Center for European, Governance and Economic Development Research

cege

Discussion Papers

Number 329 – November 2017

THE LOST RACE AGAINST THE MACHINE: AUTOMATION, EDUCATION, AND INEQUALITY IN AN R&D-BASED GROWTH MODEL

Klaus Prettner and Holger Strulik

Georg-August-Universität Göttingen

ISSN: 1439-2305

The Lost Race Against the Machine: Automation, Education, and Inequality in an R&D-Based Growth Model[*]

Klaus Prettner[†]
Holger Strulik[‡]

November 2017

Abstract. We analyze the effects of automation and education on economic growth and inequality in an R&D-based growth model with two types of labor: high-skilled labor that is complementary to machines and low-skilled labor that is a substitute for machines. The model predicts that innovation-driven growth leads to increasing automation, an increasing skill premium, an increasing population share of college graduates, increasing income and wealth inequality, and a declining labor share. In contrast to conventional wisdom, our theory predicts that faster economic growth promotes inequality. Because education and technology are endogenous, redistribution to low-skilled individuals may actually not improve disposable low-skilled income, irrespective of whether it is financed by taxes on labor income or machine input in production. We extend the model by fair wage concerns and show how automation implies involuntary low-skilled unemployment.

Keywords: Automation, R&D-Based Growth, Inequality, Wealth Concentration, Unemployment, Redistribution.
JEL: E23, E25, O31, O33, O40.

[*] We would like to thank Hans Fehr, Volker Grossmann, Andreas Irmen, Maurizio Iacopetta, Sophia Kan, Wilhelm Kohler, Martin Kukuk, Jakob Madsen, Sephora Mangin, Solmaz Moslehi, Manfred Stadler, and Doganoglu Toker for comments. The first draft of this paper was largely written while Holger Strulik was a visiting research fellow at the economics department of Monash University. Holger thanks the department for its extraordinary hospitality.
[†] University of Hohenheim, Institute of Economics, Schloss, Osthof-West, 70593 Stuttgart, Germany; email: klaus.prettner@uni-hohenheim.de.
[‡] University of Göttingen, Department of Economics, Platz der Göttinger Sieben 3, 37073 Göttingen, Germany; email: holger.strulik@wiwi.uni-goettingen.de.

1. Introduction

Common wisdom in growth and labor economics suggests that technological progress is labor-augmenting. Technological progress developed by market R&D and incorporated into new machines is intended to complement human work effort and to make workers more productive (see, for example, Jones, 2005). In this paper, we look at the dark side of R&D-driven technological change. We consider a situation where new technologies complement only high-skilled workers but substitute for low-skilled workers.

In contrast to other recent studies on automation and growth (to be discussed below), we consider an overlapping generations structure of the population and focus on endogenous education and inequality within and across generations. Since (at least at the current state of technology) high-skilled labor is more difficult to automate than low-skilled labor, people may avoid the downsides of technological progress and enjoy its benefits by upgrading their skills. We show that an increase in skill premium due to automation motivates an increasing share of people to obtain higher education in the form of a college degree. However, in a heterogeneous society, not everybody is able or willing to obtain higher education. Due to effort constraints, some individuals do not to acquire higher education and are left behind. In this way, R&D-based growth leads to increasing income and wealth inequality from one generation to the next and, in an extension of the model, to increasing involuntary unemployment of low-skilled individuals.

The most popular discussion of skill-biased technological progress is perhaps provided by Goldin and Katz (2009), who argue that America has lost the "Race between Education and Technology" because highschool completion rates have stagnated since the 1950s. However, as emphasized by Acemoglu and Autor (2009), this loss is relative because the underlying model assumes that low-skilled labor also benefits from innovation but "only" to a lesser degree than high-skilled labor. Here, in contrast, we conceptualize individuals with tertiary education (a college degree) as high-skilled workers. These workers are complements to machines and their wages as well as their share in the workforce continued to increase throughout the 20th century. The race against technology is lost by those individuals who do not to obtain a tertiary education and thus do not benefit from innovation and technological change.

Another important distinction from Goldin and Katz (2009) is that, in their study, education and technology are treated as being exogenous such that, in principle, education could win the race against technology. In our framework, by contrast, both forces are endogenous. Skill-biased technical change promotes education and more education, in turn, leads to more R&D, more innovation, and further advancements of technology. The outcome that education fails to keep pace with increasing skill-biased technological change is derived from the fact that technology advances perpetually, approaching a balanced growth path in infinite time, while the labor share of high-skilled individuals approaches an upper limit in finite time.

The idea of labor-substituting technological progress has been popularized by Brynjolfsson and McAfee's (2011) book on another race, the "Race against the Machine". Brynjolfsson and McAfee claim that technological progress, understood as automation, makes people more innovative, productive, and richer but at the cost of increasing unemployment and (wealth) inequality in society. Early quantitative evidence for this view stems from Berman et al. (1998), who show that around 70 percent of the decline in production workers' share of the wage bill can be explained by R&D and computerization. More recently, Graetz and Michaels (2016) provide evidence that industrial robots lead to a reduction in the demand for low-skilled labor; Frey and Osborne (2017) argue that the average educational attainment of an occupation and the probability of this occupation to be automated are highly negatively correlated; Arntz et al. (2016) explain that low-skilled workers perform tasks that are typically much easier to automate than the tasks performed by high-skilled workers; and Acemoglu and Restrepo (2017) find that the increase in industrial robots in U.S. manufacturing had large negative effects on wages and employment across commuting zones with the strongest wage effects on workers with a high school education or less.

To curb rising inequality of living standards, it is seemingly attractive to transfer income to the losers from automation, perhaps in the form of unconditional transfers financed by progressive labor income taxation or by taxing machine input, i.e., by imposing a so-called robot tax (Shiller, 2017; Guerreiro et al., 2017; Gasteiger and Prettner, 2017). Here, we investigate the impact of these policies when both education and technology are endogenous. We show that redistribution, aside from its repercussions on innovation and growth, may actually harm disposable income of low-skilled individuals, at least in the short and medium run. The reason is that both a robot tax and a progressive income tax reduce the potential income gain from higher education, which leads to a larger population share of low-skilled individuals. As a result, increasing low-skilled labor supply depresses the wage for low-skilled workers. By the same general equilibrium argument, low-skilled wages could benefit from a subsidy for higher education. Due to the induced additional uptake of higher education, low-skilled labor supply declines and low-skilled wages increase. Redistribution to low-skilled workers improves their disposable income unambiguously only when higher education is stationary.

Rising inequality may also trigger rising unemployment. To investigate this idea, we integrate Akerlof and Yellen's (1990) fair wage theory into an extension of the model. Automation increases the productivity and income of high-skilled workers but leaves productivity of low-skilled workers unchanged. If low-skilled workers would receive "neoclassical" wages according to their marginal product under full employment, they would perceive the increasing inequality as unfair and would not exert full effort at work. To elicit full effort, firms might let low-skilled workers participate in the productivity gains from automation and adjust their employment accordingly, causing involuntary unemployment among low-skilled workers. This mechanism, however, does not necessarily imply that unemployment increases with automation. The reason for this perhaps unexpected result is that automation also increases the skill premium and induces more higher education such that the supply of low-skilled labor declines. With both the supply and demand of low-skilled labor contracting, the effect on unemployment is ambiguous. Only when education is stationary, automation and fair wage concerns imply unambiguously rising unemployment.

Some recent articles have investigated automation in the context of long-run development. Hémous and Olsen (2016) and Acemoglu and Restrepo (2016) are perhaps most closely related to our contribution. Like us, both studies focus on R&D-based innovations and inequality in the process of economic growth. In both studies, the household side of the economy is simpler than in our case because there is no education decision and skills are taken as given by the infinitely living (representative) individual. The production side, however, is more complex in both studies. Specifically, final goods are assumed to be produced by a variety of intermediate goods (Hémous and Olsen) or a variety of tasks (Acemoglu and Restrepo). Varieties are produced by labor and potentially by (low-skilled) labor replacing machines. R&D generates new varieties that start out as un-automated. Firms may then make investments in order to automate the production of their intermediate good. As a result, (low-skilled) wages benefit from R&D-based innovations. In this setup, more productive automation could even raise (low-skilled) wages because it encourages more R&D. It is perhaps fair to say that both theories focus on the production side of the economy and on the question of under which conditions (low-skilled) workers could gain from automation. This justifies a rather stylized household side of the economy. In particular, the race between education and technology, and the impact of redistribution policies are not discussed.

While Hémous and Olsen and Acemoglu and Restrepo assume that R&D creates new intermediate inputs or tasks in production that start out un-automated, we conceptualize R&D as the process that creates the very machines that substitute for low-skilled labor in production. Acknowledging that R&D probably generates both un-automated new tasks and machines that substitute for low-skilled labor, our theory complements the existing literature. It is simpler and provides a less benign view on the role of R&D, which we think is more appropriate to address inequality concerns and their suggested remedies. Moreover, the implications seem to be more in line with the recent empirical findings of Acemoglu and Restrepo (2016) for the U.S.

An earlier related study is provided by Krusell et al. (2000), who discuss capital-skill complementarity and inequality in a growth model without R&D and TFP growth. Growth is driven by the accumulation of equipment capital, which substitutes low-skilled labor and augments high-skilled labor. The study focuses on mechanisms on the production side of the economy and considers neither household decisions nor redistribution policies.

Sachs and Kotlikoff (2012) provide a model that discusses, like us, the interaction between automation and education in an overlapping generations context. However, their framework as well as their conclusions are different from ours. In their framework, all individuals are assumed to start their working life as being low-skilled and may later in life invest in education and physical capital. When exogenous technical advances increase the productivity of machines that substitute for low-skilled labor, young individuals respond by investing less in education. Instead of a race between education and technology, the study thus investigates a case where the two "runners" move in opposite directions.[1]

The interaction between technology, wages, and education relates our paper to the unified growth literature, where one of the core mechanisms is the rise of education triggered by technological progress (Galor and Weil, 2000; Galor and Moav, 2002; Galor, 2005; 2011). In contrast to this literature, we focus on tertiary education, R&D-based growth, and automation through new technologies. In an earlier study (Strulik et al., 2013), we constructed an overlapping generations version of the Romer (1990)–Jones (1995) R&D-based growth model with an endogenous education and fertility decision to discuss long-run adjustment processes. However, we did not consider automation and the evolution of inequality.

Our paper also contributes to the long-standing debate on the interaction between inequality and economic growth. While the earlier theoretical literature focused mainly on the causality running from inequality to growth, where empirical studies found a negative association (Persson and Tabellini, 1994; Alesina and Rodrik, 1994; Aghion et al., 1999), the literature related to skill-biased technical change (cited above) argues in favor of the causality running from growth to inequality and suggests a positive association. Recently, Piketty (2014) has popularized the view that economic growth reduces inequality in the context of the neoclassical growth model. Here, we argue that R&D-based growth theory in conjunction with automation provides a "non-Pikettarian" result: ceteris paribus, faster growth is predicted to lead to more inequality in labor income and wealth. This finding, however, does

[1] Other, for various reasons, less related studies on automation and macroeconomic performance are provided by Zeira (2006), Steigum (2011), Peretto and Seater (2013), Benzell et al. (2015), Sachs et al. (2015), Abeliansky and Prettner (2017), Gasteiger and Prettner (2017), and Prettner (2017). Most of these studies do not explain technological progress endogenously. Exceptions are Zeira (2006) and Peretto and Seater (2013), which, however, do not address inequality issues and redistribution.

not imply that there is no threat from automation when the growth rate of factor productivity declines. As long as R&D-based growth is positive, automation causes inequality to rise. Along the transition, we can then observe a negative association between growth and inequality because growth is declining, while inequality is on the rise. We show this outcome by simulating a calibration of the model with U.S. data.

