm{CH}}$ and $\widehat{\mathsf{B}}_{\mathrm{dCov}}$ are almost identical throughout, and $\widehat{\mathsf{B}}_{\mathrm{nGML}}$ and $\widehat{\mathsf{B}}_{\mathrm{CvM}}$ obtain qualitatively the same results. IRFs implied by the assumption of unconditional covariance shifts show qualitative differences for three out of nine response patterns. These findings are in line with our simulation results: Detecting independent components by means of nonparametric dependence diagnostics and the GARCH identification approach perform similarly under conditional heteroskedasticity and non-Gaussianity of the structural shocks. Moreover, the relatively weak performance of $\widehat{\mathsf{B}}_{\mathrm{UH}}$ in this setting can be confirmed. Accordingly, the subsequent analysis of economic activity vs. uncertainty linkages and feedback is conditional on the structural models implied by $\widehat{\mathsf{B}}_{\mathrm{CH}}$ and $\widehat{\mathsf{B}}_{\mathrm{dCov}}$.

4.5 The relationship between uncertainty and business cycles

Related studies on the effects of uncertainty shocks identify the structural parameter matrix by the assumption of lower triangularity. However, for the specific system under scrutiny, the literature does not agree on the (best) ordering of the variables. For instance, Bloom (2009) and Bachmann et al. (2013) order uncertainty indices first, whereas Bekaert et al. (2013) and Jurado et al. (2015) order them last. The estimates of the structural parameter matrices obtained from assuming conditional heteroskedasticity or independence of structural shocks (dCov) are

$$\widehat{\mathsf{B}}_{\mathrm{CH}} = \begin{bmatrix} \underset{(-0.03;\ 0.02)}{0.035} & \underset{(-1.04;\ -35.97)}{-1.054} & \underset{(0.23;\ 3.87)}{0.218} \\ \underset{(0.85;\ 32.13)}{0.871} & \underset{(0.13;\ 2.20)}{0.183} & \underset{(0.06;\ 1.72)}{0.081} \\ \underset{(-0.06;\ -1.27)}{-0.077} & \underset{(-0.03;\ -0.73)}{-0.045} & \underset{(1.73;\ 36.96)}{1.741} \end{bmatrix} \text{ and } \widehat{\mathsf{B}}_{\mathrm{dCov}} = \begin{bmatrix} \underset{(0.05;\ 0.51)}{0.105} & \underset{(-1.06;\ -28.61)}{-1.075} & \underset{(0.20;\ 2.54)}{0.210} \\ \underset{(0.86;\ 18.98)}{0.866} & \underset{(0.20;\ 1.49)}{0.252} & \underset{(0.05;\ 0.85)}{0.039} \\ \underset{(-0.06;\ -0.46)}{-0.038} & \underset{(-0.08;\ -0.61)}{-0.070} & \underset{(1.56;\ 136.22)}{1.566} \end{bmatrix},$$

respectively, with values in parentheses $(a;\ b)$ denoting the bootstrap means (a) and t-ratios (b).[22] Since structural shocks are often labeled in terms of their impact effects on the variables of the system, it is worth

[22] Bootstrap means out of 1000 replications are close to the point estimates for both models, which we consider as an informal indication of bootstrap consistency. Bootstrap t-ratios are the ratio of parameter estimates and bootstrap standard errors. Standard errors for $\widehat{\mathsf{B}}_{\mathrm{CH}}$ could be also derived as the square root of the inverted information matrix. Both approaches obtain similar results.

