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HYPERBOLIC DISCOUNTING AND THE TIME-CONSISTENT SOLUTION OF THREE CANONICAL ENVIRONMENTAL PROBLEMS

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Hyperbolic Discounting and the Time-Consistent Solution of Three Canonical Environmental Problems*

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Abstract. In this paper I propose a time-consistent method of discounting hyperbolically that contains the discount rate implied by Gamma discounting as a special case. I apply the discounting method to three canonical environmental problems: (i) optimal renewable resource use, (ii) the tragedy of the commons, (iii) economic growth and pollution. I then compare results with those for conventional exponential discounting using the normalization that both methods provide the same present value of an infinite constant flow. I show that, irrespective of potentially high initial discount rates, time-consistent hyperbolic discounting leads always to a steady state of maximum yield, or, if the environment enters the utility function, a steady state where the Green Golden Rule applies. While (asymptotic) extinction is a real threat under exponential discounting it is impossible under time-consistent hyperbolic discounting. This result is also confirmed for open access resources. In a model of economic growth and pollution, hyperbolic discounting establishes the Golden Rule of capital accumulation and the Modified Green Golden Rule.

Keywords: discounting, time-consistency, renewable resource use, property rights, growth, pollution.

JEL: D60, D90, Q20, Q50, Q58, O40.

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1. Introduction

Economists and natural scientists agree that the appropriate choice of the discount rate is of preeminent importance in the evaluation of policies that matter for the distant future. Environmental policies are perhaps the best example for decision problems in which the costs accrue from today onwards and most of the benefits occur in the distant future. For example, any potential disagreement in the assessment of the material effects of global warming is dwarfed by the impact of the discounting method on the implied net present value in monetary terms (Weitzman, 2007; Nordhaus, 2007; Dasgupta, 2008).

Conventional economic analysis is based on exponential discounting, i.e. it applies a constant discount rate. This assumption is problematic for at least four reasons. First, one could argue with Stern (2008) that the discounting of utility experienced by future generations is unethical. The fact that, simultaneously, it appears to be fully rational for the citizens alive today to discount income streams in the near future leads immediately to the conclusion of declining discount rates. Second, while it is long accepted that individuals are present-biased and apply declining discount rates to the near future (Frederick et al., 2002), recent empirical evidence from the housing market suggests that hyperbolic discounting is also applied to payoffs in the distant future (Giglio et al., 2015). Third, if standard economics of intertemporal calculus based on a constant discount rate are extended by uncertainty concerning either future interest rates or future growth rates, one arrives axiomatically at the conclusion that discounting should be hyperbolic (Weitzman, 2007). Fourth, if experts disagree about the "correct" constant rate of discounting, the discount rate implied by the average discount factor declines over time (Weitzman, 2001). For these reasons (and others) a panel of leading economists in environmental economics recently came to the conclusion that the discounting of costs and benefits of long-horizon projects should best be done at a hyperbolic rate (Arrow et al., 2014).

An important concern, however, is that hyperbolic discounting leads to suboptimal and inconsistent decision making. In fact, the introduction of hyperbolic discounting in the economics of psychology is motivated by the need to explain preference reversals and time-inconsistent behavior (Frederick et al., 2002). Consequently, scholars tend to believe that hyperbolic discounting inevitably implies an inconsistency problem.[1] Applying hyperbolic discounting to environmental

[1]For example, Angeletos et al. (2001, p. 54) write that "When a household has a hyperbolic discount function, the household will have dynamically inconsistent preferences, so the problem of allocating consumption over time cannot be treated as a straightforward optimization problem"

decision making would then entail a continuous reversal of previous plans and suboptimal outcomes (Winkler, 2006). For example, Hepburn et al. (2010) show that naive social planners who apply hyperbolic discounting without being aware of the time-inconsistency problem can manage renewable resources into extinction. Alternatively, it has been suggested that sophisticated social planners, who are aware of the time-inconsistency problem, could achieve consistent decisions by using Markovian strategies in a dynamic game with (not yet existent) future planners (Karp, 2005). This solution, however, is not entirely satisfying either. The achieved Nash equilibrium is not pareto-optimal and, compared to a simple discounting rule, a Markovian strategy seems to be difficult to implement.

In this paper, I suggest a different solution to the problem and propose the use of *time-consistent* hyperbolic discounting. Many economists know the result from Strotz' (1956) seminal paper that only exponential discounting leads to time-consistent decisions if the discount factor is a function of the algebraic distance $(t_0 - t)$ between planning time t_0 and payoff time t. The "if"-clause, however, has sometimes been forgotten in the following literature such that the conventional wisdom evolved that non-exponential discounting necessarily entails time inconsistency. Here, I suggest a form of hyperbolic discounting to which the theorem of multiplicative separability in decision time and payoff time applies (see Burness, 1976; Drouhin, 2015). As a result, decisions are time-consistent.

Time-consistent hyperbolic discounting got so far relatively little attention in environmental economics. A notable exception is Heal (2005) who discusses a form of multiplicatively separable discounting (logarithmic discounting) and praises its attractive properties (p. 1125, p. 1138). Notwithstanding this prominent "advertisement" there are, to the best of my knowledge, no applications of time-consistent hyperbolic discounting in environmental economics. Here, I fill this lacuna and solve three canonical environmental problems for the case of time-consistent hyperbolic discounting: the optimal use of a renewable resource, the non-cooperative use of an open access resource, and the problem of optimal growth and pollution.

