

**HYPERBOLIC DISCOUNTING CAN BE  
GOOD FOR YOUR HEALTH**

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# Hyperbolic Discounting Can Be Good For Your Health\*

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**Abstract.** It has been argued that hyperbolic discounting of future gains and losses leads to time-inconsistent behavior and thereby, in the context of health economics, not enough investment in health and too much indulgence of unhealthy consumption. Here, we challenge this view. We set up a life-cycle model of human aging and longevity in which individuals discount the future hyperbolically and make time-consistent decisions. This allows us to disentangle the role of discounting from the time consistency issue. We show that hyperbolically discounting individuals, under a reasonable normalization, invest more in their health than they would if they had a constant rate of time preference. Using a calibrated life-cycle model of human aging, we predict that the average U.S. American lives about 4 years longer with hyperbolic discounting than he would if he had applied a constant discount rate. The reason is that, under hyperbolic discounting, experiences in old age receive a relatively high weight in life time utility. In an extension we show that the introduction of health-dependent survival probability motivates an increasing discount rate for the elderly and, in the aggregate, a u-shaped pattern of the discount rate with respect to age.

*Keywords:* discount rates, present bias, health behavior, aging, longevity.

*JEL:* D03, D11, D91, I10, I12.

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## 1. INTRODUCTION

According to conventional wisdom, when individuals discount future gains and losses at a hyperbolically declining discount rate, it implies time-inconsistent decisions (e.g. Angeletos, 2001). The perpetual revision of current plans by future selves may then lead to suboptimal decision making because individuals place greater emphasis on immediate pleasures while postponing previously planned beneficial behavior. With respect to health, for example, individuals may overindulge in current pleasure by eating and drinking, and postpone their planned physical exercise such that they live less healthy and shorter lives than if their decisions had been time-consistent.

In this paper, we disentangle the issue of present bias of preferences from the time inconsistency problem by investigating a plausible way of hyperbolic discounting which, perhaps surprisingly, implies time-consistent decision making. We set up a life-cycle model of health behavior and endogenous longevity and show that – under a reasonable normalization – individuals live healthier and longer lives if they discount the future hyperbolically than if they had applied a constant rate of time preference (exponential discounting).

In order to avoid a misinterpretation of our results, we would like to ascertain that we fully agree that limited self-control and time-inconsistent decision making as well as other manifestations of bounded rationality could be major causes of insufficient investment in financial assets as well as in health. The point that we wish to make is that the observation of hyperbolic discounting is *insufficient* in order to expect inferior investment decisions and health behavior. This is important because many studies observe hyperbolic discounting behavior but comparatively few studies explicitly show that individuals make time inconsistent decisions. The latter would require observing the same individuals over time and to conclude that their original plans are actually reversed at later dates. Inferring time inconsistency from the observation of hyperbolic discounting could be misleading. These considerations may be helpful for an assessment of the sometimes inconclusive studies on the impact of hyperbolic discounting on health behavior (e.g. Khwaja et al., 2007).

Specifically, we take it as a well established fact that the way we discount future gains and losses affects our behavior and that people who discount the future at a higher rate tend to invest less in future gains in favor of immediate gratification. With respect to health, it has been known since Fuchs (1982) that the way of discounting affects our health and several studies have found that individuals who discount the future heavily are more likely to be obese (e.g. Komlos et al., 2004), to

smoke (e.g. Scharff and Viscusi, 2011), and to perform fewer health maintenance activities (Bradford, 2010); for surveys, see Lawless et al. (2013) and Bradford et al. (2014).

The role of time preferences for human behavior, however, can be discussed in at least three dimensions, which are sometimes not properly distinguished in the literature: (i) the magnitude of discounting as such (measuring the degree of impatience), (ii) the method of discounting (measuring the speed of declining impatience, i.e. present bias), and (iii) the issue of time-consistency. With respect to the discounting method, there seems to be consent in the behavioral economics literature that the conventional assumption of exponential discounting (at a constant discount rate) is made purely for simplicity and that actual behavior is better described by discount rates that are declining in the time horizon (for surveys, see Frederick et al., 2002, and DellaVigna, 2009). The most popular functional forms of the latter are hyperbolic and quasi-hyperbolic discounting. Hyperbolic discounting assumes that the discount rate declines over the whole planning horizon while quasi-hyperbolic discounting assumes that the discount rate declines only in the immediate future (within the next unit of time) and stays constant afterwards. The few empirical studies that try to distinguish between discounting methods tend to find stronger support for hyperbolic discounting (Abdellaoui et al., 2010; van der Pol and Cairns, 2011).

In this paper, we try to assess the impact of the discounting method by controlling for the magnitude of discounting as such. Specifically, we compare predicted lifetime outcomes for hyperbolic and exponential discounting under the normalizing assumption that the present value of a constant flow (of, for example, income) experienced over the expected lifetime of a 20-year-old person is the same. This normalization is not arbitrary. When controlling for the magnitude of discounting in this way, any difference in behavior can be attributed to the discounting method, i.e. the feature that hyperbolic types apply high discount rates for the near future and low ones for the distant future. Notice that the normalization is a *necessary* device in order to disentangle the feature of high discounting (of which we know already that it is health damaging) from the feature of present bias (where we argue that so far the literature is inconsistent and incomplete).<sup>1</sup>

We, moreover, separate the choice of the discounting method from the issue of time inconsistency by investigating a reasonable time-consistent way of hyperbolic discounting. Strotz' (1956) seminal paper made it well-known that only exponential discounting leads to time-consistent decisions if the discount factor is a function of the algebraic distance ( $\tau - t$ ) between planning time  $t$  and

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<sup>1</sup>The normalization can be conceptualized as a finite-time application of the equivalent present-value argument of Myerson (2001), see also Caliendo and Findlay (2014) and Strulik (2015a).

payoff time  $\tau$ . The “if”-clause, however, has sometimes been forgotten in the following literature such that the conventional wisdom evolved that non-exponential discounting necessarily entails time inconsistency. For example, Angeletos et al. (2001) write that “When a household has a hyperbolic discount function, the household will have dynamically inconsistent preferences, so the problem of allocating consumption over time cannot be treated as a straightforward optimization problem” (p. 54). Likewise, with respect to health behavior, Cawley and Ruhm (2012) state in their handbook article that “Hyperbolic discounting results in time-inconsistent behavior” (p. 139). Here, we suggest a form of hyperbolic discounting to which the theorem of multiplicative separability in  $t$  and  $\tau$  applies (see Burness, 1976; Drouhin, 2015). As a result, decisions are time-consistent.

In order to appreciate the practical importance of our work, notice that behavioral studies on the present bias of preferences cannot distinguish between our method of hyperbolic discounting and the conventional method of hyperbolic discounting *unless* they provide evidence for time inconsistent behavior. The majority of studies, however, relies on one-time observations, which makes it impossible to observe the behavior of one person at different points in time (Sprenger, 2015). Therefore, time inconsistency cannot be inferred and “our” preferences of time-consistent hyperbolic discounting are equally well supported by the conventional preferences of time-inconsistent hyperbolic discounting.

In order to assess the impact of time-consistent hyperbolic discounting on health behavior, health outcomes, and longevity, we set up a life-cycle health model with hyperbolic discounting and calibrate it with data for an average (20-year-old male) American. We then perform the computational experiment of endowing the “Reference American” with an exponential discounting method. Employing the normalization introduced above, we show that the hyperbolically discounting Reference American saves more, invests more in health, and lives longer than he would with exponential discounting. Hyperbolically discounting individuals are present biased through their entire life-cycle, because their discount rate declines with age. This implies that they are excessively impatient when they are young and healthy but they also apply relatively low (but still declining) discount rates to utility experienced in old age. Hyperbolically discounting individuals thus plan to spend much on health in old age when their health deficits accumulate relatively quickly. For that purpose they save at relatively high rates from at least their “middle ages” onwards. Since these savings plans are never revised, they are conducive to a long and healthy life. In our benchmark scenario, individuals are predicted to live 4 years longer with hyperbolic discounting than they would with exponential

discounting. The longevity gap increases further when individuals discount the future more heavily (i.e. when the present value of a constant life-long stream declines for both discounting methods).