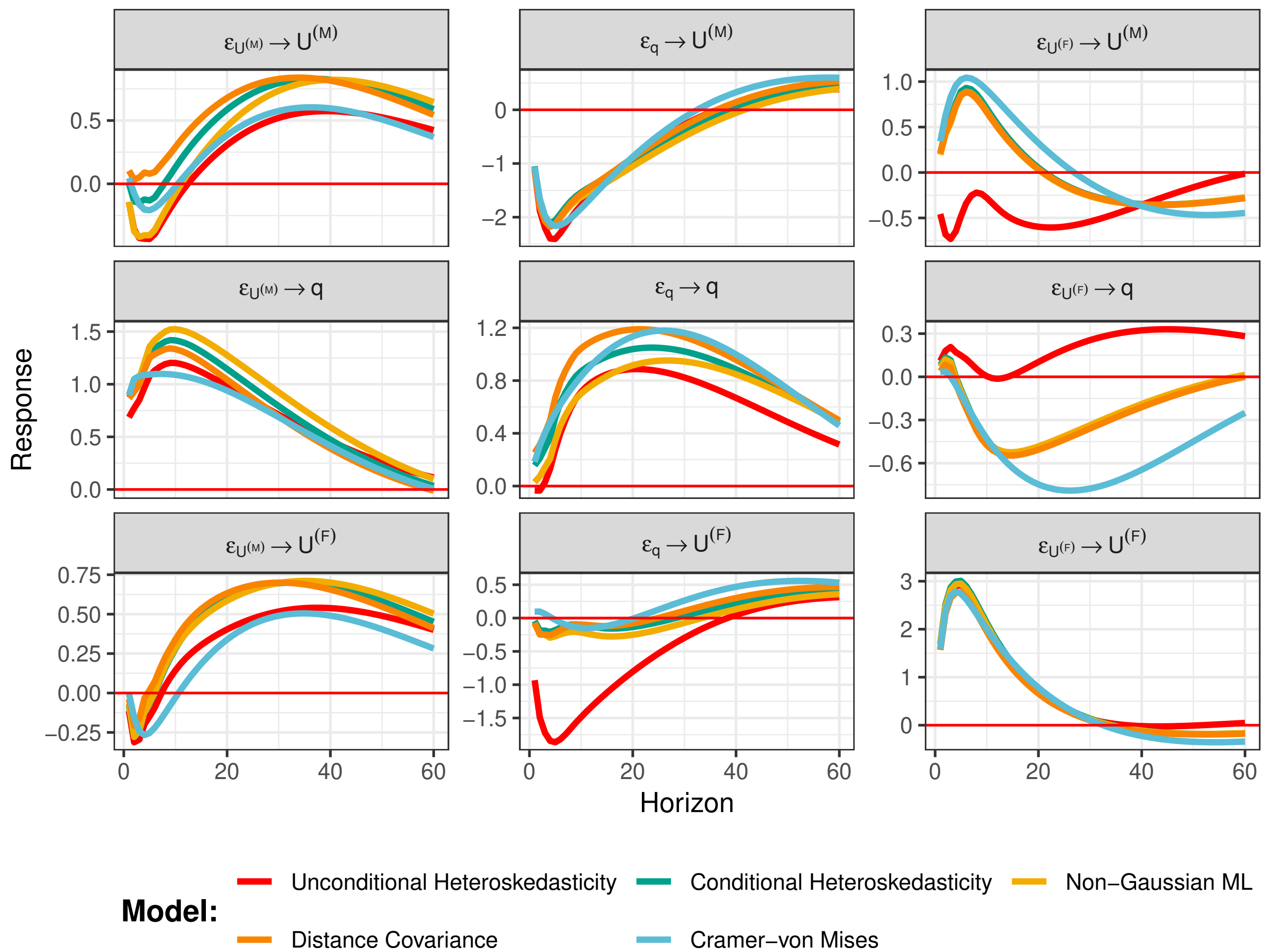

Figure 5: Point estimates of impulse response functions.

noticing that we find a characteristic sign pattern in both estimated matrices $\widehat{\mathsf{B}}_{\bullet}$, $\bullet \in \{\text{CH, dCov}\}$.

The $t-$ratios for the upper diagonal parameter estimates, $\widehat{\mathsf{b}}_{\bullet,ij}, j > i, \bullet \in \{\text{CH}, \text{dCov}\}$, indicate that contemporaneous dynamics are not supportive for the assumption of a triangular relation between structural and reduced form disturbances of the present ordering. Moreover, since both structural matrices show two significant impact effects on macroeconomic uncertainty and economic activity $\widehat{\mathsf{b}}_{\bullet,ij} \neq 0, i = 1, 2, \bullet \in \{\text{CH}, \text{dCov}\}$, there is no alternative column ordering (or variable ordering in y_t) which is in line with a lower triangular impact transmission of identified shocks on the variables of the system (see also the tests on joint insignificance of upper diagonal parameters $H_0 : \mathsf{b}_{\bullet,ij} = 0, j > i$, documented in Table A6).

To investigate the causality between uncertainty and real economic activity in an exemplary manner, we show IRFs associated with $\widehat{\mathsf{B}}_{\text{dCov}}$ in Figure 6 joint with 68% confidence intervals. It turns out that financial uncertainty shocks trigger sluggish real economic slow-downs after about six months. In contrast, positive production shocks do not reduce financial uncertainty, which indicates that enhanced financial uncertainty is not a result of economic slowdowns. While the link of financial uncertainty and production is unidirectional, we find a bidirectional relation between macroeconomic uncertainty and real economic activity. Overall, our