Time-consistent hyperbolic discounting, in contrast to time-inconsistent behavior, leads to quite benign environmental outcomes. At the steady state, it establishes the social optimum for closed access resource management (maximum sustainable yield, Green Golden Rule). I also compare with the solution obtained for exponential discounting. For that purpose I use the normalization that both discounting methods provide the same present value of a constant flow

(Myerson et al., 2001). This allows me to disentangle the effects due to the discounting method from the effects due to impatience as such. As a rule, I find that time-consistent hyperbolic discounting outperforms exponential discounting at the steady state and off the steady state. In particular, there exist situations in which exponential discounting leads to extinction while hyperbolic discounting leads always to sustainable and socially optimal outcomes.

It is perhaps less of a surprise that present-biased hyperbolic discounting performs well at or close to the steady state since, by design, the impact of discounting vanishes at the steady state. However, the result that present-bias turns out to be good for the environment when the economy starts far off the steady state is remarkable. After all, for the near future, hyperbolic discount rates exceed (by far) the present-value equivalent constant discount rate. Seemingly, they should thus lead to overexploitation of the resource. The intuition for the non-obvious result of sustainability lies in the inherent dynamics of the resource stock and the discount rate. The first order conditions exclude extinction in finite time as an optimal outcome (intuitively, utility would jump to minus infinity at the point of extinction of the resource). Thus, only asymptotic extinction could be an optimal outcome. When the discount rate declines hyperbolically, however, there exists always a finite point in time at which the discount rate falls short of any positive natural growth rate of the resource, a condition that ensures sustainability. With further declining discount rate, any resource-management strategy converges towards the long-run social optimum. For conventioal exponential discounting, however, asymptotic extinction is a real threat, in particular for common access resources. I explain these features with the help of phase diagrams and numerical implementations of the models.

The conclusion that hyperbolic discounting can be time-consistent and that its application leads to the long-run social optimum has policy relevance since a discounting rule can be easily implemented as an institution. In fact, declining discount rates are already applied in the evaluation of public projects in the United Kingdom and France (HM Treasury, 2003; Lebegue, 2005). Concrete economic and environmental policies, in contrast, are harder to govern by institutions and may change from day to day, the specific problem at hand, and the responsible policymakers (see Acemoglu et al., 2005 on the importance of institutions vs. policy). This conclusion seems to be of particular importance for the management of open access resources where non-cooperative strategies are likely. It is thus reassuring that the implementation of a hyperbolic discounting rule, given its multiplicative separability, will lead to time-consistent decisions. This means that it will not cause future policy reversals and that there is no need for commitment or the sophistication of strategies of the current policymakers.

The papers is organized as follows. In the next Section I introduce time-consistent hyperbolic discounting. In Section 3 I apply the method to the optimal management of a renewable resource. In Section 4 I discuss the exploitation of an open access resource under hyperbolic discounting and in Section 5 I discuss the problem of optimal economic growth and pollution. For all cases I illustrate off-steady-state behavior with numerical examples and establish the superiority of time-consistent hyperbolic discounting vis a vis conventional exponential discounting. Section 6 concludes the paper.

2. Welfare and Discounting

Consider a social planner who maximizes welfare understood as utility from consumption $u(c)$ experienced over an infinite time horizon and discounted to the present.

$$W(t_0) = \int_{t_0}^{\infty} D(t_0, t) \cdot u(c(t)) \mathrm{d}t, \tag{1}$$

with the discount factor $D(t_0, t)$ depending on calender time t and decision time t_0. As a reference case we consider conventional exponential discounting at a constant rate such that $D(t, t_0) = e^{-\bar{\rho}(t-t_0)}$. The main focus, however, is on hyperbolic discounting where the discount factor is given by the following multiplicatively separable function of decision time and calender time:

$$D(t_0, t) = \left(\frac{1 + \alpha t_0}{1 + \alpha t} \right)^{\beta}, \qquad \beta > 1. \tag{2}$$

Notice that any sequence of social planners starts with a discount factor of one at their individual decision time t_0 and that the discount factor declines hyperbolically with calender time. For the special case where $\alpha = \beta = 1$ the discount factor is equal to the one proposed by Ainslie (1975). The parameters α and β are useful to calibrate discounting. Below we consider, for example a calibration supported by Weitzman's (2001) Gamma discounting approach. The discount rate is defined as $\rho(t) = -(\mathrm{d}D/\mathrm{d}t)/D =$ and it is obtained as

$$\rho(t) = \begin{cases} \frac{\alpha\beta}{(1+\alpha t)} & \text{for hyperbolic discounting} \\ \bar{\rho} & \text{for exponential discounting.} \end{cases} \tag{3}$$

Notice that the hyperbolic discount rate does not depend on decision time t_0. It declines in calender time t and a larger α or β implies a higher discount rate at any time. Since the discount rate is independent from decision time, decisions are time-consistent. The crucial feature that provides time consistency is that the discount factor is multiplicatively separable in decision time and calender time (Drouhin, 2015; Burness, 1976).