The paper proceeds as follows. In the next section, we set up the basic model of R&D-driven automation. In Section 3, we take the education system as given and provide a series of analytical results on economic growth and various aspects of inequality along the balanced growth path. The full model with growth–education interaction is investigated in Section 4. We discuss two alternative scenarios, a conventional adjustment path, where the economy gradually converges towards positive balanced growth and an alternative scenario, where TFP growth is mildly declining. While our main results hold true in both scenarios, the second scenario is better suited to capture long-run trends of growth, education, and inequality. We use this version to investigate the various forms of redistribution policies mentioned above. In Section 5, we extend the model by fair wage concerns and involuntary unemployment of low-skilled workers. Section 6 concludes the paper.

2. The Model

2.1. **Basic Assumptions.** Consider an overlapping generations economy in which individuals live for two periods. Individuals enter the economy as young adults and are equipped with secondary education (high school or less). Young adults are endowed with one unit of time and decide whether or not to spend a certain amount η of their available time studying in order to acquire greater skills in the form of a college degree or more. The remaining time of young adulthood is supplied on the labor market. Young adults save for the second period of their life, when they are retired. After the second period, individuals die with certainty. Time t evolves discretely with each time step capturing one generation. The working population is of size L_t and supposed to be constant. In an earlier version of this paper (Prettner and Strulik, 2017), we showed that the main results also hold when there is a growing population. Employing the argument of finite space on earth, it can be argued that the only meaningful long-run steady state is associated with a stationary population (Strulik, 2005). Since population is exogenous, we therefore assume it to be constant at all times.[2]

Individuals are heterogeneous with respect to their ability to graduate from college, which is expressed as the disutility (or effort) entailed by studying. As a result, the society is divided into high-skilled and low-skilled workers: high-skilled labor, denoted by $L_{H,t}$, is, as

[2]See Strulik and Weisdorf (2008) for a unified growth model that endogenously generates convergence towards a stationary population.

conventionally assumed, complemented by machines, whereas low-skilled labor, denoted by $L_{L,t}$, is substituted by machines. For simplicity, we ignore the potential automation of *some* high-skilled jobs by artificial intelligence. Including this feature would provide more realism to our stylized model but would not change the main mechanisms and the results.

There are three production factors, the two types of labor described above and physical capital in the form of machines and robots. Low-skilled workers can only be employed in the final goods sector for tasks that can also be performed by machines. High-skilled workers can be employed as workers in the final goods sector responsible for tasks that cannot be easily automated or as workers in the R&D sector for developing new technologies (engineers and scientists). In the basic model, we ignore the possibility of involuntary unemployment such that all labor markets are cleared. A government taxes wage income and/or the use of machines (i.e., it imposes a "robot tax") at a constant rate and uses the revenue for lump-sum redistribution to the workforce.

2.2. **Individuals.** Individuals experience utility from consumption in working age and old age. In period t, the remaining lifetime utility of working-age individuals of type $j = L, H$ is given by

$$u_t = \log(c_{j,t}) + \beta \log(\bar{R}s_{j,t}) - v(a, \tilde{\eta}), \tag{1}$$

where $c_{j,t}$ is consumption of young adults, β is the discount factor, and $\bar{R}$ is the gross interest rate on savings $s_{j,t}$ such that $c_{j,t+1} = \bar{R}s_{j,t}$ refers to consumption in the second period of life. For simplicity, we assume that the economy is comparatively small and open to international capital flows to an extent that the interest rate is determined at the world market.

Higher education requires a constant investment of time such that the time spent on education is $\tilde{\eta} = 0$ without tertiary education and $\tilde{\eta} = \eta$ with tertiary education, where $0 < \eta < 1$. Realistically, not all members of a society are willing or capable to obtain a college degree. We model this aspect conveniently by assuming that individuals are of idiosyncratic ability a and that the disutility (or effort) to be incurred by higher education is declining in a. Formally, the disutility of education is denoted by v with $v(a, 0) = 0$, $v(a, \eta) = v(a) > 0$, and $\partial v/\partial a < 0$. Moreover, there exists a pole for ability, $\lim_{a \to \bar{a} > 0} v = \infty$, which captures the notion of a lower bound of ability below which graduation from college is associated with infinite disutility (or requires infinite effort). Consequently, there are always some individuals in society who remain without a college degree.[3]

[3] As an alternative to assuming a constant time cost of tertiary education and ability affecting disutilty, we could have assumed that the time needed to obtain a college degree depends on ability. This would qualitatively provide the same results at the micro level. At the macro level, however, it would counterfactually imply a secular increase of the average time to graduation when the skill premium rises.

Let w_j denote the wage per unit of labor supply of type $j = L, H$, τ_w the tax rate on wage income, and T_j the lump-sum transfers to type j. The budget constraint that each individual faces is then given by

$$(1 - \tau_w)(1 - \tilde{\eta})w_{j,t} + T_j = c_{j,t} + s_{j,t}. \tag{2}$$

Maximizing utility (1) subject to the budget constraint (2) leads to optimal consumption and optimal savings as

$$c_{j,t} = \frac{(1 - \tau_w)(1 - \tilde{\eta})w_{j,t} + T_j}{1 + \beta}, \qquad s_{j,t} = \frac{\beta(1 - \tau_w)(1 - \tilde{\eta})w_{j,t} + T_j}{1 + \beta}, \tag{3}$$

where $\beta/(1 + \beta)$ is the savings rate of both types of workers.

2.3. **Education Decision.** By inserting (3) into (1), we obtain the indirect utility function conditioned on education. Individuals compare utility with and without a college degree and obtain higher education if

$$v(a) \leq (1 + \beta) \log \left[\frac{(1 - \tau_w)(1 - \eta)w_{H,t} + T_H}{(1 - \tau_w)w_L + T_L} \right] \equiv \tilde{w}_t, \tag{4}$$

in which $\tilde{w}_t$ denotes the net skill premium in terms of utility. Suppose ability is distributed in the interval $(0, 1)$. Let a_t^* denote the ability level at the threshold, i.e., where (4) is fulfilled with equality. Then,

$$v^{-1}(\tilde{w}_t) = a_t^*. \tag{5}$$

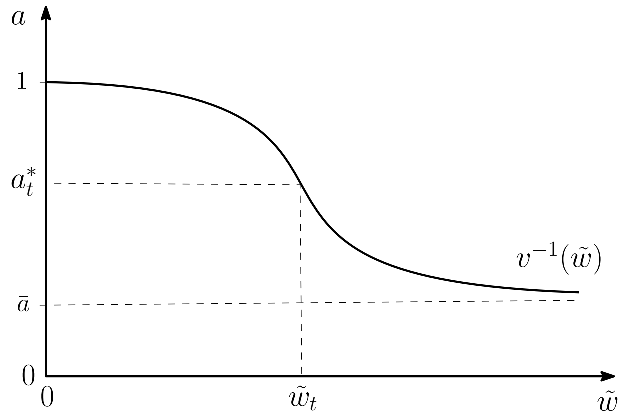
By applying the formula for the differentiation of inverse functions, we have $(v^{-1})' < 0$ and $\lim_{\tilde{w}_t \to \infty} = \bar{a}$. Moreover, logic requires that $v^{-1}(0) = 1$ such that nobody acquires higher education when there is no skill premium. Figure 1 shows the implied shape of the $v^{-1}(\tilde{w})$ curve, i.e., the threshold for higher education. At a given level $\tilde{w}_t$, individuals with ability a_t^* or higher obtain tertiary education. Individuals with lower ability remain low-skilled.

2.4. **Final Goods Production.** The production side of the economy builds upon Romer (1990) and Jones (1995). Aggregate output is produced with physical capital in the form of machines and with both types of labor according to the production function

$$Y_t = L_{H,Y,t}^{1-\alpha} \left(L_{L,t}^{\alpha} + \sum_{i=1}^{A_t} x_{i,t}^{\alpha} \right), \tag{6}$$

where $L_{H,Y,t}$ is the part of high-skilled labor that is employed in the final goods sector, $x_{i,t}$ are machines of the specific type i, $\alpha \in (0, 1)$ denotes the elasticity of output with respect to human labor that can easily be automated, and A_t is the stock of specific blueprints available for the associated machines of type i, i.e., it represents the technological frontier of the country

Figure 1: The Education Threshold



Individuals with ability a below the threshold $a_t^* = v^{-1}(\tilde{w})$ remain without higher education. There are $[1 - a_t^*]L_t$ individuals with higher education.

under consideration. We conceptualize technological progress (a growing technological frontier A, which is TFP growth) as an increase in the variety of machines in the production process.[4]

Let $p_{i,t}$ denote the net price of a unit of a machine of type i and τ_R the ad-valorem tax on machine input in production (the robot tax). The factor rewards are then given by

$$w_{H,Y,t} = (1-\alpha)L_{H,Y,t}^{-\alpha}\left(L_{L,t}^{\alpha} + \sum_{i=1}^{A_t} x_{i,t}^{\alpha}\right) \quad \Leftrightarrow \quad w_{H,t} = (1-\alpha)\frac{Y_t}{L_{H,Y,t}}, \tag{7}$$

$$w_{L,t} = \alpha(L_{H,Y,t}/L_{L,t})^{1-\alpha}, \tag{8}$$

$$(1+\tau_R)p_{i,t} = \alpha L_{H,Y,t}^{1-\alpha} x_{i,t}^{\alpha-1}, \tag{9}$$

The key difference with respect to the conventional growth literature is that technological progress (rising A) has a different impact on the two types of labor. As commonly assumed, it increases the productivity of complementing labor L_H and is in this sense quasi labor-augmenting. However, it leaves the productivity of substitutable labor L_L unaffected such that the relative importance of this type of labor declines with technological progress.

The key difference to related studies on automation (Acemoglu and Restrepo, 2016; Hemous and Olson, 2016) is that we provide an aggregate view according to which machines do not replace low-skilled labor in aggregate production. In our notation, the alternative view would be formalized by a production function $Y_t = L_{H,Y,t}^{1-\alpha}\left(L_{L,t} + \sum_{i=1}^{A_t} x_{i,t}\right)^{\alpha}$ and, depending on factor prices, production would employ either machines or low-skilled labor. Obviously, this

[4] Alternatively, we could have used a quality-ladder model (following Aghion and Howitt, 1992), which, in reduced-form, would be equivalent to the variety approach.

approach requires a disaggregated view on the production process, where labor-replacing machines are only available in some sectors of the economy.

Here, in contrast, low-skilled labor is always in demand. Intuitively, when new varieties of machines enter the production process (A rises) and perform tasks formerly performed by low-skilled labor, low-skilled workers find employment elsewhere in the economy. We thus maintain the notion of quasi labor-augmenting technological progress from conventional growth theory. The only difference is that technological progress does not increase the productivity of low-skilled labor. This can be seen in the low-skill wage (8), which is independent from technology and determined just by the relative skill supply. High-skilled labor, in contrast, benefits from technological progress and wages increase at the rate of aggregate output when labor supply stays constant [see (7)]. Finally, note that the demand for machines depends negatively on the robot tax τ_R [see (9)].