We extend the benchmark model with unhealthy consumption conceptualized as smoking behavior and calibrate it for an average American. We then show that under exponential discounting, individuals would consume about four times as much unhealthy goods as they do with hyperbolic discounting. As a result, the predicted gap in life expectancy between the two types increases to about 14 years. The mechanism behind this finding is the same as for the benchmark model. Although hyperbolic-discounting individuals are present biased, they apply relatively low discount rates to utility experienced in old age and confine unhealthy consumption in order to live a long and healthy life.

An implication of time-consistent hyperbolic discounting is that individuals become more patient as they get older. This feature is consistent with theoretical considerations on the evolution of time preference through natural selection (which also supports hyperbolic discounting, see Rogers, 1994) and it is empirically supported by the studies of Green et al. (1994), Bishay (2004), and Steinberg et al. (2009). With respect to the size of the age effect, predictions based on our model are in the same ballpark as the inferences from evolutionary theory by Rogers (1994) who predicts a (one-year ahead) discount rate of about  $\exp(-0.06) = 0.94$  for 20 years old individuals and of about  $\exp(-0.02) = 0.98$  for middle-aged and older individuals.

The literature on age and discounting, however, is not unanimous. Some studies find that discount rates rise in old age (Huffman et al., 2016, for ages 70-85), some find a u-shaped age-discounting profile (Read and Read, 2004), and others argue that the discount rate should not depend on age but on the state of health (Sozou and Seymour, 2002). In Section 5 of this paper, we integrate health-dependent survival probabilities into our model and propose a reconciliation of these literatures. Considering an aggregate discount rate, consisting of pure time preference and survival probability, the model predicts a u-shaped age-profile of the discount rate where the increasing branch is motivated by deteriorating health in old age.

Our life-cycle model of health investment and health deficit accumulation is based on Dalgaard and Strulik (2014). The health deficit model has a foundation in gerontology (Mitnitiski et al., 2002; Gavrilov and Gavrilova, 1991) and is a convenient tool to make quantitative inferences about health behavior by means of computational experiments. Compared to health capital (Grossman, 1972), which is a latent variable and hard to capture empirically (see e.g. Wagstaff, 1986), the health deficit

model allows for an easy transfer of knowledge from medicine and gerontology to economics (and vice versa), and facilitates a straightforward numerical calibration of the model. The health deficit model has recently been applied to shed new light on the Preston curve (Dalgaard and Strulik, 2017), the education gradient (Strulik, 2018); the historical evolution of retirement (Dalgaard and Strulik, 2017), adaptation to bad health (Schuenemann et al., 2017a), and the gender gap in mortality (Schuenemann et al., 2017b).

The role of exponential discounting for health behavior and longevity is briefly discussed by Ehrlich and Chuma (1990) in the context of health capital accumulation and by Strulik (2018) in the context of health deficit accumulation. The role of hyperbolic discounting for health behavior and longevity remained largely unexplored in life-cycle models of endogenous health and longevity although the issue attracts great attention in empirical health economics as well as in the medical sciences (see e.g. Story et al., 2014, for a recent review).<sup>2</sup>

The remainder of the paper is organized as follows. In the next section, we set up the basic model of health investment, health deficit accumulation, and longevity. We calibrate it with U.S. data, and derive our main results. In Section 3 and 4, we extend the model with respect to unhealthy consumption and uncertain death. We show that these extensions provide a more encompassing and plausible prediction of human life-cycle behavior and leave our main conclusions intact.

## 2. THE BASIC MODEL

**2.1. Discounting.** Consider an individual who experiences instantaneous utility  $U(\tau)$  at age  $\tau$  such that lifetime utility experienced at age  $t$  is given by  $V(t) = \int_t^T d(t, \tau)U(\tau)d\tau$ , with the discount factor  $d(t, \tau)$  depending on age  $\tau$  and initial age  $t$ . To model hyperbolic discounting, we take the popular functional form suggested by Mazur (1987) and write it in a multiplicatively separable way such that immediate rewards and costs are not discounted, irrespective of the initial age at which the individual reflects on life-time utility, implying that

$$d(t, \tau) = \frac{1 + \alpha t}{1 + \alpha \tau}. \tag{1}$$

The discount factor is one at initial age  $t$  and declines hyperbolically with age  $\tau$ . For the special case where  $\alpha = 1$  the discount factor is equal to the one proposed by Ainslie (1975). The parameter

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<sup>2</sup>Gruber and Koeszegi (2001) and Cutler et al. (2003) investigate how hyperbolic discounting affects specific health behavior (smoking and overeating) outside the life-cycle context, i.e. without addressing the impact of discounting on health investment, health deficit accumulation, and longevity.

$\alpha$  is a useful device for controlling the size of the discount factor for a given age. A larger value of  $\alpha$  reduces the discount factor, i.e. it makes individuals less patient, in particular at young age. The discount rate,  $\rho \equiv -(dd/d\tau)/d = -\dot{d}/d = \alpha/(1 + \alpha\tau)$ , in contrast, does not depend on initial age  $t$ . It declines in a hyperbolic fashion in  $\tau$  and a larger  $\alpha$  implies a higher discount rate at any age. Alternatively, we could have used a generalized discount factor based on Loewenstein and Prelec (1992),  $d_L = [(1 + \alpha t)/(1 + \alpha\tau)]^{\beta/\alpha}$ , with no implications for the results. The crucial feature that provides time consistency is that the discount factor is multiplicatively separable in  $\tau$  and  $t$ , i.e.  $d(t, \tau) = x(t)y(\tau)$ , see Drouhin (2015) and Burness (1976).

An implication of the multiplicatively separable discount rate is that older individuals discount the future less heavily. To see this, consider the discount factor for payoffs at time distance  $\Delta t$  and (planning) age  $t$ , i.e.  $d(t, t + \Delta t) = (1 + \alpha t)/(1 + \alpha(1 + \Delta t))$ , with  $\partial d/\partial t = \Delta t/((1 + \alpha(1 + \Delta t))^2) > 0$ . Older individuals apply a lower discount factor. As discussed in the introduction, the feature that the discount factor is declining in age has a foundation in evolutionary theory (Rogers, 1994).<sup>3</sup>

**2.2. Normalization.** A particularly interesting research question is the investigation of how the *method* of discounting affects behavior. Acknowledging the well known fact, discussed in the Introduction, that impatient individuals behave less healthy, we would like to understand how the *hyperbolic decline* of the discount rate affects behavior, when compared to discounting at a constant rate. In other words, we would like to separate the issue of present bias from the issue of patience. If the horizon of optimization were infinite, we could apply the equivalent-present-value argument made by Myerson et al. (2001) in order to discipline the analysis (for applications, see Caliendo and Findlay, 2014, and Strulik, 2015a). Here, inspired by Myerson et al., we suggest a similar normalization for the finite time horizon. Specifically, we consider a constant stream of one unit of, for example, income until the age of death  $T$  and compute the difference in present value under exponential and hyperbolic discounting. Let the constant rate of exponential discounting be denoted by  $\bar{\rho}$ . The difference between the two discounting methods is then given by:

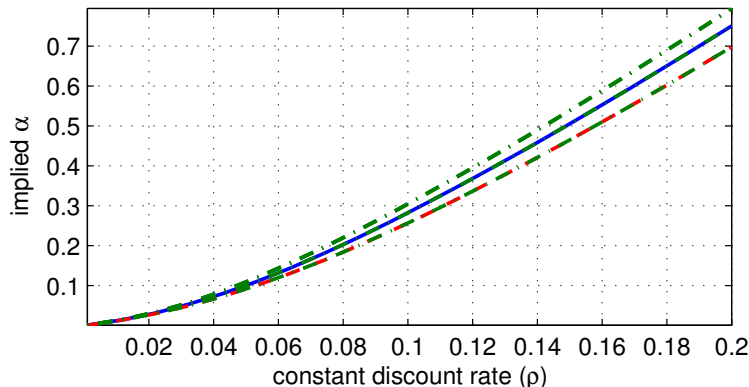
$$\Delta d = \int_t^T \exp(-\bar{\rho}(\tau - t))d\tau - \int_t^T \frac{1 + \alpha t}{1 + \alpha\tau} d\tau = \frac{1}{\bar{\rho}}(1 - \exp(-\bar{\rho}(T - t))) - \frac{1 + \alpha t}{\alpha} \log\left(\frac{1 + \alpha T}{1 + \alpha t}\right), \quad (2)$$

<sup>3</sup>Notice the difference of  $d$  compared with the more popular hyperbolic discount factor based on algebraic distance, given by  $d_A(t, \tau) = 1/(1 + \alpha(\tau - t))$ . The discount factor  $d_A$  is independent from initial age, it is not multiplicatively separable, and it implies time inconsistent decisions (Strotz, 1956).



in which the first term on the right-hand side is the present value under exponential discounting and the second term is the present value under hyperbolic discounting. In order to compare discounting methods, we compute from (2) for any given  $\bar{\rho}$  the corresponding  $\alpha(\bar{\rho})$  that leads to  $\Delta d = 0$ .