In order to make inferences, welfare has to be finite, which in turn requires $\beta > 1$. To see this, assume that a steady state of constant utility exists. Then, for $\int_{t_0}^{\infty} \left(\frac{1+\alpha t_0}{1+\alpha t}\right)^{\beta} \mathrm{d}t = \frac{(1+\alpha t_0)^{\beta}}{\alpha(\beta-1)} \left(\frac{1}{1+\alpha t}\right)^{\beta-1} |_{t_0}^{\infty}$ to be finite we need $\beta > 1$. The feature that the utility integral is finite although the discount rate vanishes in the long-run simplifies the following analysis. It allows to apply standard optimization techniques and avoids the use of less-well known methods like the catching-up criterion (Seierstad and Sydsaeter, 1987).

For a fair comparison between exponential and hyperbolic discounting we apply the equivalent-present-value argument made by Myerson et al. (2001); see also Caliendo and Findlay (2014) and Strulik (2015). This means that the value of the discounting parameters is determined such that a constant infinite stream (of, for example, utility) provides the same present value. The requirement $\int_0^{\infty} e^{-\bar{\rho}(t-t_0)}\mathrm{d}t = \int_0^{\infty} \left(\frac{1+\alpha t_0}{1+\alpha t}\right)^{\beta} \mathrm{d}t$ leads to the solution

$$\bar{\rho} = \alpha(\beta - 1). \tag{4}$$

The condition of a finite present value of an infinite utility stream is thus equivalent to the condition that the associated equivalent constant time preference rate is positive ($\beta > 1$). The normalization of parameters is needed in order to disentangle the effect of the discounting method (constant versus hyperbolically declining) from pure impatience. Here we are interested in how the *hyperbolic decline* of the discount rate affects the management of a renewable resource.

As discussed in the Introduction, Weitzman (2001) has shown that individual disagreement (or uncertainty) about the correct constant discount rate leads in the aggregate to a hyperbolically declining discount rate. The Weitzman case is represented by the calibration $\alpha = \sigma^2/\mu$ and $\beta = \mu^2/\sigma^2$ which leads to the discount rate

$$\rho(t) = \frac{\mu}{1 + \frac{\sigma^2}{\mu} \cdot t}, \tag{5}$$

where μ is the mean and σ the standard deviation of the individual discount rates. Weitzman's estimates of $\mu = 0.04$ and $\sigma = 0.03$, imply $\beta = 1.77$ and thus supports a finite utility integral.

3. Optimal Harvesting of a Renewable Resource

We first consider renewable resource growth according to the Verhulst (1938) model (see e.g. Wilen, 1985). This means that absent any harvesting (or pollution) the resource stock (or environmental quality), denoted by x, grows logistically until it reaches its carrying capacity κ. The change of the resource stock is given by $g(x) = rx(1 - x/\kappa)$ where r denotes the maximum natural growth rate. Notice that the maximum sustainable yield from the resource is attained where $g(x)$ reaches a maximum, i.e. where $g_x = 0$.[2] Consumption c is equal to the harvesting rate such that the change of stock is obtained as

$$\dot{x} = rx\left(1 - \frac{x}{\kappa}\right) - c. \tag{6}$$

The social planner maximizes (1) subject to (2) and (6), given the initial stock x_0 and the non-negativity conditions, $x \geq 0$, $c \geq 0$. The associated Hamiltonian function is given by

$$H = \left(\frac{1+\alpha t_0}{1+\alpha t}\right)^{\beta} u(c) + \lambda\left[rx(1 - x/\kappa) - c\right],$$

in which λ is the shadow price of the resource. The first order conditions are:

$$\left(\frac{1+\alpha t_0}{1+\alpha t}\right)^{\beta} u_c = \lambda, \tag{7}$$

$$\lambda g_x = -\dot{\lambda}. \tag{8}$$

Differentiating the first condition with respect to time and eliminating λ and $\dot{\lambda}$ leads to the following consumption rule:

$$\dot{c} = \frac{c}{\eta}\left(g_x - \frac{\alpha\beta}{1+\alpha t}\right), \tag{9}$$

in which $\eta \equiv -(\partial^2 u/\partial c^2)c/(\partial u/\partial c)$ is the elasticity of marginal utility, i.e. the inverse of the elasticity of intertemporal substitution. Without loss of generality we assume that η is constant. Notice that by log-differentiating of (7) the consumption rule is independent from decision time t_0, implying that consumption plans are time-consistent. This is a natural outcome of multiplicative separability. Optimal resource management is determined by resource growth (6) and consumption growth (9), together with the transversality condition $\lim_{t\to\infty} \lambda(t)x(t) = 0$. Consumption growth is high when the return on investment in the resource stock, i.e. the return

[2] For any function $y(x, z)$, I use the shorthand notation $y_x \equiv \partial y(x, z)/\partial x$. I also omit the time index whenever it is not needed for clarification.

on foregone consumption, reflected by the first term in parenthesis in (9), is high or when the discount rate, reflected by the second term, is low.