2.5. **R&D Sector.** The R&D sector produces blueprints for new machines by employing scientists, which are recruited from high-skilled labor. The production function of the R&D sector is given by

$$A_t - A_{t-1} = \bar{\delta}_t L_{H,A,t}, \tag{10}$$

where $L_{H,A,t}$ denotes scientists employed in the R&D sector and $\bar{\delta}$ is the productivity of these scientists. The productivity level of scientists itself depends on intertemporal knowledge spillovers (the standing-on-giants-shoulders externality) and on congestion effects (the stepping-on-toes externality) as described by Jones (1995). We follow the standard approach and write

$$\bar{\delta}_t = \frac{\delta A_{t-1}^{\phi}}{L_{c,A,t}^{1-\lambda}}, \tag{11}$$

where $\phi \in (0,1]$ measures the strength of intertemporal knowledge spillovers and $1-\lambda$ with $\lambda \in [0,1]$ measures the strength of the congestion externality.

Profits in the R&D sector are given by the revenue that R&D firms generate by selling the patents they developed net of the costs for the scientists that they employed,

$$p_{A,t}\bar{\delta}_t L_{H,A,t} - w_{H,A,t} L_{H,A,t}, \tag{12}$$

where $p_{A,t}$ is the price of blueprints and $w_{H,A,t}$ denotes the wage rate of scientists. Due to the competitive labor market, the wage rate of scientists attains the same level as the wage rate for high-skilled workers in the final goods sector. R&D firms maximize profits by choosing optimal R&D employment, which provides the optimality condition $w_{H,A,t} = \bar{\delta}_t p_{A,t}$. Our overlapping generations structure allows us to introduce a finite patent length of one generation, which is reasonably close to the actual patent duration of approximately 20 years (United States Patent and Trademark Office, 2017).

2.6. **Intermediate Goods Sector.** The intermediate goods sector uses physical capital as a variable input factor to produce machines. The production function is linear with a unitary capital input coefficient such that $x_{i,t} = k_{i,t}$, where $k_{i,t}$ is the amount of physical capital employed by each intermediate goods producer. There are two types of firms in the intermediate goods sector. Producers of the latest vintage of machines use a blueprint (patent) from the R&D sector as fixed input. These firms have a certain degree of market power and free entry into the intermediate goods sector implies that operating profits in period t, $\pi_{i,t}$, are equal to the entry costs consisting of the price that has to be paid up-front for the blueprint such that

$$\pi_{i,t} = p_{A,t}. \tag{13}$$

Producers of older vintages of machines are no longer protected by patent law and free entry ensures that a zero profit condition holds. Let variables associated with the latest vintage of machines be indexed by i and variables associated with earlier vintages by j. Operating profits for producers of the latest vintage are given by

$$\pi_{i,t} = p_{i,t}(x_{i,t})x_{i,t} - \bar{R}x_{i,t}. \tag{14}$$

Profit maximization implies

$$p'_{i,t}(x_{i,t})\frac{x_{i,t}}{p_{i,t}} + 1 = \frac{\bar{R}}{p_{i,t}} \qquad \Rightarrow \qquad p_{i,t} = \frac{\bar{R}}{\alpha}. \tag{15}$$

Producers of the latest vintage of machines charge a markup over marginal cost and the production of machines of type i adjusts (due to capital inflow/outflow) up to the point at which $\bar{R} = \alpha^2 L_{H,Y,t}^{1-\alpha} x_{i,t}^{\alpha-1}/(1+\tau_R)$. Producers of older vintages charge prices at marginal cost $p_{j,t} = \bar{R}$ for all j such that the production of machines of type j adjusts up to the point at which $\bar{R} = \alpha L_{H,Y,t}^{1-\alpha} x_{j,t}^{\alpha-1}/(1+\tau_R)$. Combining both demand functions provides the input ratio

$$x_{j,t} = \alpha^{\frac{1}{\alpha-1}} x_{i,t}, \tag{16}$$

implying that the demand for older vintages is higher because their price is lower. Aggregating over all vintages and using (16) we obtain

$$\sum_{j=1}^{A_{t-1}} x_{j,t}^{\alpha} + \sum_{i=A_{t-1}}^{A_t} x_{i,t}^{\alpha} = \tilde{A}_t x_{i,t}^{\alpha}, \qquad \tilde{A}_t \equiv \left[\alpha^{\alpha/(\alpha-1)} - 1\right] A_{t-1} + A_t - \alpha^{\frac{\alpha}{\alpha-1}}. \tag{17}$$

Using the new notation, we can rewrite final goods production as $Y_t = L_{H,t}^{1-\alpha}[L_{L,t}^{\alpha} + \tilde{A}_t x(i)^{\alpha}]$.

2.7. **Equilibrium.** In the basic model, we abstract from unemployment. Since ability is distributed in $(0, 1)$, the population share of low-skilled individuals is $v^{-1}(\tilde{w}_t)$ and the population share of high-skilled individuals amounts to $1 - v^{-1}(\tilde{w}_t)$, see Figure 1. Consequently, low-skilled employment is given by $L_{L,t} = v^{-1}(\tilde{w}_t)L_t$, and, since high-skilled individuals spend a fraction η of their time on education, aggregate supply of high-skilled labor amounts to $L_{H,t} = [1 - v^{-1}(\tilde{w}_t)](1-\eta)L_t$.

Labor market clearing requires that employment of high-skilled labor in the final goods sector and in R&D add up to the total supply of high-skilled labor such that

$$L_{H,t} = L_{H,Y,t} + L_{H,A,t}. \tag{18}$$

The market-clearing wage rate is given by

$$w_{H,A,t} = w_{H,Y,t} \quad \Leftrightarrow \quad p_{A,t}\frac{\delta A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}} = (1-\alpha)\frac{L_{L,t}^{\alpha} + \tilde{A}_t x_{i,t}^{\alpha}}{L_{H,Y,t}^{\alpha}}. \tag{19}$$

From Equation (9), we get demand for intermediates as

$$x_{i,t} = \left[\frac{\alpha}{p_{i,t}(1+\tau_R)}\right]^{\frac{1}{1-\alpha}} L_{H,Y,t}. \tag{20}$$

Plugging (13) and (20) into (19) provides profits of producers of the latest vintage of machines

$$\pi_{i,t} = \frac{\delta A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}} = (1-\alpha)\frac{L_{L,t}^{\alpha} + \tilde{A}_t \left[\frac{\alpha}{p_{i,t}(1+\tau_R)}\right]^{\frac{\alpha}{1-\alpha}} L_{H,Y,t}^{\alpha}}{L_{L,Y,t}^{\alpha}}. \tag{21}$$

Using (15), we obtain profits as $\pi_{i,t} = (1-\alpha)\bar{R}x_{i,t}/\alpha$. Inserting this expression, the price $p_{i,t} = \bar{R}/\alpha$, and equations (18) and (20) into (21), we obtain an implicit function for the employment level of scientists $L_{H,A,t}$. If an interior solution with R&D exists, the employment level of scientists solves the equation

$$\frac{\bar{R}}{\alpha}\left[\frac{\alpha^2}{\bar{R}(1+\tau_R)}\right]^{\frac{1}{1-\alpha}} (L_{H,t} - L_{H,A,t})\frac{\delta A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}} = \left(\frac{L_{L,t}}{L_{H,t} - L_{H,A,t}}\right)^{\alpha} + \tilde{A}_t\left[\frac{\alpha^2}{\bar{R}(1+\tau_R)}\right]^{\frac{\alpha}{1-\alpha}}. \tag{22}$$

Finally, the government runs a balanced budget. Suppose that low-skilled individuals receive a fraction κ of total transfers such that $T_{L,t} = \kappa T_t/L_{L,t}$ and $T_{H,t} = (1-\kappa)T_t/L_{H,t}$ with

$$T_t = \tau_W w_{H,t} L_{H,t} + \tau_W w_{L,t} L_{L,t} + \tau_R p_{i,t} x_{it}\left[A_t - (1-\alpha)A_{t-1} - \alpha\right], \tag{23}$$

where the last term captures the fact that older vintages of machines are sold at a lower price, thus, providing less tax revenue.

3. Analytical Results

The full model is recursive: similar to the models of unified growth theory (Galor and Weil, 2000, Galor, 2011), individuals need to form expectations on their future wages to decide upon their education. Future wages, however, depend on the education decision. Thus, the full model is not analytically accessible and we discuss the adjustment dynamics numerically in Section 4. Here, we assume that the result of the education decision, $L_{H,t}$, is given as a positive pre-determined state variable at any time t (implying $0 < L_{H,t} < L_t$). Suppose, for now, that $\tau_R = 0$. Then, solving for the equilibrium boils down to solving one equation, namely (22), for one unknown variable, employment in R&D, $L_{H,A,t}$.

3.1. **Equilibrium R&D Employment.** Inspection of (22) provides the following result.

Proposition 1. *At any time t, the equilibrium employment level in the R&D sector exists and it is positive and unique.*

For the proof notice that, due to the assumed positivity constraints on parameters and state variables $\bar{R} > 0$, $\delta > 0$, $\phi \in (0, 1]$, $\alpha \in (0, 1)$, $\lambda \in (0, 1)$, $A_{t-1} > 0$, $L_{L,t} > 0$, and $L_{H,t} > 0$, the left-hand side (LHS) of Equation (22) is *strictly decreasing* in $L_{H,A,t}$, while the right-hand side (RHS) is *strictly increasing* in $L_{H,A,t}$. Furthermore, we have that

$$\lim_{L_{H,A,t} \to 0} LHS = \infty, \qquad \lim_{L_{H,A,t} \to 0} RHS = const. > 0,$$
$$\lim_{L_{H,A,t} \to L_{H,t}} LHS = 0, \qquad \lim_{L_{H,A,t} \to L_{H,t}} RHS = \infty.$$

As a consequence, there is a unique positive level of scientists in the R&D sector. Once $L_{H,A,t}$ has been found, we can solve for all other variables.

3.2. **Balanced Growth Path.** The additional assumption that $\bar{a}$ has already been reached establishes an asymptotic balanced growth path along which the population shares of workers stay constant. The long-run economic growth rate rises if there are more scientists employed in R&D and if these scientists have a higher productivity level (δ), and it decreases with the extent of the duplication externality. Let $L_{H,A}^*$ denote the solution of (22) at $a_t^* = \bar{a}$. The balanced growth rate is obtained from (10) and (11) as $g = \delta (L_{H,A}^*)^\lambda$ for $\phi = 1$ and as $g = 0$ for $\phi < 1$. As it is well known, when the population and the education level per person stay constant, positive balanced growth exists only for a knife-edge case. Off the balanced growth path, however, the case of zero asymptotic growth is compatible with meaningful adjustment dynamics, as shown below.

3.3. **Wage inequality.** Inspection of (7) and (8) shows that high-skilled workers enjoy wage growth when the economy is growing (growing Y_t). By contrast, wages of low-skilled workers are constant on the balanced growth path because factor shares are constant. This leads directly to the next result.