Figure 1: Equivalent Discount Rates



The figure shows combinations of  $\bar{\rho}$  and  $\alpha$  implying the same present value of a life-long constant flow. Solid lines: remaining life-time of 55.5 years; dashed (red) lines: remaining life-time of 45.5 years; dash-dotted (green) lines: remaining life-time of 65.5 years.

In order to relate to the basic health deficits model of Dalgaard and Strulik (2014), we consider an individual who starts his economic life at age 20 such that the “model-age”  $t = 0$  corresponds with actual age of 20 years. We then evaluate  $\Delta d$  at  $T = 55.5$ , which was the life expectancy of a 20-year-old male U.S. American in the year 2000 (Dalgaard and Strulik’s Reference American). The normalization  $\alpha(\bar{\rho})$  ensures that any constant stream of payoffs or costs discounted over the lifetime of the Reference American has the same present value under exponential and hyperbolic discounting. Given this normalization, any difference in lifetime outcomes can be attributed to the discounting method, i.e. the fact that hyperbolically discounting individuals discount the near future at a higher rate than the distant future. As explained in the Introduction, the normalization is not arbitrary but necessary in order to identify the impact of the discounting *method* on behavior in an unbiased way (i.e. not confounded by the degree of impatience as such).

Figure 1 visualizes the normalization. It shows  $\bar{\rho}$ - $\alpha$  combinations implying the same present value of a life-long constant flow. Solid lines assume a remaining lifetime of 55.5 years (as for the Reference American). Dashed (red) and dash-dotted (green) lines show results for a shorter and longer life. At common levels of  $\bar{\rho}$  the normalization is only mildly affected by the assumed length of life. Notice that  $\alpha$  is also the discount rate at initial age. Constant discount rates in the range of the return to capital (5-10 percent) thus correspond with high initial discount rates between 12 and 30 percent.

**2.3. Dynamic Constraints and Optimization.** We assume that utility is iso-elastic in consumption and – for the benchmark model – independent from the state of health. Specifically, the utility function is given by  $U(\tau) = [c(\tau)^{1-\sigma} - 1]/(1 - \sigma)$ , in which  $1/\sigma$  is the elasticity of intertemporal substitution. As motivated by Dalgaard and Strulik (2014), health deficits  $D$  accumulate according to

$$\dot{D} = \mu [D - Ah^\gamma - a], \quad (3)$$

in which  $h$  is health expenditure. By investing in health, individuals can reduce the exponential accumulation of health deficits  $D$ . The law of motion for health deficit accumulation has a foundation in gerontology and can be approximated very well with health data (Mitnitski et al., 2002; see Dalgaard and Strulik, 2014, for a detailed discussion). Individuals are free to save and to borrow. Their instantaneous budget constraint is given by

$$\dot{k} = w + rk - c - ph, \quad (4)$$

in which  $k$  is capital,  $w$  is labor income,  $r$  is the interest rate, and  $p$  is the price of health goods. Death occurs endogenously when  $\bar{D}$  health deficits have been accumulated, that is, individuals solve a free terminal value problem with terminal time  $T$  when  $D(T) = \bar{D}$ . Delaying death is the only motive for health expenditure in the benchmark model. The other boundary conditions are  $k(0) = k_0$ ,  $D(0) = D_0$ , and  $k(T) = \bar{k}$ . For the basic model we focus on a deterministic setup. In Section 4, we introduce death as a stochastic event.

The Hamiltonian associated with lifetime utility maximization is given by

$$H(t, \tau) = \frac{1 + \alpha t}{1 + \alpha \tau} \cdot \frac{c^{1-\sigma} - 1}{1 - \sigma} + \lambda_k [w + rk - c - ph] + \lambda_D \mu [D - Ah^\gamma - a].$$

The first-order conditions and costate equations are

$$\frac{1 + \alpha t}{1 + \alpha \tau} c^{-\sigma} - \lambda_k = 0 \quad (5)$$

$$- \lambda_k p - \lambda_D \mu \gamma A h^{\gamma-1} = 0 \quad (6)$$

$$\lambda_k r = -\dot{\lambda}_k \quad (7)$$

$$\lambda_D \mu = -\dot{\lambda}_D. \quad (8)$$

Differentiating (5) with respect to age  $\tau$  and inserting  $\lambda_k$  and  $\dot{\lambda}_k$ , we obtain the Euler equation for consumption

$$\frac{\dot{c}}{c} = \frac{1}{\sigma} \left( r - \frac{\alpha}{1 + \alpha\tau} \right), \quad (9)$$

which requires that the growth rate of consumption equals the difference between the interest rate and the discount rate, weighted by the intertemporal elasticity in consumption. Notice that the growth rate of consumption depends on current age  $\tau$  but not on initial age  $t$  because the discount rate is independent from initial age. From (6) and (8) we obtain the health Euler equation:

$$\frac{\dot{h}}{h} = \frac{r - \mu}{1 - \gamma}, \quad (10)$$

which requires that the growth rate of health expenditure equals the difference between the natural rate of health deficit accumulation  $\mu$  and the interest rate, weighted by the degree of decreasing returns in health expenditure. Notice that (10) also does not depend on initial age. The ‘‘Health Euler’’ is extensively discussed in Dalgaard and Strulik (2014).

The condition for optimal terminal time is that the Hamiltonian at the time of death assumes the value of zero. Substituting  $\lambda_k$  and  $\lambda_D$  into the Hamiltonian evaluated at time  $T$  we obtain the condition

$$H(t, T) = \frac{1 + \alpha t}{1 + \alpha T} \left\{ \frac{c^{1-\sigma} - 1}{1 - \sigma} + c^{-\sigma} [w + rk - c - ph] - \frac{ph^{1-\gamma}c^{-\sigma}}{\mu\gamma A} \mu [D - Ah^\gamma - a] \right\} = 0.$$

Since the discount factor in front of the parentheses is never zero and since the terms in parentheses evaluated at time  $\tau = T$  are functions of  $T$  but not of  $t$ , the condition holds independently from initial age  $t$ . Since all dynamic equations (3), (4), (9), and (10), are independent from initial age as well, the solution is time consistent, confirming Theorem 1 of Burness (1976).<sup>4</sup> In other words, although future payoffs are discounted hyperbolically, future selves never regret the decisions made by the current self and the consumption and health plan of the current self is not revised by future selves. This feature allows us to solve the model with standard methods and to investigate how a high preference for immediate gratification (hyperbolic discounting) as such affects health expenditure, health, and longevity. In other words, we can perform the computational experiment of whether individuals would live a longer and healthier life if they did not have hyperbolic preferences.

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<sup>4</sup>See Drouhin (2009) for a general approach to hyperbolic discounting and time consistency.