Setting $\dot{c} = \dot{k} = 0$ in (6) and (9) and applying the non-negativity constraints, we obtain the trivial steady state, $(c = 0, k = 0)$, and a unique positive steady state at:

$$x = x^* \equiv \kappa/2, \qquad c = c^* \equiv \frac{r\kappa}{4}. \tag{10}$$

Here we have made use of the fact that the discount rate declines to zero in the long run, $\lim_{t\to\infty} \rho(t) = 0$. Performing the same analysis for the case of exponential discounting, we arrive at the consumption rule

$$\dot{c} = \frac{c}{\eta}\left(g_x - \bar{\rho}\right), \tag{11}$$

and the steady state

$$x = x^{**} \equiv \left(\frac{r-\bar{\rho}}{2r}\right) k, \qquad c = c^* \equiv \frac{(r^2-\bar{\rho}^2)\kappa}{4r}. \tag{12}$$

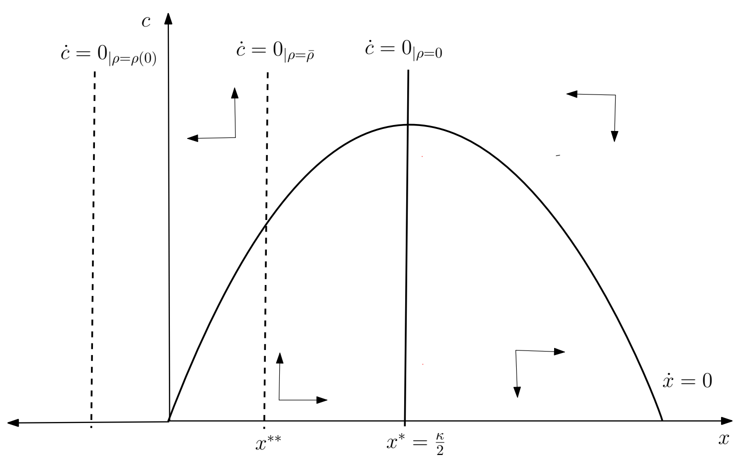
Comparing the solutions (10) and (12) and recalling that $g_x = 0$ where $x = \kappa/2$ verifies the following result.

Proposition 1. *Time-consistent hyperbolic discounting leads to steady-state consumption of the maximum sustainable yield. Exponential discounting leads to steady-state consumption of less than the maximum yield. If $\bar{\rho} > r$ there exists no sustainable steady state for exponential discounting. There exists always a sustainable steady state for hyperbolic discounting.*

Notice that this result is not due to impatience as such since we normalized the discount rates. The result reflect just the impact of the discounting methods. Intuitively, hyperbolic discounting gives more weight to the far future, which is when the steady state is (asymptotically) reached. Thus, if a steady state exists, harvesting less than maximum yield is not an expression of impatience but an expression of applying a constant discount rate.

We next consider off-steady consumption and the possibility of extinction. Inspection of (12) shows that exists no positive steady-state under exponential discounting if the discount rate exceeds the maximum natural growth rate (i.e. for $\bar{\rho} > r$). It is thus tempting to believe that extinction is more frequent under hyperbolic discounting since initially the (very) high discount rate exceeds $\bar{\rho}$. However, the opposite is true. To see this, consider the phase diagram in Figure 1.A. The $\dot{x} = 0$–isocline is given where $c = g(x)$ and displayed as the hump-shaped curve. The resource declines above the isocline and grows below it. The $\dot{c} = 0$–isocline is a horizontal line through $x = \kappa(r - \rho(t))/(2r)$. Consumption grows to the left of the isocline and declines to the right of it. The resulting arrows of motion are depicted in Figure 1.A. They show that the intersection of the isoclines is a saddlepoint.

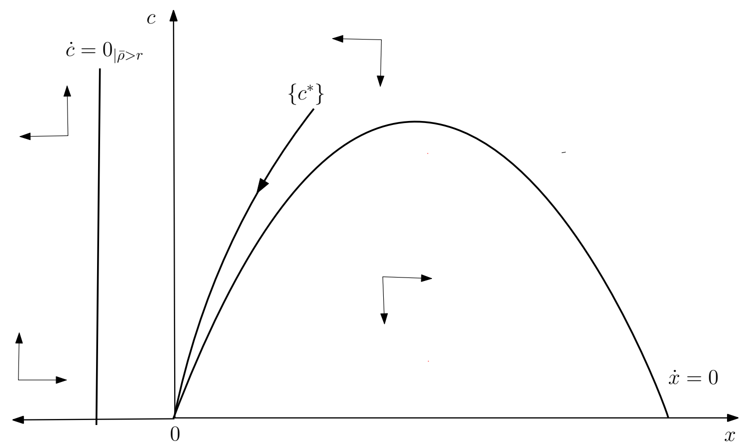
Figure 1.A: Hyperbolic Discounting and Optimal Renewable Resource Use



Strictly speaking, however, the $\dot{c} = 0$–isocline cannot be depicted in a phase diagram because it is a non-autonomous equation. As time proceeds and the discount rate declines, $x = \kappa(r - \rho(t))/(2r)$ grows and the isocline moves to the right. Figure 1 shows an initial isocline where $\rho(0)$ exceeds r, indicating an unsustainable situation. However, as time proceeds, the isocline shifts to the right and, once it crosses the ordinate, sustainability is ensured. With further proceeding time, the isocline moves to $x = \kappa/2$ and consumption converges towards the maximum sustainable yield.