Proposition 2. *Technological progress is skill-biased. In an economy populated by high-skilled workers who are complementary to machines and low-skilled workers who are substitutes to machines, higher growth implies higher wage inequality along the balanced growth path.*

The intuition for this result is straightforward. Technological progress raises the productivity of high-skilled workers by introduction of new machines. At the same time, however, new machines do not raise the productivity of low-skilled workers because these workers are substitutes for machines. Another way to illustrate the disruptive effect of technological progress on low-skilled workers is to consider the labor share in aggregate income and to decompose it between high-skilled workers and low-skilled workers.

Proposition 3. *Along a path of positive balanced growth, the total labor share is declining towards* $(1-\alpha)$*. The low-skilled labor share is declining to zero.*

For the proof, we compute the labor share as

$$(1-\alpha)+\frac{w_{L,t}L_{L,t}}{Y_t}, \tag{24}$$

where $(1-\alpha)$ is the high-skilled labor share. We then note that, along a path of positive balanced growth, $L_{L,t}$ and $w_{L,t}$ are constant because population shares are constant, whereas Y_t is growing at a positive rate. The decline in the labor share in the course of automation is consistent with the empirical evidence for the U.S. pointing out that the labor share was constant until the 1970s, but declined by almost 6 percentage points since then (Karabarbounis and Neiman, 2014).

3.4. **Wealth Inequality.** The declining relative income of low-skilled labor has, furthermore, a clear inequality-enhancing effect on the distribution of wealth.

Proposition 4. *In a growing economy without redistribution, the share of wealth held by high-skilled workers increases and converges to one asymptotically. Ceteris paribus, faster economic growth leads to a faster increase of wealth inequality.*

For the proof, we insert wages (7) and (8) into savings (3) for $T_H = T_L = 0$ and obtain relative wealth held by high-skilled workers $\tilde{s}$:

$$\tilde{s}=\frac{(1-\eta)(1-\alpha)\frac{Y_t}{L_{H,Y,t}}}{(1-\eta)(1-\alpha)\frac{Y_t}{L_{H,Y,t}}+\alpha\left(\frac{L_{H,Y,t}}{L_{L,t}}\right)^{1-\alpha}\frac{L_{L,t}}{L_{H,t}}}. \tag{25}$$

Along the balanced growth path, the population shares stay constant, while $Y_t/L_{H,Y,t}$ grows perpetually. This implies that the second term in the numerator becomes gradually less important from a quantitative point of view such that $\tilde{s}$ converges to 1. Clearly, wealth inequality increases faster when Y_t grows at a higher rate. Notice that, off the steady state, rising higher education (declining L_L and increasing L_H) reinforces wealth inequality during the transition towards the steady state.

In our stylized framework, rising wealth inequality is a product of growing wages of high-skilled workers, stagnating wages of low-skilled workers, and constant saving rates. In a less stylized framework, utility functions could take into account subsistence needs or status concerns in consumption. These mechanisms would, however, further amplify wealth inequality because they imply lower saving rates for the poor. The result of Proposition 4 deviates from with the findings of Piketty (2014), who argues that, ceteris paribus, faster economic growth reduces inequality.

4. The Race between Education and Technology

4.1. **Preliminary Considerations.** We next consider adjustment dynamics off the steady state and the interaction between education and technology. Qualitatively, it is straightforward to see the impact of technology on education.

Proposition 5. *With technological progress, the share of high-skilled labor in the population increases and converges towards $\bar{a}$.*

The proof is obvious from Proposition 2 and inspection of Equation (5) and Figure 1. As technology advances and more machines are used in production, the wage of high-skilled workers increases relative to low-skilled workers. An increasing skill premium motivates more individuals to obtain higher education. Increasing supply of high-skilled labor leads, ceteris paribus, to relatively higher wages for low-skilled workers [see Equations (7) and (8)], and, thus, suppresses the skill premium. This interplay captures the "race between education and technology" only that, in its original setup, Goldin and Katz (2009) treated education as exogenous, whereas here, education is endogenous and triggered by technological advancements. Given that technological advances are the dominating force, the skill premium is gradually increasing, and a larger share of society obtains higher education. Ultimately, the society converges towards a situation, where a population share of $\bar{a}$ remains without higher education because the learning ability of its members is too low for obtaining a college degree. In other words, in a growing economy, the threshold $\bar{a}$ is reached in *finite* time. From then onwards, the population shares of high-skilled labor and low-skilled labor stay constant and the economy converges towards a balanced growth path (which is only reached as time approaches infinity).

To fully assess the interactions in the race between education and technology, we need to solve the model numerically. We consider two different scenarios. In the first scenario, we assume, as it is common in R&D-based growth theory, that the economy converges gradually towards a steady state of positive long-run growth. In the second scenario, we consider the case of a secular decline in the growth rates of TFP and per capita GDP. This case is less frequently discussed in the literature (exceptions are Jones, 2002, and Groth et al., 2010). However, it is particularly relevant in the present case to address the question of whether increasing automation is compatible with declining productivity growth. The case of (mildly) declining productivity growth is also better suited for calibrating the model with U.S. data. In the following, we thus briefly discuss conventional adjustment dynamics towards positive balanced growth and then calibrate the model more carefully for the case of declining productivity.

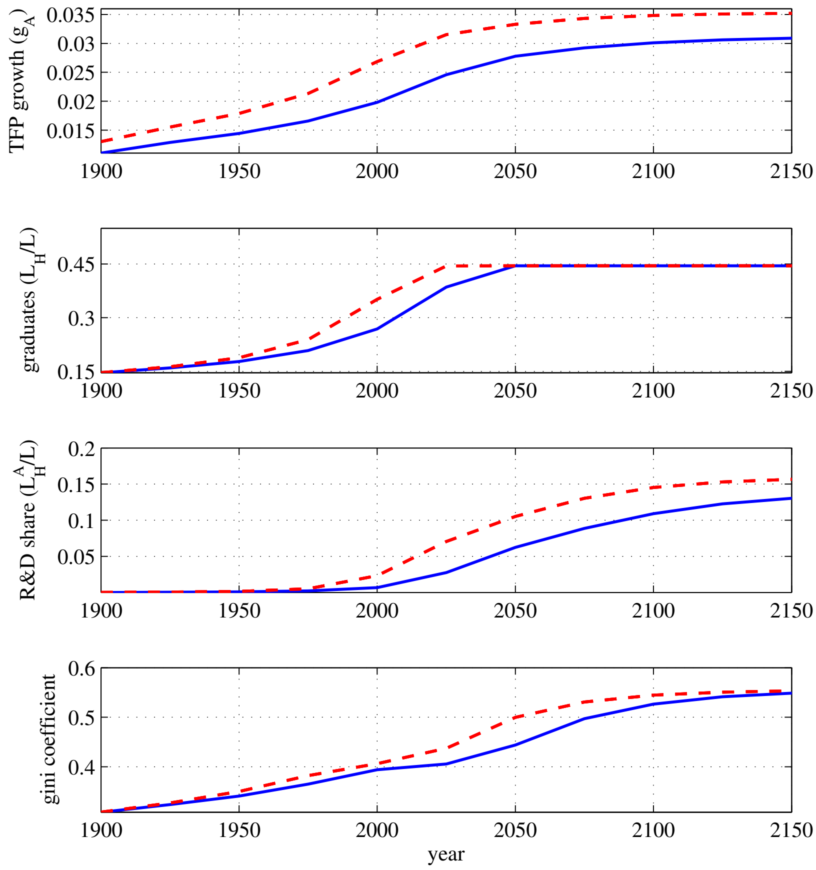
4.2. **Positive Steady-State Growth.** We start the computation of adjustment dynamics in the year 1900 and convert the predicted growth rates per generation into annual rates. To generate positive long-run growth, we need to impose $\phi = 1$. We set $\bar{R} = 2$, which implies an annual real interest rate of approximately 4.5 percent. We assume that the time preference rate equals the interest rate and set $\beta = 1/\bar{R}$. Regarding the output elasticity of machines, we assume that $\alpha = 0.6$ such that the long-run labor share is given by 0.4. Assuming that high-skilled individuals enter college at age 19, complete their education after 5 years and leave the workforce at age 63, we obtain the estimate of $\eta = 0.11 = 5/(63 - 19)$.

The distribution of ability in the population is likely to be bell-shaped. IQ, for example, is by definition approximately normally distributed. This means that the cumulated distribution function of ability is s-shaped and the cumulated distribution function for effort or disutility of education is s-shaped as well. The inverse function $v^{-1}(\tilde{w})$ is thus the inverse of an s-shaped function with a positive lower limit (as shown in Figure 1). It can be conveniently captured by the inverse logistic function $v^{-1}(\tilde{w}) = \bar{a} + (1 - \bar{a})/\{1 + \exp[\theta(\tilde{w} - \psi)]\}$. We set $\bar{a}$ to 0.5, implying that in the limit, half of the population is high-skilled (obtained a college degree).

We set the population size (arbitrarily) to 1,000 and then adjust the initial technology level $A(0)$, the technology parameters δ and λ, and the education parameters ϕ and θ such that the model predicts – for the end of the 20th century – an annual TFP growth rate of around 2 percent per year, an R&D share of around 2 percent, that around 30 percent of the population have acquired a college degree, and that the Gini coefficient of income is about 45 percent. This leads to the estimates $A(0) = 8$, $\psi = 2$, $\theta = 10$, $\lambda = 0.2$, and $\delta = 0.43$. In the benchmark case, there is no redistribution such that $\tau_W = \tau_R = 0$.

Adjustment dynamics are shown in Figure 2. Generational growth rates are converted to annual growth rates assuming that a generation lasts for 25 years. Blue (solid) lines reflect adjustment dynamics of the benchmark case. As the economy grows and skill-biased technological progress unfolds (first panel), more individuals are motivated to acquire a college education (second panel). The increase in college graduates renders high-skilled labor less scarce and more high-skilled labor is allocated to R&D (third panel). This, in turn, further amplifies technological progress such that the economy takes off with initially increasing

Figure 2: Adjustment Dynamics ($\phi = 1$)



Parameters: $\alpha = 0.6$; $\beta = 0.33$; $\delta = 0.43$, $\phi = 1$, $\lambda = 0.2$, $\theta = 0.10$, $\psi = 2$.

growth rates. After a while, however, the stepping-on-toes effect becomes noticeable and the gain in growth rates levels off as the economy adjusts towards the steady state.

During the transition, the wage rate of low-skilled labor increases somewhat due to its declining supply such that steady-state wages are around 1.4 times higher than initial wages. However, for the aggregate low-skilled wage bill, this effect is almost compensated by declining supply such that $w_L L_L$ increases only by 4 percent over a century. Compared to these minuscule changes, the wages of high-skilled labor increase drastically in conjunction with TFP growth. As a consequence, income and wealth inequality increase as the economy converges towards the steady state. The bottom panel of Figure 2 shows inequality measured by the Gini coefficient of wage income, which is, in our simple setup, easily obtained as $s_w - L_{H,t}/L_t$, where s_w is the income share of high-skilled individuals, $s_w \equiv w_{H,t}L_{H,t}/(w_{H,t}L_{H,t} + w_{L,t}L_{L,t})$. As the economy grows, the income share of high-skilled individuals converges to 100%, whereas the share of high-skilled workers converges toward 45%. To see this, notice that the share of high-skilled individuals converges to 50% and that, at each moment in time, 11% of high-skilled individuals are still in the education system and not in the workforce. Altogether, the Gini coefficient therefore converges towards 55%.