**2.4. Model Calibration.** For the benchmark run we take the calibrated model of Dalgaard and Strulik (2014) and replace exponential discounting with hyperbolic discounting. Specifically, we consider a 20-year-old male U.S. American in the year 2000 who has a life expectancy of 55.5 years (dying at age 75.5; NVSS, 2012); earning an annual labor income of \$ 35320 (BLS, 2011) and who spends about 13 percent of his lifetime income on health (the health expenditure share of GDP in the U.S. in the year 2000; World Bank, 2015). We assume  $k(0) = \bar{k} = 0$  (no bequest and no inheritance). We take from Mitnitski et al. (2002) the estimate  $\mu = 0.043$  and  $D(0) = 0.0274$  as well as  $\bar{D} = 0.1$  at the age of death (age 75.5). We normalize  $p = 1$  and set  $\gamma = 0.19$  as estimated by Dalgaard and Strulik (2014). We identify  $a$  by assuming that the role of technology in the repair of health deficits of adults was virtually zero in the year 1900, when the life expectancy of a 20-year-old U.S. American was 42 years (NCHS, 1980). For the benchmark run, we set  $\bar{\rho} = r = 0.06$ , as in Dalgaard and Strulik (2014), which implies the equivalent  $\alpha = 0.143$  for hyperbolic discounting, according to our normalization  $\Delta d = 0$  (cf. Section 2.2). Matching actual life expectancy with the health expenditure share, we obtain the estimates  $A = 0.00131$  and  $\sigma = 0.94$ . The calibrated model parameters are summarized below Figure 2. Most values are close to the estimates in Dalgaard and Strulik (2014), and we refer to that paper for a thorough discussion of them. Numerical solutions are obtained by using the Relaxation algorithm (see Trimborn et al., 2008).

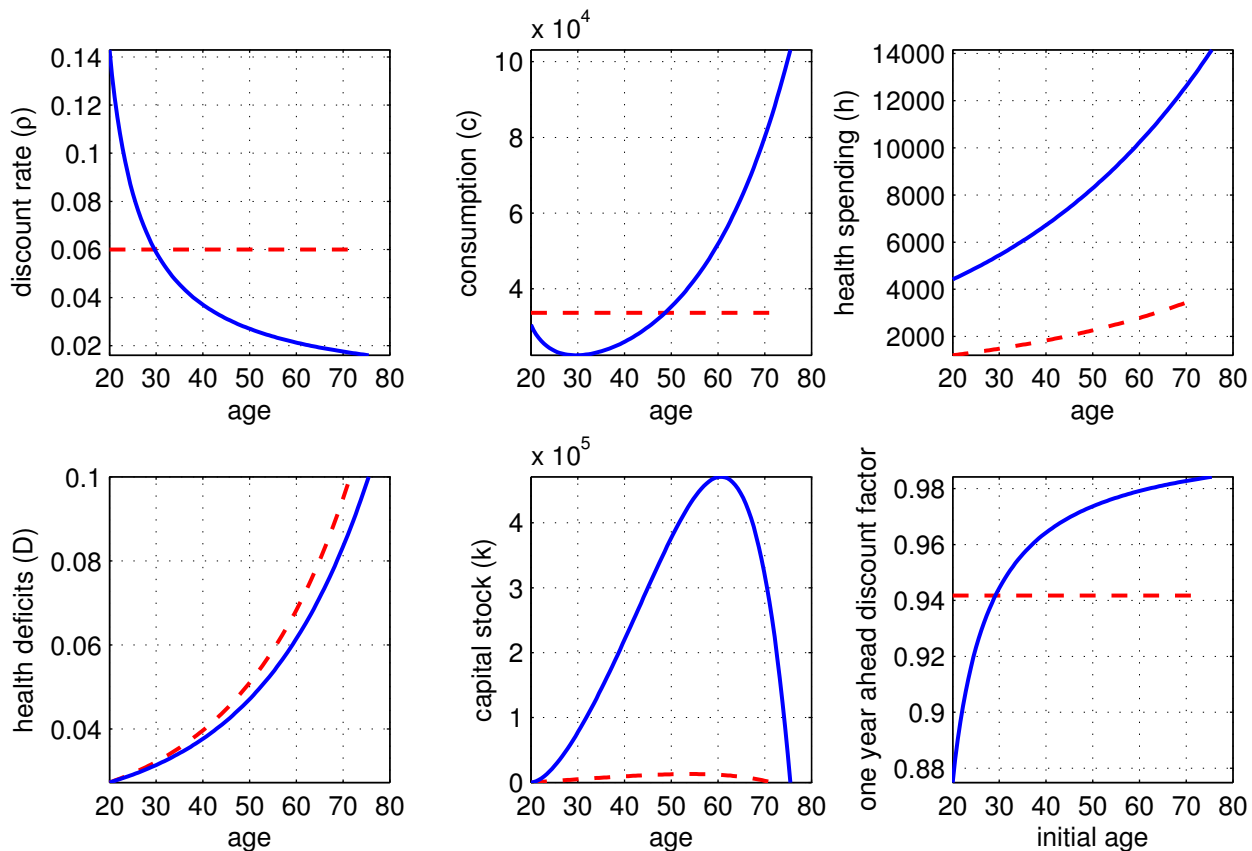
**2.5. Results.** Solid lines in Figure 2 show the predicted life-time outcomes under hyperbolic discounting. For comparison, dashed lines show the predicted life-time outcomes under exponential discounting. The upper left panel shows the discount rate which equals 14 percent at initial age 20 and declines in hyperbolic fashion towards about 2 percent in old age. Notice that the hyperbolic discount rate lies below the corresponding constant discount rate from about age 30 onwards, given the normalization  $\Delta d = 0$  (eq. 2). Compared to exponentially discounting individuals, hyperbolically discounting individuals display not only (much) higher discount rates for the near future but also lower discount rates for the more distant future. That is, utility experienced in old age, when health deficits accumulate quickly, gets a relatively larger weight under hyperbolic discounting.

The upper center panel shows the lifetime trajectory for consumption. Consumption stays constant under exponential discounting by design (since  $\bar{\rho} = r$ ). Under hyperbolic discounting, consumption declines at young age (when the discount rate exceeds the interest rate) and increases in old age (when the discount rate falls short of the interest rate). The interesting insight here is that hyperbolically discounting individuals plan to consume more in old age than exponentially discounting individuals.

The relatively large weight that old age receives in life-time utility under hyperbolic discounting is also visible in the predicted life-time trajectory for health expenditure, which is above the trajectory for exponential discounting at every age. As a result, health deficits are accumulated less quickly under hyperbolic discounting (see the lower left panel) and the individual dies about 4 years later than he would, had he applied a constant discount rate. The predicted age of death is 71.4 for exponential discounting, instead of 75.5. The lower center panel shows that individuals are predicted to save much more under hyperbolic discounting because they aspire to consume more in old age and because they plan to invest more heavily in health maintenance when they get old.

The lower right panel visualizes the age-dependency of the discount factor. It shows the discount factor applied on the utility experienced in the following year for given initial ages, i.e.  $(1 + \alpha t)/(1 + \alpha(t+1))$ . Notice that the abscissa is now scaled by initial age  $t$ , instead of age. The discount factor is about 0.88 at the beginning of economic life at age 20 (which equals “model age” zero). It increases

Figure 2: Health and Hyperbolic Discounting: Benchmark Run



Blue (solid) lines: hyperbolic discounting,  $\alpha = 0.143$ ; red (dashed) lines: exponential discounting,  $\rho = 0.06$ . Other parameters:  $a = 0.013$ ;  $A = 0.00131$ ,  $\gamma = 0.19$ ;  $\mu = 0.043$ ;  $w = 35,320$ ;  $k(0) = \bar{k} = 0$ ;  $r = 0.06$ ;  $\sigma = 0.94$ ;  $p = 1$ .

quickly and reaches 0.94 at about age 30, after which it declines more slowly towards about 0.98 in old age. As discussed in the introduction, this adjustment of the discount rate over age corresponds well with predictions from evolutionary theory (Rogers, 1994). As discussed in Rogers (1994), the feature that individuals adjust their discount factor as they age ensures the time-consistency of their original plans.