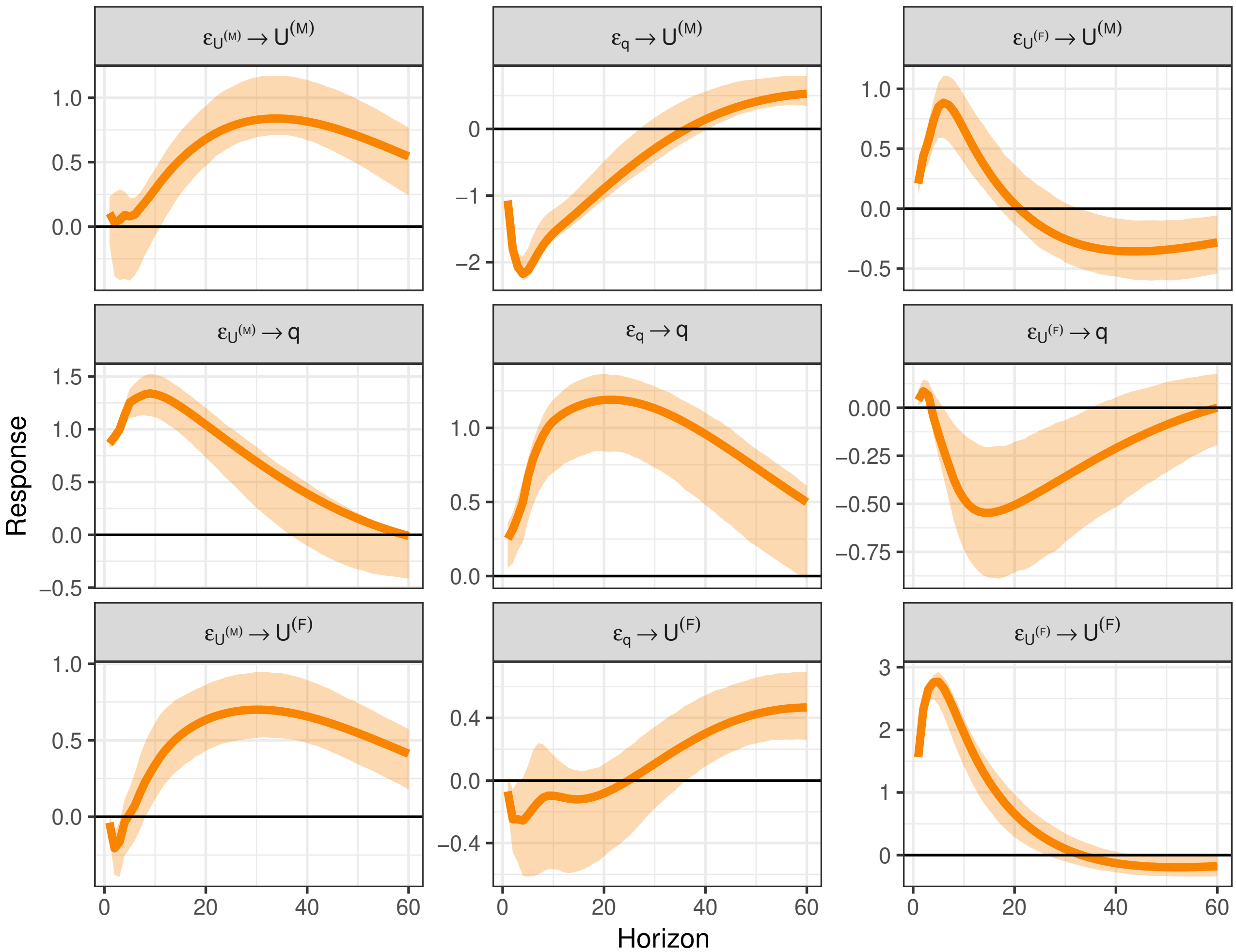

Figure 6: Impulse response functions obtained by $\widehat{\mathsf{B}}_{\text{dCov}}$ joint with point-wise 68% confidence bands obtained by 1000 bootstrap replications. Point-wise comparisons do not reveal any significant differences between the IRFs implied by $\widehat{\mathsf{B}}_{\text{dCov}}$ and $\widehat{\mathsf{B}}_{\bullet}$, $\bullet \in \{\text{CH, nGML, CvM}\}$. With regard to IRFs obtained from $\widehat{\mathsf{B}}_{\text{UH}}$, point-wise comparisons show significantly different effects in case of three response functions ($\varepsilon_{U^{(M)}} \to U^{(M)}$, $\varepsilon_{U^{(M)}} \to q$, $\varepsilon_{U^{(M)}} \to U^{(F)}$).

findings are mostly in line with those of Ludvigson et al. (2018). Nevertheless, we stress two noteworthy differences in the IRFs. Firstly, while the financial uncertainty shock in Ludvigson et al. (2018) exerts an extremely persistent effect on industrial production, the IRFs in Figure 6 signal a transitory effect, which is more in line with the associated literature (see Bloom, 2009; Bekaert et al., 2013; Jurado et al., 2015). Secondly, Ludvigson et al. (2018) find a positive instantaneous effect of a production shock on financial uncertainty, yet we detect no significant effect on impact. Interestingly, our result seems to be more in line with theoretical arguments of Ludvigson et al. (2018) which are in favour of a uni directional transmission from financial uncertainty to real economic activity.

5 Conclusions

The detection of structural shocks in SVARs relies either on economically motivated restrictions or statistical means. We compare alternative identification approaches in a simulation study and in the framework of an empirical analysis of the relationship between uncertainty and business cycles in the US economy. In specific, we focus on a stylized version of identification based on sign restrictions (representing economic restrictions), two identification approaches based on heteroskedasticity (statistical identification) and three identification procedures based on independence of the shocks.

By means of Monte Carlo simulations we confirm and specify the bias induced by classical sign restrictions to occur irrespective of the underlying distribution and sample size. Unlike theory based set identification, statistical identification schemes provide consistent estimates of the structural model parameters. Overall, we find that identification by means of (co)variance changes appears more specialized in the sense that it (i) provides precise estimation results if the data aligns with the specified/assumed type of heteroskedasticity, and (ii) obtains imprecise estimates under (co)variance misspecification. In contrast, the independence-based models (especially those which are nonparametric) are more flexible. More specifically, a non-Gaussian ML estimator performs best if the likelihood is correctly specified, while identification by means of nonparametric dependence diagnostics is the method of choice if the distribution of structural shocks is unknown, non-existent or subjected to heteroskedasticity of unknown form. Hence, in non-Gaussian models, identification of independent components is worth considering regardless of potential heteroskedasticity. In particular, targeting at independent components by means of minimizing a nonparametric independence diagnostic (the distance covariance, say) appears as a method of choice if an analyst lacks detailed and accurate information on statistical features of the data.

We apply alternative statistical identification procedures to an SVAR model on the relationship between macroeconomic and financial uncertainty and the US business cycle (Ludvigson et al., 2018). The obtained structural shocks and impulse responses largely support the results of Ludvigson et al. (2018) who argue powerfully against recursive identification schemes, and - instead - advocate a combination of narrative event constraints and nonzero correlations with external instruments. Economic narratives and external instruments are most helpful for the labelling of statistically identified shocks. The more agnostic identification by means of statistical criteria and the understanding of the statistical characteristics of the identified shocks are, however, important for the reliability and interpretation of structural model implications. For instance, under heteroskedasticity the quantitative interpretation of impulse responses to 'unit' shocks deserves particular attention, and the notion of shocks to occur in 'isolation' is best motivated within systems generated from independent structural shocks. Structural shocks determined to align with economic narratives and correlation constraints in the strongest possible form lack independence with high significance. We find that financial uncertainty is primarily a source of business cycle fluctuations, whereas macroeconomic uncertainty rises in response to economic activity shocks.