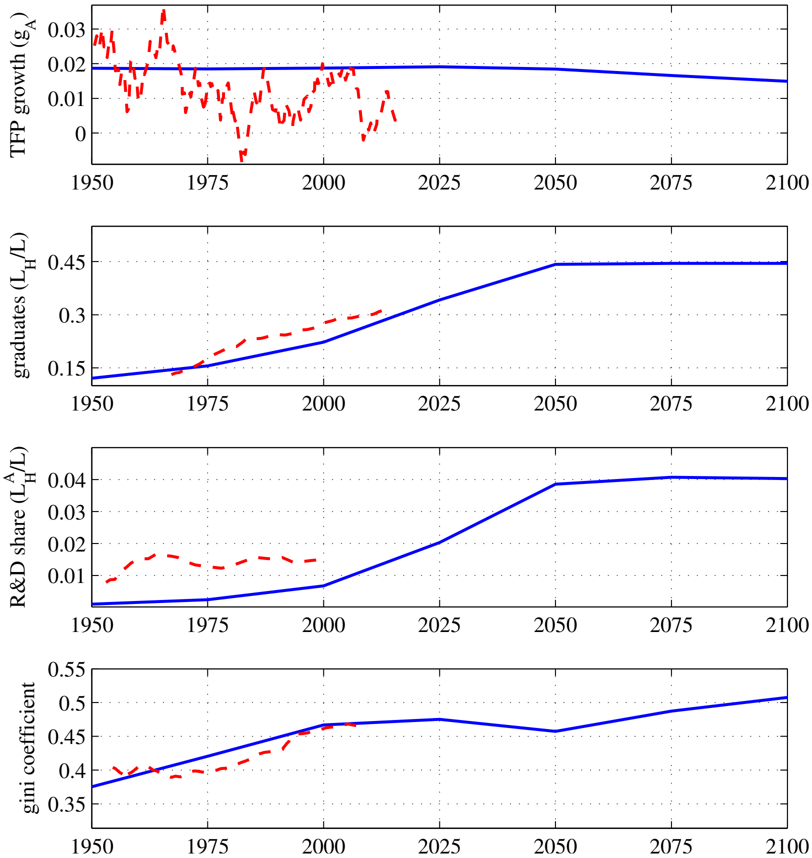
To show our "non-Piketterian" result, we next increase δ to 0.5 (from 0.43) and keep all other parameters and initial values from the benchmark run. The results are shown by dashed lines in Figure 2. Due to the assumed higher productivity in R&D, the alternative economy grows at a faster rate, initially and everywhere along the adjustment path (panel 1). The higher rate of skill-biased technological progress induces a faster growth of income for the high-skilled population and triggers more education (panel 2). A better educated workforce provides more labor supply for R&D (panel 3), which further spurs innovation and economic growth. Since low-skilled labor is not benefiting from these trends, inequality increases faster in the high-growth economy (panel 4). Individuals who suffer from ability constraints in learning, and fail to achieve college graduation are left behind earlier than in the benchmark run.

4.3. **Automation and Declining TFP Growth.** The numerical exercise of the previous section is in line with many related studies in quantitative growth economics in the sense that TFP growth continued to rise in the second half of the 20th century. However, in actuality, TFP growth declined mildly during this period. While the counterfactual prediction of the model may be regarded as harmless in a different context, it is of particular importance for the issue of automation because it has been argued that, if automation were indeed strongly affecting the economy, it should be observed in conjunction with *rising* TFP growth and rising investment rates.

In an attempt to improve the last section's calibration, we try to fit U.S. trends for the second half of the 20th century and beyond for TFP growth (Fernald, 2015), the population share with a college degree (U.S. Census, 2015), the Gini coefficient of before-tax monetary income (U.S. Census, 2015; computation taken from Berruyer, 2012), the R&D expenditure to GDP ratio (Ha and Howitt, 2007), and the investment rate (World Bank, 2017). The parameters used in the calibration are provided below Figure 3. The most notable change as compared to the previous exercise is the estimate of $\phi = 0.7$. A value of ϕ below unity is needed to fit a mildly declining TFP growth trend. It implies slow convergence towards a steady state of zero exponential growth. We should say that we do not perform this exercise because we endorse zero long-run growth in the very distant future, but do so in order to make the model consistent with declining TFP growth in the recent history and in the near future. Other parameter changes are an increase of δ to 2.0 to make up for the reduced effect of the standing-on-shoulders externality in the 20th century and increases of β to 0.64 and ψ

to 3.3 to generate a steeper rise of college graduates in the 20th century. All other parameter values are kept from the benchmark run.

Figure 3: Adjustment Dynamics ($\phi < 1$)



Parameters: $\alpha = 0.6$; $\beta = 0.64$; $\delta = 2.0$, $\phi = 0.7$, $\lambda = 0.2$, $\theta = 0.10$, $\psi = 3.3$.

Solid lines in Figure 3 refer to the predicted adjustment dynamics. Dashed lines refer to the underlying data. The first panel shows that the calibration supports an almost constant and mildly falling trend of TFP growth and approximates TFP growth in the late 20th and early 21st century fairly well. It also shows that decreasing returns in learning from previous innovations ($\phi = 0.7$) still support 1.4 percent TFP growth at the end of the 21st century.

Although productivity growth is not increasing, all the previously established mechanics of the model are at work. The reason is that they require only *positive* TFP growth but not increasing or constant TFP growth. In the second panel, we see how the rising skill premium induces an increasing share of the population to acquire higher education. The model approximates the actual increase of college graduates well, from a population share of 13 percent in the early 1970s to a population share of 33 percent in the year 2015.

The middle panel in Figure 3 shows that the model captures a mildly rising trend in the R&D-share but predicts a level of R&D that is somewhat too low for the 20th century. This shortcoming is due to the simple structure of the model which provides no degrees of freedom to adjust TFP growth independently from investment in R&D. The difference with respect to the economy illustrated in Figure 2 is that the rising employment in R&D does not spur further increases in the innovation rate and in economic growth because it is counter-balanced by decreasing returns in learning from previous innovations.

The fourth panel shows that the model matches the increasing trend in inequality well, which is predicted to rise further and to converge to 0.55 percent when the relative income and wealth of low-skilled individuals converge to zero. The computational experiment clearly refutes the view that declining or constant productivity growth and declining investment is incompatible with increasing automation and increasing inequality. As explained above, for these trends to be simultaneously observed, we only need positive TFP growth, i.e., further innovation in automation.

4.4. **Redistributive Taxation.** Since only high-skilled individuals benefit from technological progress, it seems appropriate to support the losers in the race between education and technology by means of redistributive taxation. Aside from redistribution through progressive income taxation, the implementation of a robot tax has recently been proposed. However, when technology and education are endogenous, redistributive taxation can entail less benign outcomes in a dynamic context than suggested by thought experiments that treat education and technology as given. In a fully dynamic context, redistribution can actually be counterproductive and hurt low-skilled labor more than a laissez-faire policy. Before we discuss redistribution in the calibrated economy, we first briefly develop some intuition for this result by means of an analytical discussion.

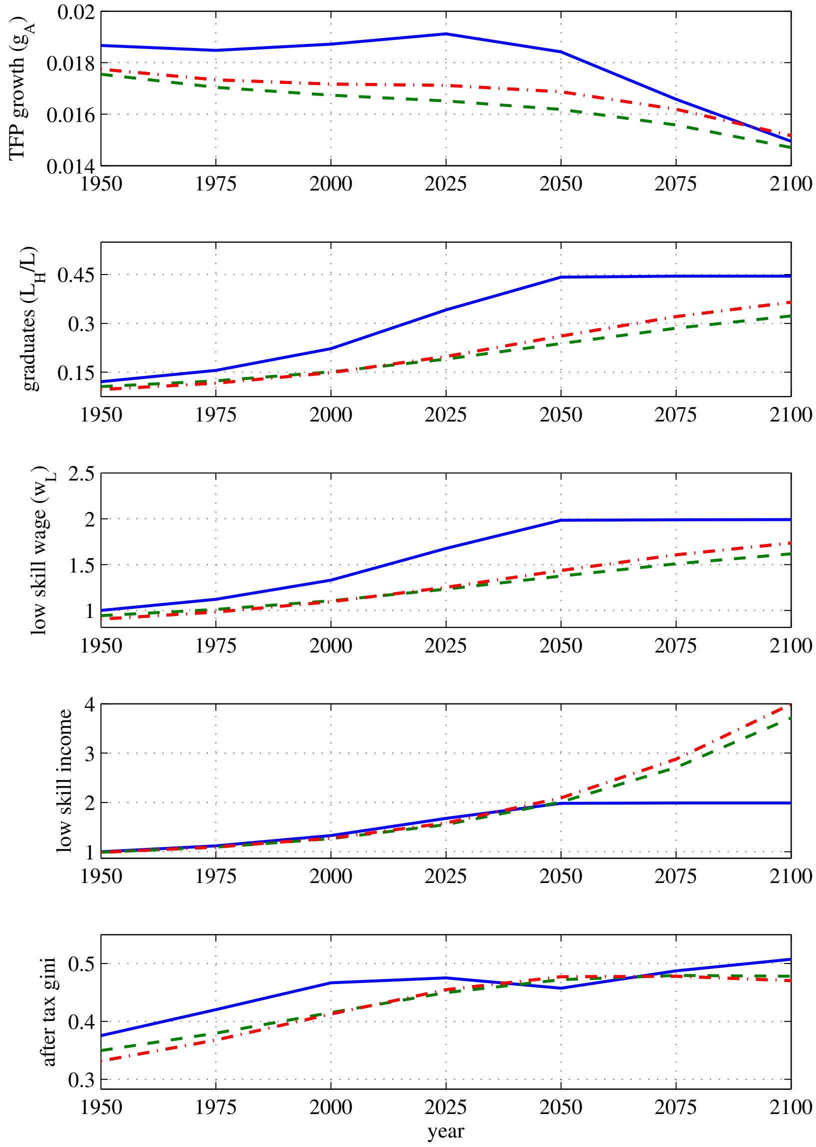
Lemma 1. *As long as $a_t^* > \bar{a}$, an increasing wage tax or an increasing transfer to low-skilled labor reduces education.*

The proof is obtained by differentiating the wage premium in monetary terms, $\hat{w} \equiv [(1-\tau_w)(1-\eta)w_H - T]/[(1-\tau_w)(1-\eta)w_H - T]$, for the case where $T_L \geq T_H > 0$. Intuitively, higher taxes on wage income or higher transfers T_L reduce the incentive to increase wages through higher education. In a dynamic context, this means that along the transition path, i.e., as long as the low-skilled labor share has not yet settled at $\bar{a}$, redistribution slows down the take up of higher education, and thereby innovation and economic growth.

The response employment in R&D (and thus economic growth) to an increase in the robot tax can be discussed with the help of the equilibrium condition (22). From visual inspection, the response is seemingly ambiguous. On the one hand, an increase of τ_R leads to less machine production, less profits in the machine producing sector, a lower equilibrium price for blueprints of new machines; thus, lower wages and employment in R&D. Formally, this implies that the left-hand side of (22) declines. On the other hand, the reduction of machine input leads to lower wages for high-skilled workers in goods production and therefore to a shift of high-skilled labor towards R&D. Formally, this implies that the right-hand side of (22) declines. By implicitly differentiating (22), it can be shown that the negative effect through profits in the machine sector dominates.