Figure 1.B illustrates the case of asymptotic extinction for exponential discounting. In this case, $\bar{\rho} > r$ and the $\dot{c} = 0$–isocline lies to the left of the c-axis. Aside from the trajectory that goes through the origin, all other trajectories hit either the c-axis or the x-axis in finite time. Any trajectory that hits the c-axis is suboptimal because it leads to a jump of c when the resource gets extinct and $c \geq 0$ becomes binding with equality (the trajectory violates the Weierstrass-Erdmann condition). Any trajectory that hits the x-axis is suboptimal because it violates the transversality condition. To see this note that at the intersection $c = 0$, $x > 0$, and $g_x < 0$. From (8) the shadow price λ grows at rate $-g_x$. Thus with x constant and λ growing at a positive rate, the transversality condition $\lim_{t\to\infty} \lambda(t)x(t)$ is violated. The outcome that extinction in finite time is suboptimal is very intuitive. Given the arrows of motion in Figure

Figure 1.B: Exponential Discounting and Asymptotic Extinction



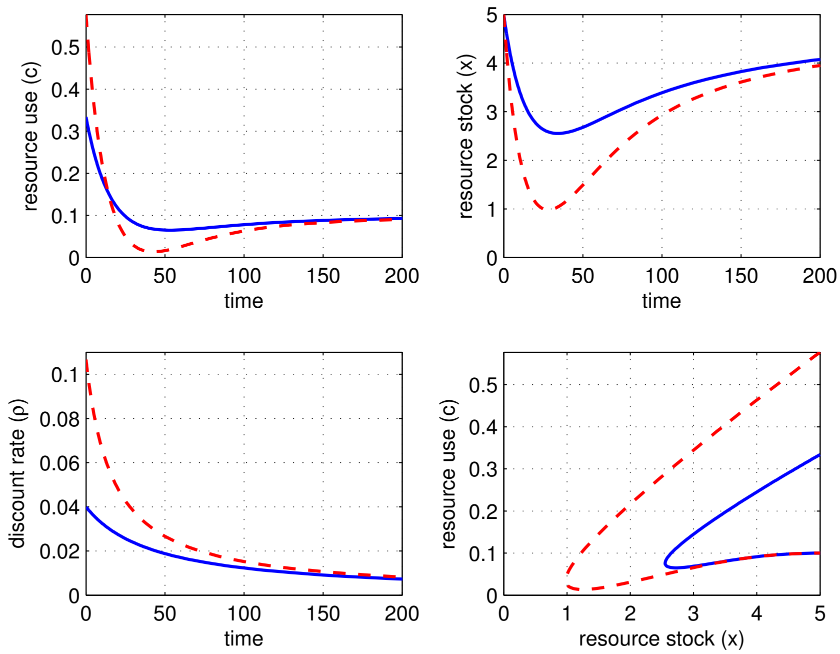
1.B., the only solution left is asymptotic extinction. As the economy moves along a path to the origin (along path $\{c^*\}$ in Figure 1.B), both x and c vanish in sync sucht that the resource becomes extinct asymptotically. Along the trajectory $g_x > 0$ such that λ declines continuously. With both x and λ converging to zero, the transversality condition is fulfilled.

Figure 1.B. can also be employed to understand why (asymptotic) extinction is impossible under hyperbolic discounting. As long as the $\dot{c} = 0$–isocline lies to the left of the c-axis, the only optimal solution is given by a trajectory like $\{c^*\}$ along which x and c decline at declining rates. However, the origin is reached only in infinite time while the rightward moving $\dot{c} = 0$–isocline crosses the c-axis in finite time at the moment when $\rho(t)$ falls short of r. This means that there exists always a finite moment of time at which an initially unsustainable harvesting policy becomes sustainable. Then, the rightward shift of the $\dot{c} = 0$–isocline continues until it comes to a rest at the long-run optimum x^* in Figure 1. The following proposition summarizes these observations.

Proposition 2. *For sufficiently high discount rate, $\bar{\rho} > r$, exponential discounting implies (asymptotic) extinction. Extinction is impossible under time-consistent hyperbolic discounting irrespective of (potentially large) initial discount rates. Instead, any optimal harvesting policy converges to the maximum sustainable yield.*

We next illustrate adjustment dynamics with an example.[3] For that purpose I set $r = 0.04$, $\kappa = 10$ and $\eta = 2$. As a benchmark case, we consider a calibration according to Weitzman (2001). Using (5), I set $\mu = 0.04$ and $\sigma = 0.03$ (i.e. $\alpha = 0.0225$ and $\beta = 1.777$) Using (4), the equivalent constant discount rate is obtained as $\bar{\rho} = 0.0175$. Blue (solid) lines in Figure 2 show the optimal trajectories for the benchmark hyperbolic case. The discount rate declines continuously from 4 percent to zero (lower left panel). Consumption c is initially high due to the high present bias and then declining towards its steady state level (upper left panel). The resource stock first declines due to the high initial consumption and then recovers gradually to the level of $\kappa/2 = 5$ (upper right panel) where consumption equals the maximum sustainable yield of 0.1 (upper left panel).