Table 1: Results: Hyperbolic vs. Exponential Discounting

	$\rho(20)/\rho(30)$	$c(20)/c(30)$	$h$ -share	$T$
hyperbolic ( $\alpha = 0.143$ )	2.44	1.36	0.13	55.5
exponential ( $\bar{\rho} = 0.06$ )	1.00	1.00	0.03	51.5
hyperbolic ( $\alpha = 0.373$ )	4.77	2.57	0.13	55.5
exponential ( $\bar{\rho} = 0.12$ )	1.00	1.79	0.01	48.3
hyperbolic ( $\alpha = 1.01$ )	11.1	5.28	0.13	55.5
exponential ( $\bar{\rho} = 0.25$ )	1.00	5.80	< 0.01	47.3
hyperbolic( $\epsilon = 0.5, \alpha = 0.143$ )	2.44	1.46	0.13	55.5
exponential( $\epsilon = 0.5, \bar{\rho} = 0.06$ )	1.00	1.00	0.06	52.8

$\rho(20)/\rho(30)$  is the discount rate at age 20 relative to age 30;  $c(20)/c(30)$  is consumption at age 20 relative to age 30;  $h$ -share is the discounted life-time income share of health expenditure;  $T$  is life expectancy at age 20.

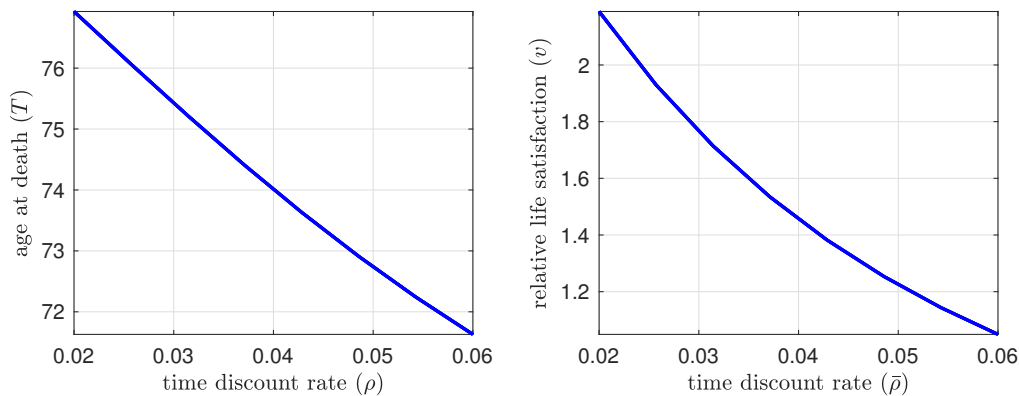
We next consider some robustness checks. Results are summarized in condensed form in Table 1. The term  $\rho(20)/\rho(30)$  computes, as a rough measure of present bias, the discount rate applied at age 20 relative to the discount rate applied at age 30. The term  $c(20)/c(30)$  provides consumption at age 20 compared to age 30,  $h$ -share is the life-time income share of health expenditure in present-value terms and  $T$  is the life expectancy at age 20 (death at age  $T + 20$ ).

The first couple of rows in Table 1 reiterates results for the benchmark calibration discussed above and displayed in Figure 2. The second couple of rows shows results when the exponential discount rate is doubled to 0.12. Given the normalization of net present value,  $\Delta d = 0$  this implies  $\alpha = 0.373$ . We take this value and recalibrate the model such that we predict the same age at death and the same health expenditure share as in the benchmark case. This leads to the new estimates  $A = 0.00130$  (instead of 0.00131) and  $\sigma = 1.01$  (instead of 0.93). Considering lifetime outcomes for the counterfactual of exponential discounting, we see that the gap gets larger. Life expectancy would be only 48.3 years if individuals would discount exponentially. The third couple of rows considers a discount rate  $\bar{\rho}$  of 0.25, a value that appears to be ridiculously high from a macroeconomic perspective but is not unusual from the viewpoint of experimental studies. It implies a high degree of impatience and, in the case of hyperbolic discounting, a very strong present bias,  $\rho(20)/\rho(30) = 11.1$ .

We adjust  $A = 0.0128$  and  $\sigma = 1.08$  in order to accurately obtain the lifetime predictions for the case of hyperbolic discounting. In the counterfactual case of exponential discounting, the very impatient individual is predicted to invest almost nothing in health such that he would die at age 67.3, i.e. 8.2 years earlier than under hyperbolic discounting. In summary, hyperbolic discounting is good for our health when it is done in a time-consistent way.

The final couple of rows considers a generalization of the model towards health in utility. For that purpose we augment the instantaneous utility function such that  $U(\tau) = [D_0/D(\tau)]^\epsilon [c(\tau)^{1-\sigma} - 1]/(1 - \sigma)$ , as in Strulik (2017). Utility is unaffected by health in the state of best health ( $D = D_0$ ) and declines as the state of health deteriorates. The elasticity  $\epsilon$  measures how steeply the marginal utility of health declines when individuals accumulate health deficits (Finkelstein et al., 2013). Here, we consider  $\epsilon = 0.5$ , a value that corresponds with the upper bound of the estimates in Finkelstein et al. (2013). Aside from the strife for a long life, individuals now have a second motive for making health investments and this feature somewhat closes the gap between hyperbolic and exponential discounting. The lifetime health expenditure share predicted for exponential discounting is now 0.06 (instead of 0.03) and life expectancy increases to 52.8. Qualitatively, however, our main result remains robust, hyperbolically discounting individuals are healthier and live longer (for about 2.7 years) when they discount in a time-consistent way.

Figure 3: Life Expectancy and Relative Life Satisfaction for Alternative  $\bar{\rho}$



Other parameters as for benchmark run (Figure 2). The right panel show the experienced value of life (life satisfaction) at age 20 relative to the hyperbolically discounting Reference American.

Finally, for illustrative purposes, we give up the normalization and ask the reverse question: what constant time preferences rate  $\bar{\rho}$  would be needed in order to achieve the same life expectancy as the hyperbolically discounting Reference American? In Figure 3 we look at the association between  $\bar{\rho}$

and life length in general. All other parameters are taken from the benchmark specification (Figure 2). The lower the impatience of the individual the higher the investments in health maintenance and repair and the higher the age at death (left panel). For about  $\rho = 0.029$ , exponential discounting provides the same life expectancy as hyperbolic discounting (death at age 75.5).

In order to assess the implications of abandoning the normalization, we calculate the evaluation of a life-long constant stream by the exponentially discounting individual relative to benchmark (the hyperbolically discounting Reference American). This value is given by  $v \equiv 1/\bar{\rho}(1 - \exp(-\bar{\rho}55.5))/[\log(1 + \alpha 55.5)/\alpha]$ . Since the utility integral from age 20 to death measures life satisfaction in terms life time utility,  $v$  provides life satisfaction of the exponentially discounting individual relative to the hyperbolically discounting individual, when consumption and life-length are held constant. For  $\rho = 0.029$ ,  $v$  is about 1.8. This means that the exponentially discounting individual needs to experience 1.8-fold higher value of life (life satisfaction) from the same fundamentals in order to live as long as the hyperbolically discounting individual. Notice how these results run counter simple intuitive reasoning. Intuitively, one might expect that hyperbolically discounting individuals, since they put more weight on immediate gratification, need a greater value of life in order to live as long as exponentially discounting individuals. Actually, however, it is the other way round.