Lemma 2. *An increasing robot tax* τ_R *leads to less employment in R&D and slower growth.*

Figure 4: Adjustment Dynamics Redistributive Taxation ($\kappa = 1$)



Parameters as for Figure 3: blue (solid) lines: $\tau_R = \tau_W = 0$ (replication of Figure 3); green (dashed) lines: $\tau_R = 0.1$; red (dashed-doted) lines: $\tau_W = 0.1$.

The proof of Lemma 2 is stated in the Appendix. While the static responses of education and innovation to redistributive taxation can be assessed analytically, the impact on low-skilled income and income inequality can only be assessed numerically in the dynamic equilibrium. For that purpose, we take the model as calibrated in Figure 3 and first discuss the border case in which only low-skilled individuals receive income transfers, i.e., $\kappa = 1$ such that $T_L = T$ and $T_H = 0$. The results are shown in Figure 4. To facilitate comparisons, solid (blue) lines replicate the benchmark case without redistribution from Figure 3.

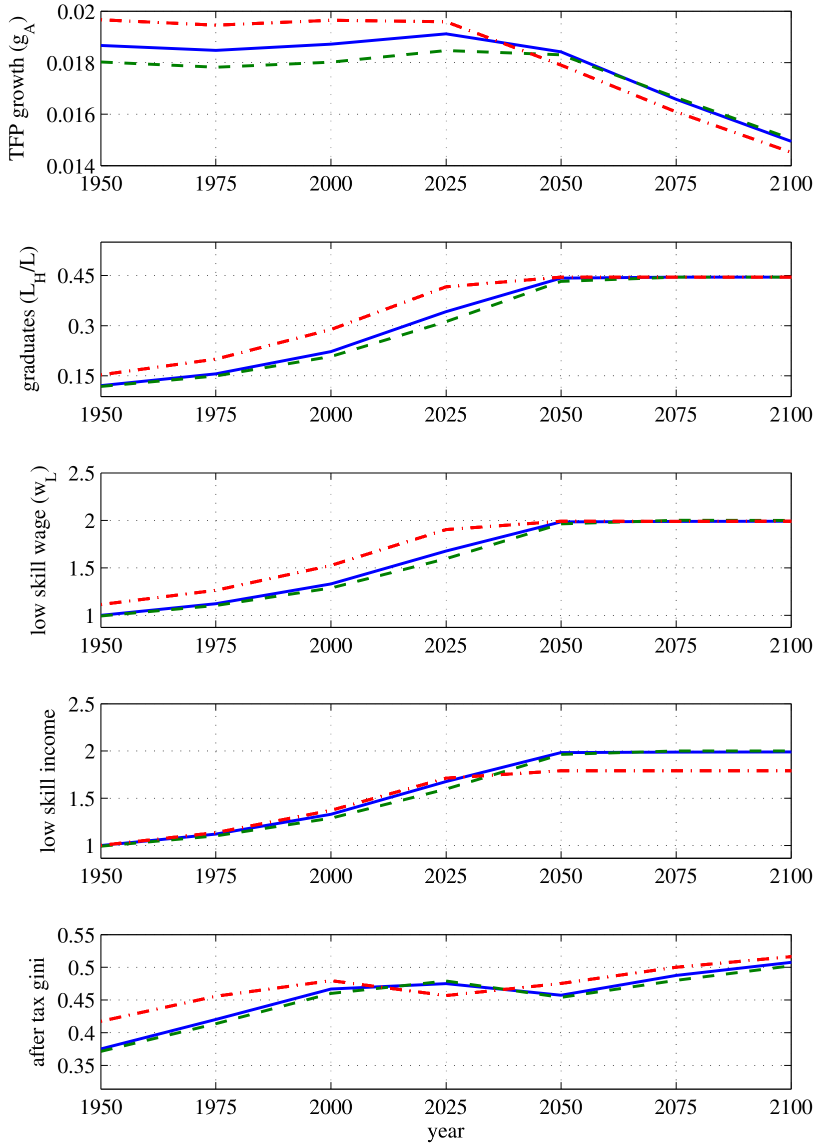
Adjustment dynamics for a robot tax of $\tau_R = 0.1$ are shown by green (dashed) lines in Figure 4. Confirming Lemma 2, the robot tax depresses R&D and TFP growth (top panel). As a result, wages for high-skilled labor increase at a lower rate, which causes a slower high-skilled labor expansion (second panel). Redistribution reduces the high-skilled labor supply also in the long-run. Since, in the limit, transfers are growing at the rate of technological progress, some individuals with abilities above but close to $\bar{a}$ are permanently discouraged from obtaining a college degree. In the limit, L_H converges to 0.45 (instead of to 0.5).

Redistribution also depresses low-skilled wages (middle panel). Because there is less higher education, there is a more abundant supply of low-skilled labor compared to the benchmark run such that $L_{H,t}/L_{L,t}$ is lower in (8). Even after-tax income of low-skilled individuals, shown in the fourth panel, is somewhat lower than without redistribution until the mid-21st century (wages are normalized such that the initial wage without redistribution equals 1). As time passes, however, the effect of redistribution dominates the general equilibrium effect on gross wages. In the limit, transfers T_L grow at the rate of high-skilled income, whereas low-skilled wages stagnate such that low-skilled income is dominated by transfers, $\lim_{t\to\infty} w_L/T_L = 0$. Already at the turn of the 21st century, low-skilled individuals are twice as rich with redistribution than without.

The bottom panel in Figure 4 shows that redistribution quite strongly affects the after-tax Gini coefficient. Until the mid-21st century, the reduction in the Gini was mostly a product of the lower income of high-skilled labor (compared to the benchmark simulation) rather than by improvements in disposable income of low-skilled individuals. These temporary effects, however, vanish as the economy converges towards the balanced growth path. In the long-run, the after-tax Gini coefficient settles at a lower level, because, through redistribution, low-skilled individuals participate in the gains from automation and economic growth. The before-tax Gini, however, is higher than in the laissez-fair benchmark run because the population share of high-skilled individuals declines. In our example, the before-tax Gini increases towards 0.6 (from 0.55), whereas the after-tax Gini is 0.46.

Alternatively, transfers could be financed by the labor income tax. Red (dash-dotted) lines in Figure 4 show the outcome for $\tau_w = 0.1$ (and $\tau_R = 0$). In contrast to the robot tax, the wage tax has a direct effect on education and an indirect effect on technological progress and growth. Lower net wages reduce the gains from education such that higher education increases at a lower rate with economic development. As a result of reduced high-skilled labor supply, there is less employment in R&D, and thus less innovation and lower productivity growth. In effect, the labor income tax reduces the progress of technology and education similarly to the robot tax and it therefore has similar consequences on the adjustment dynamics for low-skilled income and inequality (panels 3 to 5 in Figure 4).

Figure 5: Adjustment Dynamics: Education Subsidy ($\kappa = 0$)



Parameters as for Figure 3: blue (solid) lines: $\tau_R = \tau_W = 0$ (replication of Figure 3); green (dashed) lines: $\tau_R = 0.1$; red (dashed-doted) lines: $\tau_W = 0.1$.

Next, we consider the case of $\kappa = 0$ such that all transfers are channeled to high-skilled individuals. Since, these transfers are designed to be granted conditionally on a college degree, they could be conceptualized as an education subsidy. The results are shown in Figure 5, where again blue (solid) lines replicate the benchmark run from Figure 3, green (dashed) lines reflect the case of $\tau_R = 0.1$ and red (dash-dotted) lines reflect the case of $\tau_W = 0.1$.

Figure 5 shows that an education subsidy financed by a robot tax is counterproductive because it *reduces* education. As explained above, the robot tax reduces the demand for machines and innovations and slows down TFP growth (top panel). The effect from reduced high-skilled income through lower growth dominates the positive direct effect of the transfer on education such that the supply of high-skilled labor increases at a *lower* rate than in the benchmark run (second panel). An education subsidy financed by labor income taxation, in contrast, has no direct impact on technology such that it indeed motivates more education (red lines in the second panel). As a result of the greater supply of high-skilled labor, employment in R&D rises and productivity grows at a higher rate than in the benchmark case.

As shown in the middle panel, gross wages of low-skilled labor actually improve by subsidizing high-skilled labor because low-skilled labor becomes relatively scarce in the production process. Because of this counterbalancing general equilibrium response, the effect of the subsidy on the net wages of low-skilled workers is relatively small during the transition. Eventually, however, low-skilled wages stagnate and are thus permanently lower than without the education subsidy (fourth panel). The Gini coefficient is higher in the early phase of the transition because of the faster increasing population share with high skills and high income. In the long-run, when population shares and before-tax income of low-skilled individuals are constant, the Gini settles at 0.55, as in the benchmark case (bottom panel in Figure 5).

5. Automation, Fair Wages, and Involuntary Unemployment

In principle, there are several gateways for rising unemployment due to automation in such a framework. For example, in an earlier version of this paper (Prettner and Strulik, 2017), we introduced unemployment via the social welfare system and a reservation wage. The notion that automation causes voluntary unemployment through frictions on the labor supply side is, however, not entirely compelling. In the following, we integrate involuntary unemployment based on Akerlof and Yellen's (1990) fair wage theory into our model. Akerlof and Yellen motivate the fair wage theory from different perspectives, including equity theory, relative deprivation theory, and social exchange theory, which all seem particularly relevant in the present context, where only one group of workers benefits from technological progress. Since it turns out that unemployment occurs only for low-skilled labor, we can speak of involuntary technological unemployment.

The basic idea is that workers compare their payment with that of coworkers and exert full effort at work only when they perceive their remuneration as fair. Specifically, effort at work e_j is given by $e = \min(w_j/w_j^*, 1)$, in which w_j^* is the wage that is perceived as fair by workers of group j. In our context, fair wage considerations are made only in the final goods sector because this is the sector in which high-skilled workers and low-skilled workers meet. In equilibrium, high-skilled workers receive the same wage in R&D as in goods production. To include effort considerations, we rewrite the goods production function such that e_j is taken into account

$$Y_t = (e_{H,t} \cdot L_{H,Y,t})^{1-\alpha} \left[(e_{L,t} \cdot L_{L,t})^{\alpha} + \sum_{i=1}^{A_t} x_{i,t}^{\alpha} \right].$$

Following Akerlof and Yellen, we assume that the fair wage of group j is a weighted average of the wage received by the reference group and the market-clearing wage

$$w_{L,t}^* = \mu w_{H,t} + (1-\mu) w_{L,t}^c, \qquad w_{H,t}^* = \mu w_{L,t} + (1-\mu) w_{H,t}^c, \tag{26}$$

where $w_{j,t}^c$ is the market clearing wage of group j and μ measures the strength of wage comparisons. The market-clearing wage is defined as the wage that would clear the market for group-j labor in the neoclassical case (i.e., when workers unconditionally exert full effort).