Figure 2: Hyperbolic Discounting: Adjustment Paths



Parameters: $r = 0.04$; $\kappa = 10$; $\eta = 2$. Blue (solid) lines: discount rate from Weitzman (2001): $\mu = 0.04$, $\sigma = 0.03$ (i.e. $\alpha = 0.0225$, $\beta = 1.777$). Red (dashed) lines: higher initial discount rate $\alpha = 0.06$ (implying unsustainability for exponential discounting at $\bar{\rho} = \alpha(\beta - 1)$). The lower right panel shows the phase diagram.

Now consider an increase of α to 0.06. Applying (5), the equivalent constant discount rate is obtained as $\bar{\rho} = 0.0467$. This value exceeds the maximum natural growth rate ($r = 0.04$) and thus extinction cannot be avoided when a constant discount rate is applied. The resulting trajectories for hyperbolic discounting are shown by red (dashed) lines in Figure 2. Although

[3]For the solution of this problem and all further numerical experiments in the paper I use the relaxation algorithm of Trimborn et al. (2008).

the social planner initially uses a very high discount rate of almost 11 percent, extinction is avoided. Before the resource is completely exploited, the discount rate has fallen below the natural growth rate (lower left panel) and the resource starts growing again (upper right panel).

Finally, we extend the model by considering utility experienced from the resource stock (capturing, for example an interest in species diversity or a clean environment). The welfare functional (1) is replaced by $W(t_0) = \int_{t_0}^{\infty} D(t_0, t)u(c, x)\mathrm{d}t$ and maximization with respect to (2) and (6) provides the costate equation,

$$\left(\frac{1+\alpha t_0}{1+\alpha t}\right)^{\beta} u_x\lambda + \lambda g_x = -\dot{\lambda}, \tag{13}$$

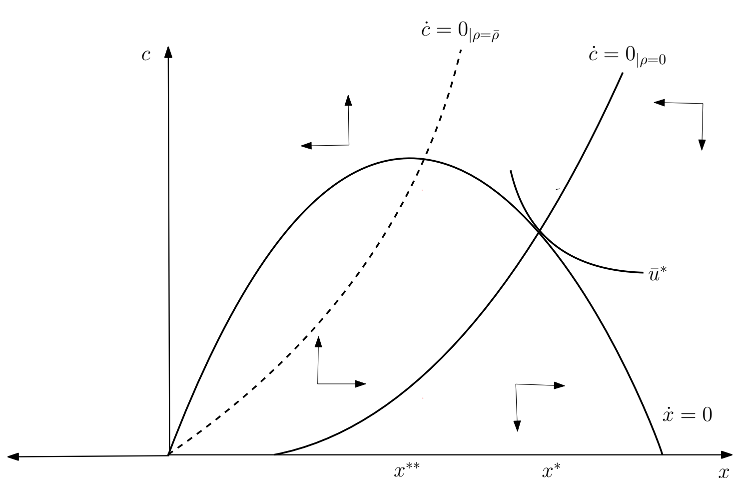
which replaces (8). The first order condition (7) remains valid. Eliminating λ we obtain from (7) and (13):

$$-\frac{\dot{u}_c}{u_c} = \left\{ g_x + \frac{u_x}{u_c} - \frac{\alpha\beta}{1+\alpha t} \right\}. \tag{14}$$

Proposition 3. *Time-consistent hyperbolic discounting establishes the Green Golden Rule as the steady state solution of optimal renewable resource management.*

For the proof we exploit that consumption and the resource stock stay constant at the steady state such that the left hand side of (14) is zero. Then, at the steady state, $g_x = -u_x/u_c = \mathrm{d}c/\mathrm{d}x$. The slope of the resource growth function is tangent to the planners indifference curve at the highest achievable utility $\bar{u}^*$, see Figure 3. This outcome is known as the Green Golden Rule (Heal, 2005).

Figure 3: Hyperbolic Discounting and the Green Golden Rule



To proceed further, assume that utility is separable in c and x and isoelastic, $u = c^{1-\eta}/(1-\eta) + x^{1-\psi}/(1-\psi)$. Then, (14) implies the $\dot{c} = 0$–isocline is given by $c = (2rx/\kappa + \rho(t) - r)^{1/\eta} x^{\psi/\eta}$ with consumption increasing above and falling below the isocline. The $\dot{x} = 0$–isocline remains as for the simple model. We obtain the phase diagram shown in Figure 3. Since x is non-negative, extinction is no threat. As time proceeds and $\rho(t) = \alpha\beta/(1+\alpha t)$ declines, the $\dot{c}$–isocline shifts to the right towards an intersection with the resource curve $g(x)$ where $g(x)$ is tangent to the highest attainable indifference curve $\bar{u}^*$ where the Green Golden Rule holds. Thus, hyperbolic discounting generalizes the Green Golden Rule. For exponential discounting the Green Golden Rule applies only for the knife-edge case of $\bar{\rho} = 0$. For time-consistent hyperbolic discounting the Green Golden Rule applies for any discounting function $\rho(t)$.