### 3. UNHEALTHY CONSUMPTION

We next extend the model with respect to unhealthy consumption because unhealthy behavior has been suggested as one major gateway through which hyperbolic preferences affect health and longevity (Cawley and Ruhm, 2012). For simplicity, we conceptualize consumption as a convex combination of consumption of health neutral goods  $\tilde{c}$  and unhealthy goods  $u$ ,  $c = \theta\tilde{c} + (1 - \theta)u$ . One advantage of the simple additive sub-utility function is that it allows for a preemptively high price at which households abstain from unhealthy consumption (see below). Let  $q$  denote the price of unhealthy goods such that the budget constraint becomes

$$\dot{k} = w + rk - \tilde{c} - ph - qu. \quad (11)$$

Unhealthy consumption speeds up health deficit accumulation as in Strulik (2016) and the law of motion for health deficits becomes

$$\dot{D} = \mu [D - Ah^\gamma + Bu^\omega - a], \quad (12)$$



in which  $B$  measures the general unhealthiness of the unhealthy good and  $\omega > 1$  measures the degree of increasing marginal damage. Setting up the maximization problem for lifetime utility as in Section 2, we obtain from the first order conditions an algebraic equation for unhealthy consumption:

$$u = \begin{cases} \left\{ \left[ \frac{1-\theta}{\theta} - q \right] \frac{A\gamma}{B\omega p} h^{\gamma-1} \right\}^{\frac{1}{\omega-1}} & \text{for } q < \frac{1-\theta}{\theta} \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

For unhealthy consumption to prevail, the relative utility weight of unhealthy goods,  $(1-\theta)/\theta$ , has to exceed the price  $q$ . If unhealthy consumption exists, then condition (13) predicts that its extent is large if the resulting health damage is low ( $B$  is low), if medical efficiency in repairing damage is large ( $A$  is large), or if the price of health goods  $p$  is low.

In order to eliminate one variable we rewrite the budget constraint (11) in terms of  $c$  and  $u$ :

$$\dot{k} = w + rk - ph - \frac{c}{\theta} - \left[ q - \frac{1-\theta}{\theta} \right] u. \quad (14)$$

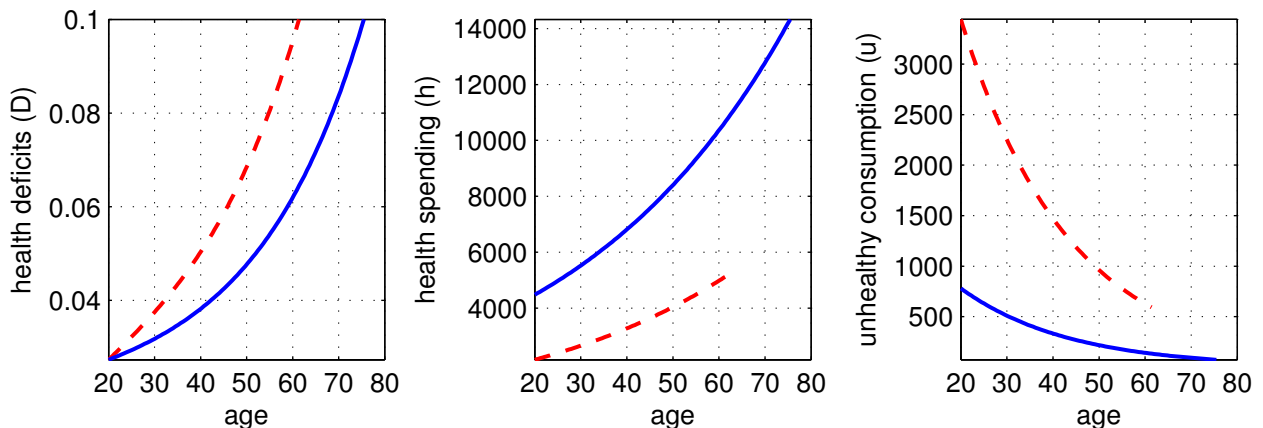
Moreover, we obtain from the first order conditions, as for the simple model, the Euler equations (9) and (10). The dynamics of individual life-cycle behavior are thus described by (9) and (10) and (12)–(14). Notice that unhealthy consumption, perhaps surprisingly, does not depend directly on the discount rate but only indirectly through its inverse relation with health expenditure  $h$  and the health Euler equation.

For the calibration of unhealthy behavior, we focus on smoking because most of the available empirical literature on consumption of unhealthy goods is on cigarettes and tobacco. In the benchmark scenario, smoking of the Reference American leads to a reduction of life expectancy of 2.5 years (as in Preston et al., 2010). Moreover, we require that over all ages, the Reference American spends about 300 dollars on cigarettes (the average expenditure in the year 2000; BLS, 2002). We normalize the price of unhealthy goods consumption to unity and set  $\omega = 1.4$ , as in Strulik (2016). Since we know relatively little about the marginal damage of unhealthy consumption, we perform a sensitivity analysis with respect to  $\omega$ . We keep from the calibration of the basic model all biological parameters as well as  $r = \bar{r} = 0.06$  and the implied value for  $\alpha$  (0.143). We estimate the remaining parameters  $A$ ,  $B$ ,  $\sigma$ , and  $\theta$  such that the Reference American has a life expectancy of 55.5 years (dies at age 75.5), spends on average 13 percent of his income on health and about 300 dollars (less than 1 percent) on cigarettes, and lives 2.5 years less than he could without smoking. This leads to the estimates  $A = 0.00146$ ,  $\sigma = 0.90$ ,  $\theta = 0.08$ , and  $B = 1.6 \cdot 10^{-7}$ . The implied price elasticity of

demand for cigarettes is about -0.2, a value at the lower end of the empirical estimates compiled by Chaloupka and Warner (2000). All parameter values are summarized below Figure 4.

**3.1. Results.** Solid (blue) lines in Figure 4 show results for the benchmark case where  $\alpha = 0.143$  and dashed (red) lines show results for the corresponding case of a constant discount rate ( $\bar{\rho} = 0.06$ ). Individuals with hyperbolic preferences spend less on unhealthy consumption, more on health, develop health deficits at a slower pace, and live longer. The consideration of unhealthy consumption widens the gap between lifetime behavior with hyperbolic and exponential discounting. The reason is the negative association between unhealthy consumption and health expenditure shown in (13). Intuitively, an optimal allocation of expenditure requires that individuals balance the marginal damage of unhealthy consumption and the marginal gain from health expenditure. The marginal gain from health expenditure is low when health expenditure is high, due to decreasing returns ( $\gamma < 1$ ). This requires low marginal damage, which is achieved for low levels of unhealthy consumption, due to increasing health costs ( $\omega > 1$ ). As a result, high health expenditure is observed in conjunction with low unhealthy consumption. Since both unhealthy consumption and health investments matter mostly for health and utility later in life, hyperbolic agents exhibit healthier behavior in both dimensions because they place more weight on late-life utility.

Figure 4: Health and Hyperbolic Discounting: Unhealthy Consumption



Blue (solid) lines: hyperbolic discounting,  $\alpha = 0.143$ ; red (dashed) lines: exponential discounting,  $\rho = 0.06$ .  
 Other parameters:  $a = 0.013$ ;  $A = 0.00146$ ;  $B = 1.6 \cdot 10^{-7}$ ;  $\gamma = 0.19$ ;  $\omega = 1.4$ ;  $\theta = 0.08$ ;  $\mu = 0.043$ ;  $w = 35, 320$ ;  
 $k(0) = \bar{k} = 0$ ;  $r = 0.06$ ;  $\sigma = 0.9$ ;  $p = q = 1$ .