Let $\overline{L}j,t$ denote labor supply of group j. We then have

$$w_{H,t}^c = (1-\alpha)(\overline{L}_{H,t})^{-\alpha}(L_{L,t}^{\alpha} + \tilde{A}_t x_t^{\alpha}), \qquad w_{L,t}^c = \alpha L_{H,t}^{1-\alpha}(\overline{L}_{L,t})^{\alpha-1}, \tag{27}$$

where we made use of the shorthand notation $\tilde{A}_t$ according to (17). In principle, multiple equilibria are possible on the labor market. Here, we follow Akerlof and Yellen (1990) and consider the so-called integrated equilibrium as the most relevant one. At this equilibrium, all workers exert full effort. High-skilled workers receive a wage that exceeds the fair wage and full employment results such that $L_{H,t} = \overline{L}_{H,t}$. Low-skilled workers receive a fair wage that exceeds the market-clearing wage and low-skilled unemployment results. The indirect demand function for low-skilled labor is given by

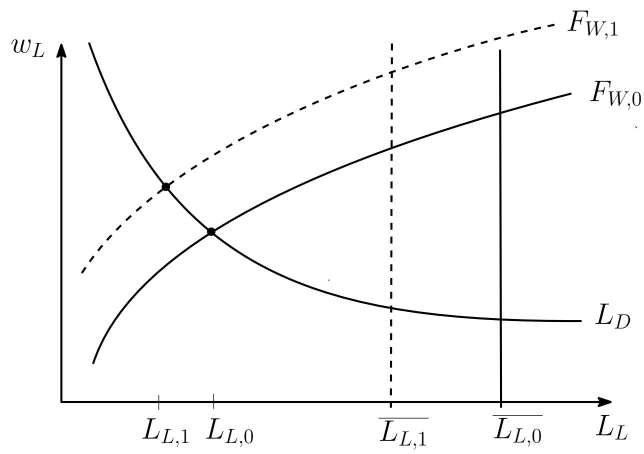
$$w_{L,t} = \alpha(\overline{L}_{H,t})^{\alpha} L_{L,t}^{\alpha-1} \equiv L_D(w_{L,t}). \tag{28}$$

Inserting (27) into (26) and using $w_{H,t} = w_{H,t}^c$ provides the "fair wage constraint", i.e., the wage at which low-skilled workers exert full effort:

$$w_{L,t} = \mu(1-\alpha)(\overline{L}_{H,t})^{-\alpha}(L_{L,t}^{\alpha} + \tilde{A}_t x_t^{\alpha}) + (1-\mu)\alpha(\overline{L}_{H,t})^{1-\alpha}(\overline{L}_{L,t})^{\alpha-1} \equiv F_w(w_{L,t}). \tag{29}$$

Labor market equilibrium prevails when (28) and (29) hold simultaneously. It is depicted in Figure 6. Labor demand according to (28) is represented by the falling curve L_D. The upward-sloping curve F_W represents the fair wage constraint. Low-skilled workers perceive it as fair to earn more as their employment rises. The full-employment constraint is represented by the vertical $\overline{L}_L$ line. The initial labor market equilibrium is obtained at $L_{L,0}$ and initial unemployment is obtained as $\overline{L}_{L,0} - L_{L,0}$.

Figure 6: The Fair Wage Constraint and Equilibrium Unemployment



Technological progress shifts the fair wage constraint F_W upwards and the low-skilled workforce $\overline{L}_L$ inwards.

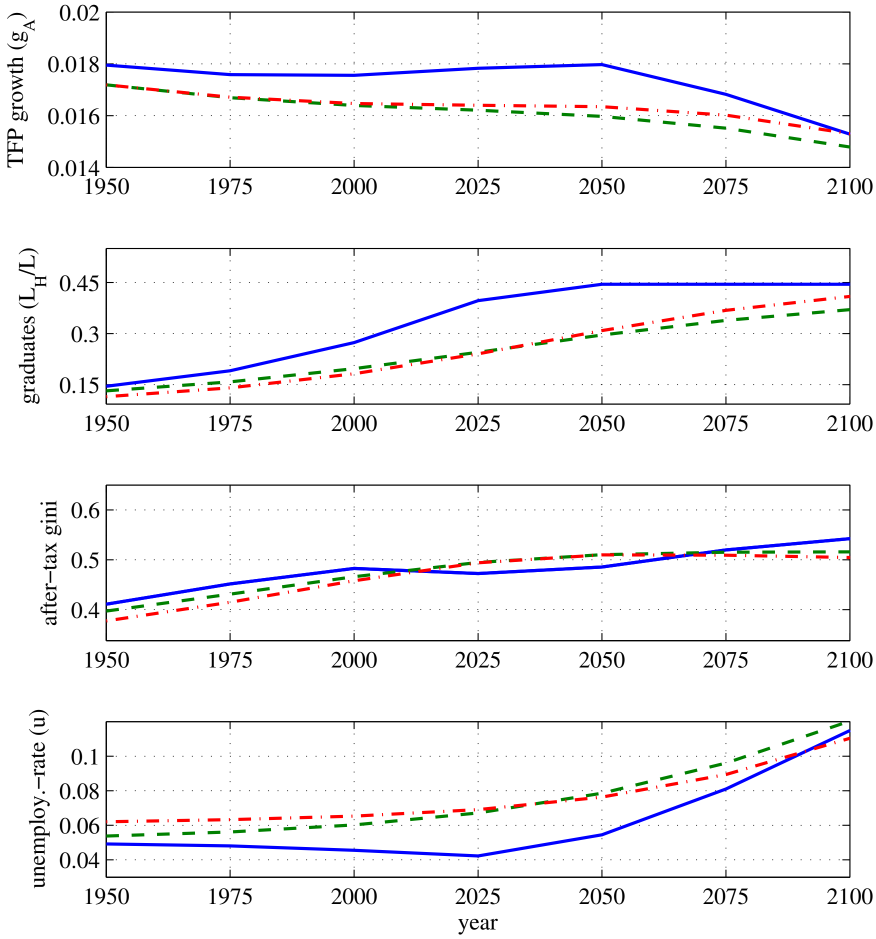
The implications from the race between education and technology for unemployment can be qualitatively deduced from Figure 6. Technological progress (rising $\tilde{A}_t$) shifts the fair wage constraint upwards to $F_{W,1}$. Low-skilled workers demand a higher wage because they notice that wages for high-skilled workers rise with increasing automation. Due to the rising reference level, they perceive a higher wage for themselves as fair. Intuitively, low-skilled workers want to take part in the advances from technology, although their labor productivity does not improve. Without a pay rise, however, labor productivity would deteriorate due to declining effort. The new equilibrium employment level is at $L_{L,1}$.

Technological advances, however, also trigger higher education such that the workforce of low-skilled individuals declines. The workforce constraint shifts to the left to $\bar{L}_{L,1}$. Taking both moves together, it is generally ambiguous as to whether low-skilled unemployment, given by $\bar{L}_L - L_L$, increases or declines due to high-skilled labor-augmenting technological progress. However, once education has converged to its upper bound, only the effect of technology on fair wages remains and low-skilled unemployment increases inevitably as automation technology advances.

Proposition 6. *Once higher education converged towards its upper bound, more innovation and faster economic growth lead to more involuntary unemployment.*

The proof is obvious from Figure 6. When education is stationary, the vertical L_L-line does not move. More innovation leads to a faster increasing TFP level $\tilde{A}$ and a faster upward movement of the fair wage constraint. As a result, low-skilled wages increase at a higher rate and employment declines faster. The result is intuitive. More innovation and higher growth of high-skilled wages leads to a faster increase of the reference level for fair wage considerations and low-skilled workers need a faster increase of wages to induce full effort. Since low-skilled productivity does not rise, firms respond with reduced demand for low-skilled labor.

Figure 7: Adjustment Dynamics (Unemployment)



Parameters as for Figure 3 and $\mu = 0.006$: blue (solid) lines: $\tau_R = \tau_W = 0$, replication of Figure 3; green (dashed) lines: $\tau_R = 0.1$; red (dashed-doted) lines: $\tau_W = 0.1$.

Finally, we consider the evolution of unemployment along the transition to balanced growth. For that purpose, we employ the model as calibrated for Figure 3 and set μ such that fair wage considerations generate an unemployment rate of 5 percent at the end of the 20th century. This leads to the estimate of $\mu = 0.004$. The relative importance of the reference wage is thus estimated to be relatively small. Figure 7 shows the implied adjustment dynamics.

Blue (solid) lines in Figure 7 reflect the case without income redistribution. The Gini coefficient in the 20th century is about 5 percent higher than in the benchmark case because there is a (small) group of individuals receiving no wage income. In the long run, however, the Gini coefficient is not affected by unemployment since relative income of low-skilled labor converges to zero irrespective of whether these individuals are employed or not. The bottom panel shows that unemployment actually declines mildly during the 20th century, indicating that the effect of reduced low-skilled labor supply (the inward movement of the $\overline{L}_L$-curve in Figure 6) dominates the effect of reduced labor demand due to technological progress and rising reference levels of the fair wage. Eventually, however, before labor supply becomes stationary, unemployment begins to rise as the level of TFP and wages of high-skilled individuals increase further.

We next reconsider the effects of redistributive taxation and assume that individuals compare gross wages and not their disposable income after redistribution. We assume that all transfers go to low-skilled labor and again consider a 10 percent robot tax (dashed, green lines) and a 10 percent labor income tax (dash-dotted, red lines). The response of TFP growth and education to redistribution is familiar and is analyzed in detail in the discussion of Figure 4. Here, it is interesting to see that redistribution increases unemployment. This response is not driven by labor supply considerations of low-skilled individuals because transfers are granted irrespective of the employment status and labor supply is inelastic by assumption. Instead, low-skilled unemployment rises because of the depressing effect of redistribution on education. As a result of reduced high-skilled labor supply, high-skilled wages are higher at any level of technology, and thus low-skilled individuals consider a higher wage as fair. In order to elicit full effort, firms pay higher wages and reduce labor demand. In contrast to the case without redistribution, unemployment is increasing during the 20th century.

6. Conclusion

In this paper, we proposed a model of endogenous technological progress and economic growth according to which R&D-based innovations in machine technology lead to more automation, a higher skill premium, and more inequality in terms of income and wealth. The model predicts that more sophisticated technology induces more education but only to a certain degree because, eventually, some individuals will be left behind who do not manage to obtain higher education (a college degree) due to ability constraints. The feature that low-skilled labor does not benefit from automation creates rising inequality because the wages of high-skilled individuals increase at the rate of technological progress. Considering the other big race mentioned in the introduction, the model suggests that it could be difficult and eventually impossible to "run with the machine" instead of against it (as suggested by Brynjolfsson and McAfee, 2011).

Similar to the related R&D-based growth literature, we focused on the manufacturing sector, which, in principle, leaves the loophole that non-routine, low-skilled labor finds employment in an expanding service sector. However, one could argue that the service sector is included in the reduced-form of our model. According to the aggregate production function, low-skilled labor is not benefiting from automation, but it is also not made redundant by the arrival of new machines. It could be conceptualized as moving to different tasks. Moreover, in general equilibrium, low-skilled labor benefits indirectly from technological progress. Skill-biased technological progress induces more higher education in the population, and thus a relative decline of low-skilled labor supply and an increase of low-skilled wages. Of course, this positive indirect effect vanishes when technology no longer triggers increasing higher education. A more serious simplification in our model is perhaps the assumption that (at least some) high-skilled labor is non-automatable. For future research, the model could be generalized by assuming that more recent vintages of machines are able to substitute, to an increasing degree, high-skilled labor.