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Appendix

A Dependence diagnostics

Distance covariance (Matteson and Tsay, 2017)

For a K-dimensional vector of structural shocks ε_t at time $t = 1, \ldots, T$ the distance covariance $\mathcal{V}^2$ detects dependence between two subsets of the components. Between the kth component $\varepsilon_{t,k}$, $k \in \{1, \ldots, K\}$ and all subsequent ones ε_{t,k^+} with $k^+ = k+1, \ldots, K$, dependence is measured by $\mathcal{V}^2(\varepsilon_{t,k}, \varepsilon_{t,k^+})$ which is the distance between the characteristic functions $\varphi_{\varepsilon_{t,k},\varepsilon_{t,k^+}}$ and $\varphi_{\varepsilon_{t,k}}\varphi_{\varepsilon_{t,k^+}}$, the joint characteristic function and the one under independence, respectively. To measure mutual dependence, i.e. dependence of all possible combinations between the variables $\varepsilon_{t,1}, \ldots, \varepsilon_{t,K}$, the dependence criterion reads as

$$\mathcal{U}_T(\varepsilon_{t,1}, \ldots, \varepsilon_{t,K}) = T \cdot \sum_{k=1}^{K-1} \mathcal{V}^2(\varepsilon_{t,k}, \varepsilon_{t,k+}). \tag{16}$$

The distance covariance $\mathcal{U}_T(\hat{\varepsilon}_{t,1}, \ldots, \hat{\varepsilon}_{t,K})$ is then minimized to identify $\hat{\varepsilon}_t = \mathsf{B}^{-1}\hat{u}_t$ with least dependent components.

Cramér-von Mises statistic (Genest and Rémillard, 2004)

Mutual dependence within a K-dimensional vector of structural shocks ε_t at time $t = 1, \ldots, T$ can be measured by the Cramér-von Mises functional

$$\mathcal{C}_T = \int_{(0,1)^K} T\Big(C_T(\epsilon) - \prod_{k=1}^{K} U_T(\epsilon_k)\Big)^2 d\epsilon$$

with cumulative distribution function U_T of a uniformly distributed variable on $\{1/T, \ldots, T/T\}$ and the empirical copula C_T. Apparently, the functional $\mathcal{C}$ measures the distance between the empirical copula based on the vector of structural shocks ε_t and the copula under independence. Genest and Rémillard (2004) describe the estimation of the copula and the explicit statistic in more detail. Minimizing $\mathcal{C}$ with respect to B (i.e., considering an empirical copula C_T determined by $\hat{\varepsilon}_t = \mathsf{B}^{-1}\hat{u}_t$) provides the HL estimates and the corresponding least dependent components.

B The trinity DSGE model

For a detailed description of the underlying DSGE model and the parametrization see Herwartz and Plödt (2016a). The derivation of the log linearized SVAR from the underlying DSGE model relies on a first order Taylor series expansion. Under deviations from Gaussian innovations, the numerical solution provided in (13) is not robust with regards to higher order moments of the true shocks (i.e., with regards to the solution of higher order Taylor series expansions). Since our interest is not in the most accurate dynamic description of economic optimization solutions and to avoid distribution specific DGPs, we abstract from this point to take advantage of scenario-independent 'true' parameter values in simulated DGPs. For simulation purposes we employ a simple 3-equation dynamic stochastic general equilibrium (DSGE) model that has been widely used as a baseline framework for monetary policy analysis (Gertler et al., 1999; Carlstrom et al., 2009; Castelnuovo, 2013, 2012, 2016). The consideration of trivariate systems is also common practice in the SVAR literature.

The log-linearized version of the model reads as

$$x_t = \gamma E_t x_{t+1} + (1-\gamma) x_{t-1} - \delta_x (r_t - E_t \pi_{t+1}) + \omega_{x,t}, \quad (17)$$

$$\pi_t = (1+\alpha\beta)^{-1} \beta E_t \pi_{t+1} + (1+\alpha\beta)^{-1} \alpha \pi_{t-1} + \kappa x_t + \omega_{\pi,t}, \quad (18)$$

$$r_t = \tau_r r_{t-1} + (1-\tau_r)(\tau_\pi \pi_t + \tau_x x_t) + \omega_{r,t}, \quad (19)$$

$$\omega_{\bullet,t} = \rho_\bullet \omega_{\bullet,t-1} + \varepsilon_{\bullet,t}, \bullet \in \{x, \pi, r\},\ t = 1, \ldots, T, \quad (20)$$

where x_t, π_t and r_t denote the output gap, inflation and the nominal interest rate, respectively, and E_t indicates expectations formed at period t. Accordingly, the equations (17) to (19) represent a New Keynesian IS equation, a hybrid New Keynesian Phillips curve, and a Taylor rule with interest rate smoothing. First order autoregressive shock processes are summarized in equation (20), with subscripts $\bullet \in \{x, \pi, r\}$ indicating a demand shock, a supply shock and a monetary policy shock, respectively.

The employed parameter settings correspond to common calibration assumptions drawn from the macroeconomic literature. The model is calibrated with common settings, i.e., $\beta = 0.99$ (discounting), $\kappa = 0.05$ (slope of Phillips curve), $\alpha = 0.5$ (indexation of past inflation), $\delta_x = 0.1$ (impact of real interest), $\gamma = 0.5$ (effect of output expectations), $\tau_\pi = 1.8, \tau_x = 0.5, \tau_r = 0.6$ (Taylor rule). The autoregressive parameters in (20) are set to $\rho_x = \rho_\pi = \rho_r = 0.5$.

C Detailed simulation results

Table A1: Simulation results for sample size $T = 100$. A labeling ratio of 1 means that the true sign pattern of B from the DGP in equation (12) (or single shocks respectively) was found in every Monte Carlo iteration.