Table 2 reports robustness checks for these results. The first two rows reiterate the benchmark case from Figure 4. The gap in life-expectancy increases from 4 years, according to the basic model, to almost 14 years when unhealthy behavior is taken into account. The reason is that individuals

Table 2: Extended Model: Hyperbolic vs. Exponential Discounting

	$\rho(20)/\rho(30)$	$c(20)/c(30)$	$h$ -share	$u$ -share	$T$
hyperbolic ( $\alpha = 0.143$ )	2.44	1.38	0.13	0.01	55.5
exponential ( $\bar{\rho} = 0.06$ )	1.00	1.00	0.05	0.04	41.3
hyperbolic ( $\alpha = 0.373$ )	4.77	2.60	0.13	0.01	55.5
exponential ( $\bar{\rho} = 0.12$ )	1.00	1.81	0.02	0.37	39.9
hyperbolic ( $\omega = 1.2$ ; $B = 7.1 \cdot 10^{-7}$ )	2.44	1.38	0.13	0.01	55.5
exponential	1.00	1.00	0.07	0.06	38.7
hyperbolic ( $B = 3.2 \cdot 10^{-7}$ ; $A = 0.00173$ )	2.44	1.49	0.13	0.01	55.5
exponential	1.00	1.00	0.05	0.05	32.9

$\rho(20)/\rho(30)$  is the discount rate at age 20 relative to age 30;  $c(20)/c(30)$  is consumption at age 20 relative to age 30;  $h$ -share and  $u$ -share are the discounted lifetime income share of health expenditure and unhealthy consumption;  $T$  is life expectancy at age 20.

increase their unhealthy consumption fourfold if they discount future payoffs at a constant rate. The next couple of rows confirms the result from the basic model that the gap widens when the level of discounting increases. The third couple of rows considers a case in which marginal damage is less steeply increasing ( $\omega = 1.2$  instead of 1.4). In order to maintain an average expenditure level of 300 dollars for unhealthy consumption,  $B$  is increased to  $7.1 \cdot 10^{-7}$ , implying that unhealthy consumption is more damaging at low doses with slower increasing damage at high doses. Consequently, individuals with exponential discounting are predicted to consume even more unhealthy goods than in the benchmark case with further deteriorating consequences on their health and longevity.

The estimate of Preston et al. (2010) of the death toll from smoking is conservative. The studies by Doll et al. (2004) and Jha et al. (2013), for example, suggest that smoking reduces life expectancy by up to 10 years. The last couple of rows thus considers the case of a much larger health damage from smoking by increasing  $B$  twofold. In order to fit a life expectancy of 55.5 years and a health expenditure share of 13 percent, the power of medical technology is increased to  $A = 0.00017$  (from  $A = 0.00014$ ) and  $\sigma$  is reduced to 0.7 (from 0.9). Altogether, this means that smoking (at benchmark levels) reduces life expectancy of the Reference American by 9.2 years, a value at the upper bound of the empirical estimates. As a result, health outcomes are predicted to be even more severe under exponential discounting. If the Reference American had a constant discount rate, life would end at age 53 due to the high damage of unhealthy consumption, which would not be taken sufficiently into account by young adults. While these predictions may appear extreme, the general takeaway is that including unhealthy behavior increases the estimated loss in health and longevity that results from discounting the future at a constant rate.

#### 4. UNCERTAIN DEATH AND U-SHAPED DISCOUNTING

As discussed in the Introduction, the literature on the age-pattern of discounting is not unanimous. Focussing on elderly individuals between ages 70 and 85, Huffman et al. (2016) observe an increasing discount rate with age. Read and Read (2004) consider individuals from a larger range of ages between 19 and 89 and find the lowest discount rate for individuals of middle age, and thus, a u-shaped age-pattern of discounting. Sozou and Seymour (2002) refine Rogers' (1994) evolutionary theory by including a realistic aging process and show that declining health and reproductive capacity motivates such a u-shaped discounting pattern. In order to reconcile these results with our previous findings on declining discount rates, we next integrate health dependent survival probabilities into the model.

Suppose that survival probability at any age depends negatively on the accumulated health deficits at that age,  $S(D)$ ,  $S' < 0$ . This means that there are now two motives to discount the future: pure time preference and survival risk (as in Halevy, 2008). Following Kamien and Schwartz (1980, Section 9, Part I), the present value of expected lifetime utility can then be represented as

$$\int_t^T S(D) \cdot \frac{1 + \alpha t}{1 + \alpha \tau} \cdot U(\tilde{c}, u) d\tau. \quad (15)$$

Notice that health deficits  $D(\tau)$  are predetermined at age  $\tau$ . This means that the solution remains time consistent even after including health-dependent survival probability. The mortality rate  $m$  is defined as the rate of change of the survival rate,  $m \equiv -\dot{S}/S = (\partial S/\partial D)(\dot{D}/D)$ .

Assuming perfect annuity markets, the interest rate is now a compound of the return on capital and the mortality rate and individuals inherit no wealth and leave no bequests. Capital left over at death is distributed among the survivors by the annuity suppliers. We thus implicitly assume that our Reference-American is surrounded by sufficiently many other individuals of the same age. The adjusted budget constraint is given by

$$\dot{k} = w + (r + m)k - \tilde{c} - ph - qu. \quad (16)$$

The rest of the model is carried over from the previous sections. The consideration of mortality risk affects neither the Euler equation (9) nor the condition for unhealthy consumption (13). It does,

however, modify the Health Euler equation (10) which becomes:

$$\frac{\dot{h}}{h} = \frac{1}{1-\gamma} \left\{ r + m - \mu + \frac{\mu\gamma Ah^{\gamma-1}c^\sigma}{p\theta S(D)} \cdot \frac{c^{1-\sigma} - 1}{1-\sigma} \cdot \frac{\partial S(D)}{\partial D} \right\}. \quad (17)$$

Intuitively, the individual now takes precautionary savings for late-life health expenditure into account. The presence of annuities  $m$  makes the health expenditure path steeper, whereas the risk of death makes it flatter (since  $S' < 0$ ).

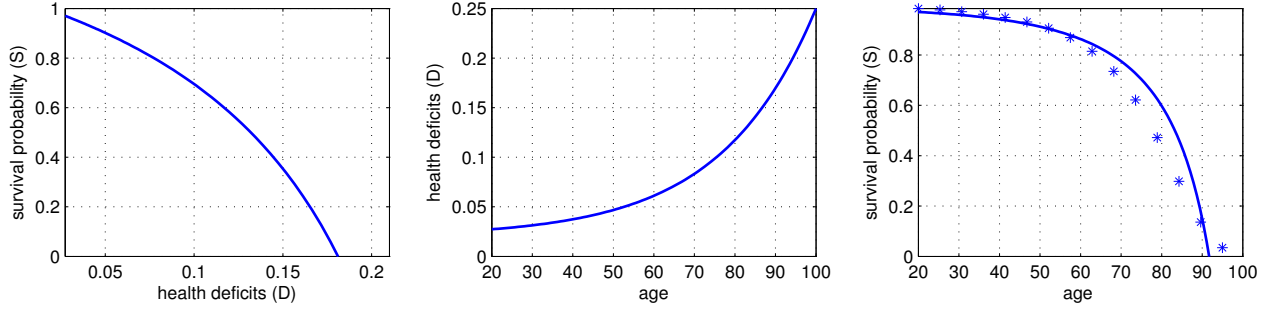
**4.1. Calibration.** According to our theory, survival does not directly depend on age but on the state of health at any age. This approach nicely captures the biological approach to aging, which aspires to replace age as a proximate determinant of death by the loss of bodily function as a deep determinant (“Only if we can substitute the operation of the actual physiological mechanism for time we have a firm idea of what we are talking about.”, Arking, 2006, p. 10). This feature can be exploited for the calibration of the model in three steps: We impose a particular parametrization of the survival function  $S(D)$ . Then, we feed in the estimates from Mitnitski et al. (2002) on the association between age and health deficits,  $D(\tau)$  and predict the association of age and survival  $S(\tau) = S(\tau^{-1}(D))$ . Finally, we confront the prediction with estimates of  $S(\tau)$  from life tables.

A parsimonious representation of the survival function is given by the concave function:

$$S(D) = \psi - \frac{\nu}{1 - \chi D}. \quad (18)$$

The panel on the left-hand side of Figure 5 shows the association between  $D$  and  $S$  implied by (18) for  $\psi = 1.75$ ,  $\nu = 0.7$ , and  $\chi = 3.1$ . The survival probability declines at an increasing rate as more health deficits are accumulated and nobody survives  $(1 - \nu/\psi)/\chi$  health deficits. The middle panel shows the association between age and accumulated deficits estimated by Mitnitski et al. (2002) for 19-75 years old Canadian men ( $R^2 = 0.95$ ). When we feed these data into the  $S(D(\tau))$  function, we get the “reduced form”,  $S(t)$ , which shows survival as a function of age. The implied functional relationship is shown on the right-hand side of Figure 5. Stars in the panel on the right-hand side indicate the survival probability estimated from life tables for U.S. American men in 1975-1999, taken from Strulik and Vollmer (2013). The approximation somewhat overestimates the survival of the elderly and underestimates the survival of the oldest old but altogether, it fits the data reasonably well.