4. Hyperbolic Discounting and the Tragedy of the Commons

In this section we explore the use of a renewable resource when property rights are not defined (or cannot be enforced). Suppose that the resource x is harvested by $i = 1, \ldots, n$ agents. The agents are symmetric, i.e. they share all parameters but follow their own resource management calculus. Here we are interesting in the non-cooperative solution (since the cooperative solution coincides with the one for simple model from the previous section). To simplify the problem, we return to the assumption that the resource does not enter the utility function. In a stylized way, the scenario could be interpreted as a game between social planners who agreed on a discounting rule but who cannot commit to any particular policy. As discussed in the Introduction, a discounting rule can be imagined as implementation of a hard-to-change institution while the actual policy (the "daily" use of the resource) cannot be contracted. We are interested in time-consistent policies, such that the planners play Markovian (feedback) strategies $c^i(x)$.[4] Summarizing, planner i maximizes

$$\int_{t_0}^{\infty} D(t_0, t) u(c^i) \qquad s.t. \tag{15}$$

$$\dot{x} = g(x) - \sum_{j=1}^{n} c^j(x). \tag{16}$$

[4]These kind of "fish war" games are based on Levhari and Mirman (1980). See Van Long (2010, Chapter 3) for a recent survey of common-access differential games.

This leads to the following first order condition and costate equation:

$$D(t_0, t)u_c^i = \lambda, \tag{17}$$

$$\lambda \left[g_x - \sum_{j=1, j\neq i}^{n} \frac{\partial c^j}{\partial x} \right] = -\dot{\lambda}. \tag{18}$$

Eliminating the costate variable and applying symmetry, (17) and (18) can be summarized as

$$-\frac{\dot{u}_c}{u_c} = \left\{ g_x - (n-1)\frac{\partial c}{\partial x} - \rho(t) \right\}, \tag{19}$$

in which $c = c^i$ for all $i = 1, \ldots, n$.

Equation (19) is hard to interpret since it involves the solution of a partial differential equation. For a special case, however, we can obtain a closed-form solution for the optimal harvesting strategy. Suppose the utility function is iso-elastic, $u = c^{1-\eta}/(1-\eta)$, and consider the case that $\eta = 2$. Fortunately, this assumption has empirical relevance. A recent meta-analysis of 2735 published estimates of the intertemporal elasticity of substitution found the world average of η at 2.0 (Havranek et al., 2015). As shown in the Appendix, for $\eta = 2$ we obtain the optimal consumption strategy

$$c(x) = \left(\frac{r + \rho(t)}{n+1} \right) x, \tag{20}$$

with $\rho(t) = \alpha\beta/(1 + \alpha t)$ in the case of hyperbolic discounting and $\rho(t) = \bar{\rho}$ in the case of exponential discounting. As shown in (20), the consumed share of the resource is large if the resource regenerates quickly (for high r) or if individuals discount at a high rate. Comparing discounting methods we see that exponentially discounting individuals consume a constant share of the resource at all times whereas hyperbolically discounting individuals consume a declining share of the resource.

The consumed share of the resource declines in the number of competing players but the totally consumed resource, nc, increases in the number of players. Two players consume the share $2/3(r+\rho)$, three players consume the share $3/4(r+\rho)$ etc. The feature that the total use of the resource increases in the number of players is an expression of non-cooperative behavior. For any player, the opportunity cost of harvesting a unit less is that the other players are induced to consume more when there are more resources. Formally this is reflected by the $\partial c^j/\partial x$ term in (18). This feedback effect drives down the value of the resource λ for every player and increases consumption. The increased consumption due to non-cooperative behavior leads to overuse of the resource and potentially to extinction, i.e. the tragedy of the commons (Hardin, 1968). Extinction, however, depends on the method of discounting.

Proposition 4. *When a renewable resource is harvested competitively without property rights, there exists always a sustainable steady state $x^{**} > 0$ in the case of hyperbolic discounting, for any number of competing individuals. For exponential discounting there exists no sustainable steady state if the discount rate is sufficiently high or if the number of competing individuals is sufficiently high, i.e. for $r < n\bar{\rho}$.*

For the proof, insert (20) into (16) and solve for $\dot{x} = 0$. This provides the steady state solution:

$$x^{**} = \frac{\kappa(r - n\rho)}{r(1+n)}, \tag{21}$$

with $\rho = 0$ in case of hyperbolic discounting and $\rho = \bar{\rho}$ in case of exponential discounting. Hyperbolic discounting eliminates extinction as a steady-state phenomenon because $\rho(t)$ vanishes in the long-run. Hyperbolic discounting, however, does not solve the overuse of the commons. For $n > 1$, the steady state resource stock x^{**} falls short of the x^*, the level that provides the maximum sustainable yield.