In our overlapping generations setting, increasing wage inequality explains a secular decline of the aggregate savings rate and a secular increase of wealth inequality. These effects are stronger at higher rates of technological progress. Our theory therefore refutes the view that high economic growth is conducive to lower inequality (Piketty 2014). We have also shown that it is difficult to improve income of low-skilled individuals as long as both technology and education are endogenous. This is true irrespective of whether redistribution is financed by progressive wage taxation (which reduces higher education and growth through lower high-skilled labor supply) or by a robot tax (which reduces demand for machines and growth through less R&D). Only when higher education is stationary, does redistribution unambiguously benefit the poor. Designing redistribution policies that circumvent the repercussions through adjustments of education and technology appears to be a serious challenge for the future.

In an extension of the model, we considered the impact of automation on involuntary unemployment of low-skilled workers through fair wage constraints. This approach appears to be natural when automation benefits only one group at the workplace. If workers refuse to exert full effort when they are not allowed to share in the gains from technological progress, unemployment results. Interestingly, as long as higher education is non-stationary, technological progress does not necessarily lead to more technological unemployment. The reason is that it also triggers more higher education, and thus reduces the low-skilled workforce. For brevity, we focused on the integrated equilibrium of the fair wage model. In future work, it could be interesting to investigate segregated equilibria as well, i.e., conditions where automation under consideration of fair wage and effort concerns motivates firms to (completely) abandon the employment of low-skilled labor. It would also be interesting to investigate a case where workers compare disposable income rather than wages, which could motivate the introduction of a universal basic income or other redistribution schemes from the winners to the losers of the race between education and technology.

Appendix

Proof of Lemma 2. To analyze the impact of education on R&D and economic growth we re-write (22) as the implicit function

$$F \equiv \frac{\bar{R}}{\alpha}\left[\frac{\alpha^2}{\bar{R}(1+\tau_R)}\right]^{\frac{1}{1-\alpha}}(L_{H,t}-L_{H,A,t})\frac{\delta A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}} - \left(\frac{L_{L,t}}{L_{H,t}-L_{H,A,t}}\right)^{\alpha} - \tilde{A}_t\left[\frac{\alpha^2}{\bar{R}(1+\tau_R)}\right]^{\frac{\alpha}{1-\alpha}} = 0.$$

Differentiation of F provides and $\partial F/\partial L_{H,A,t} < 0$ and

$$\begin{aligned}
\frac{\partial F}{\partial(1+\tau_R)} &= -\frac{1}{1-\alpha}\frac{1}{1+\tau_R}\frac{\bar{R}}{\alpha}\left[\frac{\alpha^2}{\bar{R}(1+\tau_R)}\right]^{\frac{1}{1-\alpha}}(L_{H,t}-L_{H,A,t})\frac{\delta A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}} \\
&+ \frac{\alpha}{1-\alpha}\frac{1}{1+\tau_R}\tilde{A}_t\left[\frac{\alpha^2}{\bar{R}(1+\tau_R)}\right]^{\frac{\alpha}{1-\alpha}} \\
&< -\frac{1}{1-\alpha}\frac{1}{1+\tau_R}\left\{\frac{\bar{R}}{\alpha}\left[\frac{\alpha^2}{\bar{R}(1+\tau_R)}\right]^{\frac{1}{1-\alpha}}(L_{H,t}-L_{H,A,t})\frac{\delta A_{t-1}^{\phi}}{L_{H,A,t}^{1-\lambda}} - \tilde{A}_t\left[\frac{\alpha^2}{\bar{R}(1+\tau_R)}\right]^{\frac{\alpha}{1-\alpha}}\right\} \\
&= -\frac{1}{1-\alpha}\frac{1}{1+\tau_R}\left(\frac{L_{L,t}}{L_{H,t}-L_{H,A,t}}\right)^{\alpha} < 0
\end{aligned}$$

and thus, by the implicit function theorem, $\mathrm{d}L_{H,A,t}/\mathrm{d}\tau_R < 0$. Then, from (10) and (11), $\mathrm{d}g_{A,t}/\mathrm{d}\tau_R < 0$.

References

Abeliansky, A., Martinez-Zarzoso, I., and Prettner, K. (2015). The Impact of 3D Printing on Trade and FDI. cege Discussion Paper 262.

Abeliansky, A. and Prettner, K. (2017). Automation and demographic change. Hohenheim Discussion Papers in Business, Economics, and Social Sciences. Discussion Paper 05-2017.

Acemoglu, D. (2002). Technical change, inequality, and the labor market. *Journal of Economic Literature* 40(1), 7-72.

Acemoglu, D., and Autor, D. (2012). What Does Human Capital Do? A Review of Goldin and Katz's The Race between Education and Technology. *Journal of Economic Literature* 50(2), 426-463.

Acemoglu, D. and Restrepo, P. (2016). The race between machine and man: Implications of technology for growth, factor shares and employment. National Bureau of Economic Research. Working Paper w22252.

Acemoglu, D. and Restrepo, P. (2017). Robots and Jobs: Evidence from US Labor Markets. National Bureau of Economic Research. Working Paper w23285.

Aghion, P. and Howitt, P. (2005). *Handbook of Economic Growth, Volume 1A*, chapter 2: "Growth with Quality-Improving Innovations: An Integrated Framework", pages 68–110.

Akerlof, G.A. and Yellen, J.L. (1990). The fair wage-effort hypothesis and unemployment. *Quarterly Journal of Economics* 105(2), 255-283.

Arntz, M., Gregory, T., and Zierahn, U. (2016). The risk of automation for jobs in OECD countries: A comparative analysis. OECD Social, Employment, and Migration Working Papers (189).

Autor, D. H. and Dorn, D. (2013). The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market. *American Economic Review* 103(5), 1553–1597.

Berman, E., Bound, J., and Machin, S. (1998). Implications of skill-biased technological change: international evidence. *Quarterly Journal of Economics* 113(4), 1245-1279.

Bernstein, J. (2017). Before you blame the robots, look to the policy (and the data). On the Economy 8 February 21, 2017.

Brynjolfsson, E. and McAfee, A. (2011). *Race against the machine.* Digital Frontier, Lexington, MA.

Brynjolfsson, E. and McAfee, A. (2016). *The Second Machine Age: Work, Progress, and Prosperity in a Time of Brilliant Technologies.* Norton & Company.

Cervellati, M. and Sunde, U. (2005). Human capital formation, life expectancy, and the process of development. *American Economic Review* 95(5), 1653–1672.

Frey, C.B. and Osborne, M.A. (2017). The future of employment: how susceptible are jobs to computerisation? *Technological Forecasting and Social Change* 114, 254-280.

Galor O. (2005). From Stagnation to Growth: Unified Growth Theory in Aghion, P. and Durlauf, S. Handbook of Economic Growth, 171-293.

Galor, O., 2011, *Unified Growth Theory*, Princeton University Press.

Galor O. and Moav, O. (2002). Natural selection and the origin of economic growth. *Quarterly Journal of Economics* 117, 1133-1191.

Galor, O. and Weil, D. (2000). Population, technology, and growth: from Malthusian stagnation to the demographic transition and beyond, *American Economic Review* 90, 806-828.

Gasteiger, E. and Prettner, K. (2017). A Note on Automation, Stagnation, and the Implications of a Robot Tax. Economics Discussion Paper 17/2017, Free University of Berlin. Available at `http://edocs.fu-berlin.de/docs/receive/FUDOCS_document_000000027258`.

Goldin, C. and Katz, L.F. (2009). *The Race Between Education and Technology.* Harvard University Press.

Graetz, G. and Michaels, G. (2015). Robots at Work. CEPR Discussion Paper 10477.

Groth, C., Koch, K.J., and Steger, T.M. (2010). When economic growth is less than exponential. *Economic Theory* 44(2), 213-242.

Guerreiro, J., Rebelo, S., and Teles, P. (2017). Should Robots be Taxed? w23806 National Bureau of Economic Research. Working Paper w23806.

Hémous, D. and Olsen, M. (2016). The Rise of the Machines: Automation, Horizontal Innovation and Income Inequality. Mimeo.

Jones, C. I. (1995). R&D-based models of economic growth. *Journal of Political Economy* 103(4), 759–783.

Jones, C.I. (2002). Sources of US economic growth in a world of ideas. *American Economic Review* 92(1), 220-239.

Jones, C.I. (2005). The Shape of Production Functions and the Direction of Technical Change. *Quarterly Journal of Economics* 120(2), 517–549.

Karabarbounis, L. and Neiman, B. (2014). The Global Decline of the Labor Share. *The Quarterly Journal of Economics* 129(1), pages 61–103.

Murray K. (2016). Wage Stagnation. SAGE Business Researcher. Url: `http://businessresearcher.sagepub.com/sbr-1775-101222-2760442/20161107/wage-stagnation`

Peretto, P.F. and Seater, J.J. (2013). Factor-eliminating technical change. *Journal of Monetary Economics* 60(4), 459-473.

Piketty, T. (2014). *Capital in the Twenty-First Century.* The Belknap Press of Harvard University Press.

Prettner, K. (2014). The non-monotonous impact of population growth on economic prosperity. *Economics Letters* 124:93–95.

Prettner, K. (2017). A note on the implications of automation for economic growth and the labor share. *Macroeconomic Dynamics* (forthcoming).

Prettner, K. and Strulik, H. (2017). The Lost Race Against the Machine: Automation, Education, and Inequality in an R&D-based Growth Model (with Klaus Prettner), Hohenheim Discussion Papers in Business, Economics and Social Sciences, 08-2017.

Romer, P. (1990). Endogenous technological change. *Journal of Political Economy* 98(5), 71–102.

Sachs, J. D., Benzell, S. G., and LaGardia, G. (2015). Robots: Curse or blessing? A basic framework. NBER Working Paper 21091.

Sachs, J. D. and Kotlikoff, L. J. (2012). Smart machines and long-term misery. NBER Working Paper 18629.

Segerström, P. S. (1998). Endogenous growth without scale effects. *American Economic Review* 88(5), 1290–1310.

Shiller, R.J. (2017). Robotization Without Taxation? Project Syndicate 3/26/2017.

Steigum, E. (2011). *Frontiers of Economics and Globalization: Economic Growth and Development*, chapter 21: Robotics and Growth, pages 543–557. Emerald Group.

Strulik, H. (2005). The role of human capital and population growth in R&D-based models of economic growth. *Review of International Economics* 13(1), 129–145.

Strulik, H., Prettner, K., and Prskawetz, A. (2013). The past and future of knowledge-based growth. *Journal of Economic Growth* 18(4), 411-437.

Strulik, H. and Weisdorf, J. (2008). Population, food, and knowledge: a simple unified growth theory. *Journal of Economic Growth* 13(3), 195.

Tinbergen, J. (1975). Income Distribution, Amsterdam: North-Holland Publishing Company BV.

The United States Patent and Trademark Office (2017). Url: http://www.uspto.gov [accessed on March 4 2017].

U.S. Census (2015). U.S. Census Bureau, Current Population Survey, Annual Social and Economic Supplements, Income Gini Ratio of Families by Race of Householder, All Races.

Young, A. (1998). Growth without scale effects. *Journal of Political Economy* 106(5), 41–63.

Zeira, J. (2006). Machines as Engines of Growth. Mimeo.