	Homoskedasticity					Unconditional Heteroskedasticity					Conditional Heteroskedasticity				
		Labeling ratio					Labeling ratio					Labeling ratio			
Distribution	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$
Sign Restrictions															
$\mathcal{N}$	2.48	.	.	.	.	5.51	.	.	.	.	2.46	.	.	.	.
$t_{(4)}$	2.45	.	.	.	.	5.33	.	.	.	.	2.42	.	.	.	.
$t_{(8)}$	2.49	.	.	.	.	5.41	.	.	.	.	2.45	.	.	.	.
χ^2	2.49	.	.	.	.	5.43	.	.	.	.	2.38	.	.	.	.
IG	2.50	.	.	.	.	5.39	.	.	.	.	2.25	.	.	.	.
Unconditional Heteroskedasticity															
$\mathcal{N}$	4.85	0.14	0.58	0.72	0.2	2.95	0.37	0.71	0.61	0.49	3.22	0.3	0.71	0.64	0.34
$t_{(4)}$	3.80	0.23	0.67	0.68	0.31	3.22	0.37	0.73	0.62	0.46	3.02	0.27	0.7	0.65	0.32
$t_{(8)}$	4.35	0.18	0.59	0.73	0.25	3.03	0.37	0.74	0.58	0.51	3.07	0.27	0.72	0.65	0.31
χ^2	4.04	0.24	0.66	0.68	0.33	3.29	0.37	0.75	0.63	0.48	2.84	0.37	0.74	0.67	0.41
IG	3.32	0.38	0.73	0.69	0.46	3.19	0.41	0.72	0.68	0.51	2.72	0.31	0.71	0.65	0.36
Conditional Heteroskedasticity															
$\mathcal{N}$	4.64	0.19	0.75	0.72	0.23	4.97	0.18	0.72	0.71	0.22	1.99	0.73	0.91	0.86	0.76
$t_{(4)}$	4.16	0.24	0.73	0.73	0.29	4.36	0.2	0.69	0.68	0.27	2.24	0.68	0.87	0.84	0.72
$t_{(8)}$	4.50	0.21	0.75	0.65	0.27	4.52	0.2	0.74	0.71	0.24	1.96	0.73	0.91	0.86	0.77
χ^2	4.17	0.27	0.75	0.72	0.33	4.09	0.28	0.74	0.73	0.35	2.15	0.65	0.87	0.83	0.69
IG	3.79	0.38	0.77	0.71	0.44	3.94	0.35	0.74	0.71	0.42	2.31	0.62	0.85	0.81	0.67
Non-Gaussian ML															
$\mathcal{N}$	4.70	0.22	0.75	0.73	0.36	12.41	0.28	0.72	0.61	0.42	5.30	0.47	0.87	0.7	0.51
$t_{(4)}$	2.64	0.59	0.87	0.78	0.65	8.47	0.44	0.75	0.68	0.56	5.03	0.51	0.86	0.72	0.56
$t_{(8)}$	3.70	0.41	0.77	0.72	0.52	9.64	0.32	0.66	0.6	0.49	5.08	0.52	0.89	0.73	0.53
χ^2	2.77	0.51	0.8	0.72	0.59	6.84	0.4	0.68	0.64	0.54	4.25	0.55	0.89	0.74	0.6
IG	1.77	0.67	0.9	0.78	0.71	5.57	0.51	0.78	0.7	0.61	3.14	0.61	0.92	0.76	0.62
Distance Covariance															
$\mathcal{N}$	3.60	0.16	0.57	0.62	0.27	5.99	0.17	0.5	0.54	0.31	2.66	0.31	0.72	0.64	0.39
$t_{(4)}$	2.38	0.46	0.78	0.71	0.55	4.15	0.35	0.62	0.68	0.47	2.34	0.38	0.76	0.67	0.44
$t_{(8)}$	3.23	0.25	0.64	0.7	0.37	4.96	0.27	0.57	0.6	0.4	2.41	0.33	0.72	0.67	0.42
χ^2	1.28	0.79	0.95	0.84	0.81	3.53	0.56	0.78	0.73	0.64	1.91	0.54	0.87	0.74	0.57
IG	1.06	0.87	0.99	0.89	0.88	3.11	0.66	0.88	0.8	0.72	1.85	0.69	0.94	0.81	0.69
Cramer-von Mises															
$\mathcal{N}$	5.03	0.15	0.57	0.72	0.2	8.41	0.09	0.49	0.56	0.13	3.84	0.15	0.59	0.6	0.21
$t_{(4)}$	4.16	0.24	0.66	0.69	0.33	6.71	0.19	0.52	0.6	0.25	3.15	0.22	0.67	0.61	0.25
$t_{(8)}$	4.86	0.17	0.58	0.7	0.24	7.72	0.12	0.46	0.59	0.17	3.49	0.18	0.63	0.59	0.23
χ^2	1.96	0.57	0.87	0.76	0.62	5.11	0.33	0.63	0.73	0.42	2.49	0.32	0.75	0.67	0.36
IG	1.56	0.7	0.93	0.81	0.73	4.14	0.43	0.72	0.74	0.51	2.17	0.45	0.83	0.71	0.48

Table A2: Simulation results for sample size $T = 200$. For further notes see Table A1.