Figure 5: Health-Dependent Survival and Survival by Age

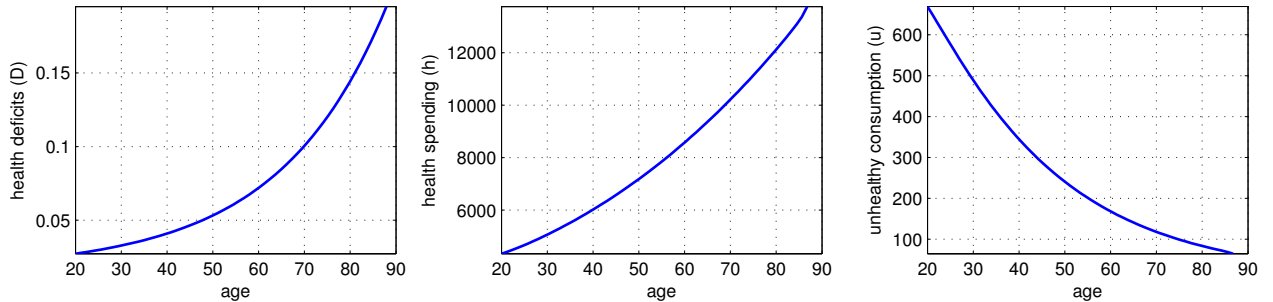


$S(t)$  is the unconditional probability of surviving until age  $t$ . Left: Assumed function  $S(D)$ . Middle: Estimated association  $D(t)$  (Mitnitski et al., 2002). Right: Predicted (line) and estimated (stars) association between age and survival probability (estimates from Strulik and Vollmer, 2013). Implied life expectancy at 20: 55.5 years.

The rest of the model is calibrated as before with the exception that the deterministic outcome age at death is replaced by the stochastic life expectancy at 20, which was 55.5 years for American males in the year 2000. This leads to mild adjustments of parameter estimates:  $A = 0.00105$ ,  $\sigma = 1.03$ , and  $\theta = 0.065$ . All other parameters are taken from the calibration from Section 3.

**4.2. Results.** Health behavior and outcomes for the stochastic model are summarized in Figure 6. Visually, the results differ indiscernibly from the deterministic trajectories (solid lines in Figure 4), corroborating the result from Strulik (2015) that uncertainty modifies outcomes of the health deficit model only marginally. Notice, however, the different interpretation of results. The stochastic life trajectories display age-specific behavior and outcomes *conditional* on reaching the specific age. While the Reference American dies at age 75.5, other individuals live substantially longer such that the age trajectories do not stop at age 75.5.

Figure 6: Health-Dependent Survival: Health Behavior and Outcomes



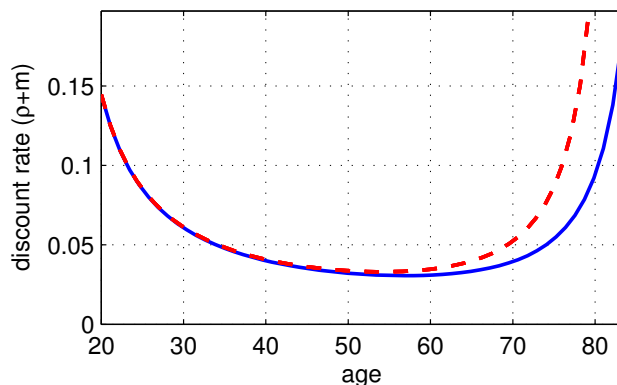
Parameters:  $\alpha = 0.143$ ;  $a = 0.013$ ;  $A = 0.00105$ ;  $B = 1.6 \cdot 10^{-7}$ ;  $\gamma = 0.19$ ;  $\omega = 1.4$ ;  $\theta = 0.065$ ;  $\mu = 0.043$ ;  $w = 35, 320$ ;  $k(0) = \bar{k} = 0$ ;  $r = 0.06$ ;  $\sigma = 1.03$ ;  $p = q = 1$ .

While the stochastic model adds more realism and verifies the robustness of our earlier results, it also provides new insights on age-specific discounting. Subsequently, we compute the aggregate

discount rate  $\rho+m$ , consisting of discounting for pure time preference (captured by  $\rho$ ) and discounting because of uncertain survival (captured by  $m$ ). The feature that  $\rho$  is declining with age while the mortality rate  $m$  is increasing with age motivates a u-shaped pattern of the aggregate discount rate.

Figure 7 displays the age-dependent discount rate for the calibrated model. At young ages, the pure time preference part dominates and the discount rate declines as individuals get older. From about age 60 onwards, the uncertainty part dominates and the discount rate rises again with increasing age. The theory thus provides an explanation for a u-shaped pattern of the discount rate. The explanation is based on human aging. Age, however, is only a proximate determinant of survival prospects. As argued in conjunction with Figure 4, the state of health, represented by the accumulated health deficits, is the fundamental cause of death and thus of survival-dependent discounting.

Figure 7: Age and the Aggregate Discount Rate ( $\rho + m$ )



Solid (blue) line: calibrated model ( $D_0 = 0.027$ ). Dashed (red) line: less healthy individual ( $D_0 = 0.029$ )

As a testable out-of-sample prediction, the model thus provides the result that unhealthier individuals of the same age (i.e. biologically younger individuals) discount the future more heavily. To demonstrate this, we modify initial health deficits to 2.9 percent (from 2.7). In contrast to the health capital model, the health deficit model predicts that small initial health deficits are amplified in the course of life; small initial health differences are thus sufficient for predicting large differences late in life (see Almond and Currie, 2011, for a critique of the health capital model in this regard). The implied discount rate predicted for the unhealthier individual with a life expectancy of 70.1 years (instead of 75.5) is shown by dashed (red) lines in Figure 7. Unhealthier individuals, controlling for age, are predicted to discount the future more heavily, in line with the evidence provided in Huffman et al. (2016).

## 5. CONCLUSION

In this study, we proposed a way of hyperbolic discounting that does not imply time-inconsistent decisions. We have integrated this hyperbolic discounting behavior into a life-cycle model and shown that it is conducive to a healthy and long life. A calibrated Reference American lives about four years longer than he would if he had a constant discount rate. The reason is that hyperbolic discounting places relatively higher weight on late-life utility, i.e. at an age when health deteriorates quickly. Hyperbolic discounting thus motivates the aspiration for a long life, less indulgence in unhealthy consumption, and more savings in young age in order to finance more health expenditure in old age and to prevent an early accumulation of health deficits.

We have shown that an extension of the basic model with respect to uncertain survival motivates a u-shape age pattern of the discount rate. The reason is that future utility is now also discounted with the mortality rate. In young age, when mortality is low, the pure time preference motive dominates the survival motive and hyperbolic discounting implies that the discount rate declines with age. In old age, in contrast, the survival motives dominates and the discount rate rises with increasing age and mortality.

The finding that time-consistent hyperbolic discounting is conducive to a healthy and long life challenges the interpretation of empirical results in behavioral health economics. It implies that hyperbolic discounting could be bad for health (and other life-cycle outcomes) if and only if it leads to time-inconsistent decisions and the revision of future plans by impatient current selves. While it is obvious how hyperbolic discounting in this case of procrastination of health-promoting behavior is detrimental to health and longevity, the if-clause seems to be frequently forgotten in empirical research (Sprenger, 2015). Many studies are not able to distinguish between hyperbolic discounting and time inconsistent behavior by construction, because they have to rely on one-time observations (instead of longitudinal analysis). Since the intertemporal decision problem is not repeatedly solved at different points of time (for different ages of the person), time inconsistency cannot be inferred. This may be one reason for the currently inconclusive empirical evidence for a negative impact of present bias on health behavior and health outcomes. Future empirical studies should take these findings into account and aim for a better distinction between present biased preferences and time inconsistent behavior.



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