	Homoskedasticity					Unconditional Heteroskedasticity					Conditional Heteroskedasticity				
		Labeling ratio					Labeling ratio					Labeling ratio			
Distribution	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$
Sign Restrictions															
$\mathcal{N}$	2.48	.	.	.	.	5.49	.	.	.	.	2.44	.	.	.	.
$t_{(4)}$	2.46	.	.	.	.	5.41	.	.	.	.	2.40	.	.	.	.
$t_{(8)}$	2.48	.	.	.	.	5.47	.	.	.	.	2.45	.	.	.	.
χ^2	2.46	.	.	.	.	5.41	.	.	.	.	2.43	.	.	.	.
IG	2.46	.	.	.	.	5.40	.	.	.	.	2.34	.	.	.	.
Unconditional heteroskedasticity															
$\mathcal{N}$	4.82	0.17	0.6	0.73	0.22	2.00	0.53	0.82	0.65	0.67	2.79	0.37	0.78	0.68	0.43
$t_{(4)}$	3.80	0.32	0.68	0.74	0.39	2.32	0.54	0.8	0.68	0.68	2.62	0.38	0.75	0.71	0.42
$t_{(8)}$	4.50	0.19	0.62	0.71	0.25	2.04	0.56	0.83	0.68	0.68	2.97	0.38	0.74	0.67	0.41
χ^2	3.92	0.3	0.65	0.71	0.36	2.29	0.51	0.83	0.65	0.62	2.79	0.38	0.76	0.68	0.43
IG	3.08	0.4	0.76	0.71	0.47	2.55	0.51	0.8	0.66	0.62	2.53	0.45	0.79	0.73	0.49
Conditional Heteroskedasticity															
$\mathcal{N}$	4.55	0.21	0.76	0.75	0.24	5.83	0.44	0.79	0.61	0.57	1.44	0.91	0.99	0.95	0.91
$t_{(4)}$	4.01	0.3	0.75	0.73	0.35	5.89	0.41	0.77	0.66	0.51	1.89	0.87	0.96	0.93	0.88
$t_{(8)}$	4.50	0.21	0.74	0.72	0.27	5.78	0.38	0.78	0.57	0.54	1.66	0.89	0.98	0.94	0.89
χ^2	4.36	0.26	0.76	0.74	0.32	6.58	0.36	0.78	0.6	0.46	1.72	0.86	0.95	0.93	0.87
IG	3.89	0.36	0.78	0.72	0.39	6.86	0.38	0.72	0.65	0.45	2.10	0.79	0.94	0.89	0.8
Non-Gaussian ML															
$\mathcal{N}$	4.53	0.23	0.75	0.69	0.43	10.27	0.36	0.76	0.62	0.54	4.06	0.68	0.94	0.79	0.71
$t_{(4)}$	1.36	0.82	0.96	0.87	0.83	7.16	0.68	0.9	0.79	0.73	3.18	0.77	0.97	0.83	0.77
$t_{(8)}$	2.69	0.52	0.84	0.74	0.61	7.25	0.48	0.8	0.66	0.61	3.26	0.67	0.97	0.78	0.68
χ^2	1.68	0.66	0.9	0.78	0.7	4.32	0.5	0.77	0.68	0.62	3.01	0.72	0.95	0.8	0.73
IG	0.98	0.86	0.98	0.89	0.87	3.92	0.7	0.89	0.77	0.78	2.61	0.75	0.98	0.81	0.75
Distance Covariance															
$\mathcal{N}$	3.58	0.17	0.56	0.65	0.27	4.74	0.29	0.61	0.56	0.46	1.90	0.59	0.88	0.77	0.64
$t_{(4)}$	1.72	0.63	0.87	0.77	0.68	3.28	0.62	0.85	0.75	0.7	1.79	0.65	0.93	0.77	0.67
$t_{(8)}$	2.77	0.31	0.7	0.63	0.44	3.90	0.38	0.7	0.59	0.55	1.82	0.57	0.9	0.71	0.6
χ^2	0.78	0.93	1	0.94	0.93	2.84	0.77	0.93	0.87	0.8	1.61	0.77	0.98	0.82	0.77
IG	0.70	0.97	1	0.97	0.97	2.67	0.86	0.98	0.9	0.88	1.69	0.84	0.99	0.87	0.84
Cramer-von Mises															
$\mathcal{N}$	5.01	0.16	0.58	0.74	0.21	7.01	0.19	0.59	0.57	0.25	2.97	0.31	0.76	0.65	0.36
$t_{(4)}$	3.58	0.37	0.69	0.72	0.45	5.46	0.3	0.67	0.62	0.37	2.53	0.33	0.74	0.69	0.37
$t_{(8)}$	4.68	0.2	0.61	0.71	0.27	6.33	0.2	0.62	0.58	0.28	2.81	0.32	0.75	0.65	0.36
χ^2	1.06	0.84	0.98	0.88	0.86	3.56	0.53	0.81	0.79	0.6	1.91	0.58	0.91	0.74	0.58
IG	0.92	0.9	0.99	0.91	0.91	3.24	0.67	0.89	0.83	0.7	1.84	0.66	0.94	0.78	0.67

Table A3: Simulation results for sample size $T = 500$. For further notes see Table A1.

	Homoskedasticity					Unconditional Heteroskedasticity					Conditional Heteroskedasticity				
		Labeling ratio					Labeling ratio					Labeling ratio			
Distribution	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$
Sign Restrictions															
$\mathcal{N}$	2.46	.	.	.	.	5.50	.	.	.	.	2.46	.	.	.	.
$t_{(4)}$	2.44	.	.	.	.	5.42	.	.	.	.	2.38	.	.	.	.
$t_{(8)}$	2.45	.	.	.	.	5.47	.	.	.	.	2.46	.	.	.	.
χ^2	2.45	.	.	.	.	5.46	.	.	.	.	2.43	.	.	.	.
IG	2.45	.	.	.	.	5.42	.	.	.	.	2.40	.	.	.	.
Unconditional Heteroskedasticity															
$\mathcal{N}$	5.00	0.13	0.61	0.74	0.18	1.20	0.81	0.96	0.82	0.84	2.47	0.52	0.85	0.72	0.55
$t_{(4)}$	3.64	0.32	0.71	0.67	0.4	1.35	0.74	0.92	0.77	0.82	2.38	0.5	0.81	0.77	0.53
$t_{(8)}$	4.58	0.21	0.65	0.72	0.26	1.29	0.77	0.93	0.79	0.82	2.47	0.48	0.79	0.74	0.52
χ^2	4.00	0.29	0.69	0.71	0.34	1.32	0.77	0.92	0.8	0.83	2.44	0.49	0.81	0.73	0.52
IG	2.97	0.45	0.81	0.72	0.5	1.58	0.73	0.91	0.77	0.8	2.33	0.5	0.83	0.73	0.54
Conditional Heteroskedasticity															
$\mathcal{N}$	4.74	0.21	0.71	0.74	0.24	3.95	0.64	0.86	0.72	0.78	1.14	0.98	0.99	0.98	0.98
$t_{(4)}$	3.82	0.31	0.78	0.73	0.36	4.92	0.61	0.83	0.7	0.73	1.43	0.97	1	0.98	0.97
$t_{(8)}$	4.47	0.27	0.77	0.74	0.31	4.20	0.57	0.85	0.65	0.71	1.30	0.97	0.99	0.98	0.97
χ^2	3.97	0.35	0.8	0.75	0.39	4.58	0.58	0.82	0.7	0.73	1.36	0.95	0.98	0.98	0.95
IG	3.74	0.36	0.78	0.73	0.41	5.70	0.49	0.79	0.72	0.58	1.56	0.96	1	0.98	0.96
Non-Gaussian ML															
$\mathcal{N}$	4.60	0.2	0.76	0.68	0.36	6.78	0.43	0.81	0.62	0.63	2.32	0.87	1	0.88	0.87
$t_{(4)}$	0.76	0.95	1	0.96	0.95	6.57	0.89	0.99	0.9	0.9	2.34	0.91	1	0.92	0.91
$t_{(8)}$	1.63	0.7	0.94	0.77	0.73	4.86	0.64	0.88	0.71	0.74	2.42	0.87	1	0.88	0.87
χ^2	1.00	0.86	0.98	0.89	0.87	3.50	0.69	0.87	0.75	0.8	1.88	0.88	1	0.9	0.88
IG	0.61	0.96	1	0.96	0.96	3.12	0.89	0.98	0.9	0.91	1.78	0.92	1	0.92	0.92
Distance Covariance															
$\mathcal{N}$	3.58	0.14	0.56	0.63	0.25	3.78	0.4	0.72	0.57	0.62	1.37	0.81	0.99	0.84	0.82
$t_{(4)}$	0.92	0.9	0.99	0.92	0.9	2.61	0.89	0.98	0.9	0.9	1.47	0.89	1	0.9	0.89
$t_{(8)}$	2.17	0.43	0.78	0.67	0.5	3.03	0.64	0.86	0.7	0.74	1.37	0.84	0.99	0.86	0.85
χ^2	0.47	1	1	1	1	2.53	0.96	1	0.97	0.96	1.37	0.88	1	0.89	0.88
IG	0.44	1	1	1	1	2.44	0.98	1	0.98	0.98	1.55	0.92	1	0.92	0.92
Cramer-von Mises															
$\mathcal{N}$	5.04	0.13	0.58	0.72	0.18	4.89	0.33	0.72	0.57	0.44	2.08	0.53	0.89	0.73	0.55
$t_{(4)}$	2.42	0.53	0.81	0.75	0.59	3.45	0.6	0.85	0.74	0.66	1.82	0.6	0.93	0.76	0.61
$t_{(8)}$	4.29	0.22	0.67	0.68	0.28	4.29	0.38	0.76	0.6	0.48	1.84	0.58	0.9	0.75	0.59
χ^2	0.63	0.98	1	0.98	0.98	2.82	0.83	0.98	0.88	0.84	1.44	0.78	0.99	0.84	0.78
IG	0.54	0.99	1	0.99	0.99	2.72	0.84	0.99	0.91	0.84	1.58	0.86	0.99	0.88	0.86

Table A4: Simulation results for sample size $T = 1000$. For further notes see Table A1.

	Homoskedasticity					Unconditional Heteroskedasticity					Conditional Heteroskedasticity				
		Labeling ratio					Labeling ratio					Labeling ratio			
Distribution	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$	$\widehat{\text{MSE}}$	All	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$
Sign Restrictions															
$\mathcal{N}$	2.45	.	.	.	.	5.49	.	.	.	.	2.45	.	.	.	.
$t_{(4)}$	2.44	.	.	.	.	5.44	.	.	.	.	2.39	.	.	.	.
$t_{(8)}$	2.45	.	.	.	.	5.47	.	.	.	.	2.44	.	.	.	.
χ^2	2.45	.	.	.	.	5.47	.	.	.	.	2.44	.	.	.	.
IG	2.46	.	.	.	.	5.49	.	.	.	.	2.40	.	.	.	.
Unconditional Heteroskedasticity															
$\mathcal{N}$	4.96	0.16	0.61	0.74	0.19	0.83	0.9	0.98	0.9	0.92	2.18	0.6	0.9	0.75	0.63
$t_{(4)}$	3.44	0.34	0.73	0.71	0.4	0.97	0.89	0.97	0.89	0.93	2.22	0.54	0.88	0.71	0.57
$t_{(8)}$	4.57	0.18	0.61	0.73	0.23	0.91	0.89	0.98	0.89	0.91	2.27	0.57	0.88	0.74	0.6
χ^2	4.01	0.27	0.65	0.7	0.34	0.89	0.9	0.97	0.91	0.93	2.18	0.59	0.88	0.75	0.62
IG	3.08	0.44	0.77	0.72	0.5	1.05	0.85	0.95	0.86	0.9	2.13	0.58	0.88	0.73	0.62
Conditional Heteroskedasticity															
$\mathcal{N}$	4.74	0.19	0.73	0.72	0.23	3.23	0.82	0.91	0.83	0.91	1.04	0.98	0.98	1	0.98
$t_{(4)}$	4.16	0.34	0.78	0.71	0.39	4.11	0.76	0.89	0.82	0.84	1.31	0.98	0.99	0.99	0.98
$t_{(8)}$	4.61	0.25	0.74	0.73	0.3	3.64	0.77	0.88	0.79	0.87	1.18	0.97	0.98	1	0.97
χ^2	4.06	0.3	0.78	0.74	0.35	3.75	0.76	0.89	0.79	0.87	1.15	0.99	0.99	1	0.99
IG	3.86	0.38	0.8	0.75	0.44	4.92	0.68	0.85	0.79	0.77	1.43	0.97	0.99	0.99	0.97
Non-Gaussian ML															
$\mathcal{N}$	4.45	0.19	0.77	0.63	0.42	5.28	0.53	0.82	0.62	0.71	1.64	0.96	1	0.96	0.96
$t_{(4)}$	0.51	0.99	1	0.99	0.99	6.72	0.97	1	0.97	0.97	1.94	0.99	1	0.99	0.99
$t_{(8)}$	1.02	0.87	0.99	0.89	0.87	3.78	0.85	0.97	0.86	0.88	1.76	0.97	1	0.97	0.97
χ^2	0.69	0.93	1	0.93	0.93	2.88	0.83	0.94	0.85	0.88	1.46	0.96	1	0.96	0.96
IG	0.42	1	1	1	1	2.90	0.98	1	0.98	0.99	1.52	0.96	1	0.96	0.96
Distance Covariance															
$\mathcal{N}$	3.74	0.12	0.5	0.62	0.22	3.18	0.6	0.79	0.67	0.76	1.17	0.93	1	0.93	0.93
$t_{(4)}$	0.60	0.98	1	0.98	0.98	2.45	0.96	1	0.96	0.96	1.34	0.96	1	0.96	0.96
$t_{(8)}$	1.48	0.71	0.9	0.81	0.76	2.67	0.84	0.96	0.86	0.87	1.20	0.93	1	0.94	0.93
χ^2	0.33	1	1	1	1	2.42	0.99	1	0.99	0.99	1.22	0.97	1	0.97	0.97
IG	0.31	1	1	1	1	2.41	1	1	1	1	1.47	0.97	1	0.97	0.97
Cramer-von Mises															
$\mathcal{N}$	5.03	0.12	0.55	0.72	0.18	3.61	0.49	0.8	0.61	0.6	1.51	0.71	0.95	0.8	0.71
$t_{(4)}$	1.65	0.68	0.92	0.8	0.71	2.76	0.77	0.97	0.81	0.79	1.49	0.76	0.98	0.83	0.76
$t_{(8)}$	3.23	0.42	0.75	0.72	0.48	3.16	0.64	0.88	0.71	0.71	1.47	0.73	0.97	0.79	0.73
χ^2	0.36	1	1	1	1	2.64	0.9	1	0.93	0.9	1.26	0.9	1	0.91	0.9
IG	0.36	1	1	1	1	2.60	0.94	1	0.96	0.94	1.48	0.95	1	0.95	0.95

D Further results from the empirical application

Table A5: Separate test results on kurtosis and skewness for $\hat{\varepsilon}_\bullet, \bullet \in$ {UH, CH, nGML, dCov, CvM}. Tests on non-Gaussian components can be rejected for $k_1 = 1$ and $k_1 = 2$ Gaussian components.

		$\hat{\varepsilon}_{U(M)}$	$\hat{\varepsilon}_q$	$\hat{\varepsilon}_{U(F)}$	H_0 :	$k_1 = 1$	$k_1 = 2$
$\widehat{\mathsf{B}}_{\text{UH}}$	Kurtosis:	10.324 (0.000)	5.423 (0.000)	13.673 (0.000)	Gaussian components	50523.7 (0.001)	4790.12 (0.001)
	Skewness:	−0.323 (0.001)	0.309 (0.001)	−1.184 (0.000)			
$\widehat{\mathsf{B}}_{\text{CH}}$	Kurtosis:	9.361 (0.000)	5.161 (0.000)	21.259 (0.000)			
	Skewness:	−0.116 (0.208)	0.15 (0.104)	−1.844 (0.00)			
$\widehat{\mathsf{B}}_{\text{nGML}}$	Kurtosis:	10.073 (0.000)	5.078 (0.000)	21.337 (0.000)			
	Skewness:	−0.28 (0.001)	0.056 (0.176)	−1.853 (0.000)			
$\widehat{\mathsf{B}}_{\text{dCov}}$	Kurtosis:	8.388 (0.000)	5.258 (0.000)	21.363 (0.000)			
	Skewness:	0.054 (0.548)	0.099 (0.294)	−1.855 (0.000)			
$\widehat{\mathsf{B}}_{\text{CvM}}$	Kurtosis:	9.091 (0.000)	5.184 (0.000)	21.001 (0.000)			
	Skewness:	−0.066 (0.454)	0.13 (0.15)	−1.822 (0.000)			

Table A6: Test results on the null hypothesis $H_0 : \mathsf{b}_{\bullet,ij} = 0, j > i, \bullet \in \{\text{CH}, \text{dCov}\}$, against the alternative that at least one of the coefficients is nonzero for all six permutations of the vector of variables. We evaluate the set of restrictions by means of likelihood-ratio (LR) tests with the SVAR-CH model and by means of joint significance χ^2 tests with the distance covariance approach. The χ^2 statistics and corresponding p-values are based on 1000 bootstrap iterations.

	CH		dCov	
	LR statistic	p-value	χ^2 statistic	p-value
$(U_t^{(M)}, q_t, U_t^{(F)})'$	51.642	< 0.0001	6695	< 0.0001
$(U_t^{(M)}, U_t^{(F)}, q_t)'$	84.151	< 0.0001	1091	< 0.0001
$(q_t, U_t^{(M)}, U_t^{(F)})'$	38.034	< 0.0001	5922.3	< 0.0001
$(q_t, U_t^{(F)}, U_t^{(M)})'$	14.639	0.0022	4239.8	< 0.0001
$(U_t^{(F)}, q_t, U_t^{(M)})'$	134.36	< 0.0001	4202.6	< 0.0001
$(U_t^{(F)}, U_t^{(M)}, q_t)'$	20.858	0.001	362.45	< 0.0001