Figure 4: Hyperbolic Discounting and the Tragedy of the Commons

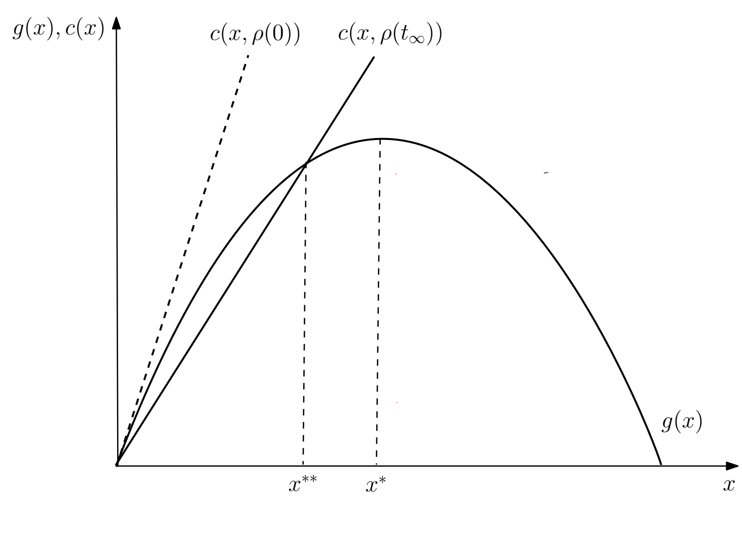


Figure 4 illustrates these results. The steady state is obtained where the linear consumption strategy $c(x)$ intersects the natural growth rate $g(x)$. As time proceeds and the discount rate declines, the consumption profile gets flatter and converges towards the intersection at x^{**} in the long run. Under exponential discounting, (asymptotic) extinction occurs for $n\bar{\rho} > r$, a relatively mild condition when there are many players. In this case consumption policy $c(x)$ lies everywhere above the $g(x)$ curve and x and c decline at declining rates. With the intuition developed in the context of Proposition 2, it is clear that extinction is impossible for time-consistent hyperbolic discounting. While x can be only exhausted asymptotically there exists always a finite time at which $n\rho(t) < r$, which guarantees long-run sustainability.

I next illustrate adjustment dynamics with an example. As for the simple model, I set $r = 0.04$; $\kappa = 10$; $\eta = 2$ and consider first the case of two players ($n = 2$). Blue (solid) lines in Figure 5 show adjustment dynamics for the Weitzman (2001) case, i.e. for $\mu = 0.04$ and $\sigma = 0.03$ (implying $\alpha = 0.0225$ and $\beta = 1.777$). The resource approaches quickly the steady state (below the maximum yield of $\kappa/2 = 5$) as the discount rate declines. The lower right panel shows the linear consumption strategy $c(x)$.

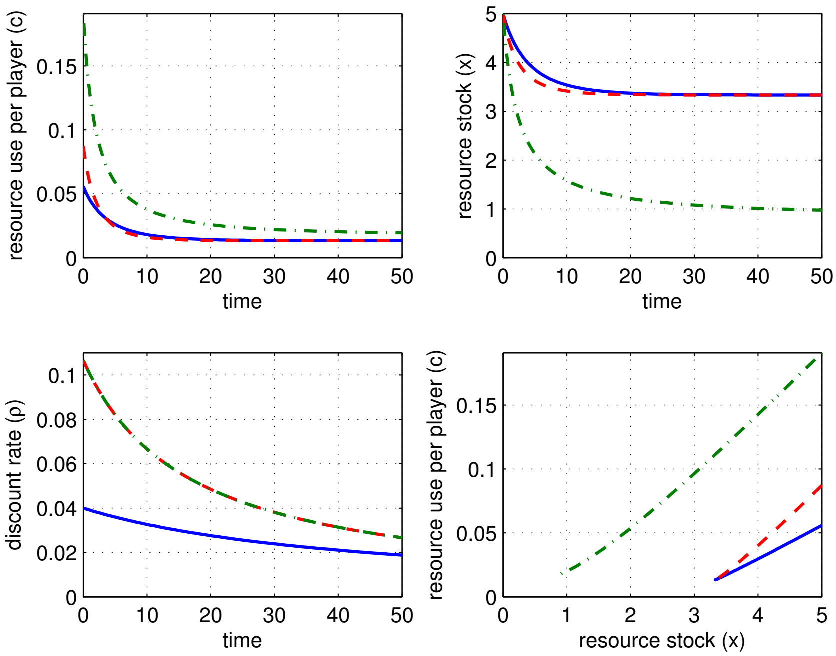
When α rises to 0.06, implying an equivalent constant discount rate of $\bar{\rho} = 0.0467$, extinction becomes inevitable with exponential discounting. Hyperbolic discounting however leads to sustainable consumption. Results are shown by red (dash-dotted) lines in Figure 5. The initial discount rate is now above 10 percent and far above the constant equivalent rate. The consumption strategy $c(x)$ gets steeper (lower right panel) and individuals consume more initially (upper left panel). However, as time proceeds, consumption declines at a faster rate than the resource stock due to the declining interest rate. In the long-run, the system approaches the same steady state as in the benchmark case.

Finally, we increase the number of competing individuals to $n = 10$ and keep the high initial discount rate ($\alpha = 0.06$). Results are shown by green (dash-dotted) lines in Figure 5. Initial consumption is now much higher because of the increased opportunity cost of foregone consumption (upper left panel and lower right panel) and the resource declines steeply. However, since the discount rate declines as well and faster, sustainability is ensured and the system converges toward the steady state at $x^{**} = \kappa/11 = 0.9$. Summarizing, missing property rights for non-renewable resources (the environment) are less of a tragedy when non-cooperative individuals apply time-consistent hyperbolic discounting.

5. Economic Growth, Pollution, and the Environmental Kuznets Curve

In this section we investigate the role of time-consistent hyperbolic discounting in the neoclassical growth model augmented by pollution and environmental quality based on Brock (1977)

Figure 5: Tragedy of the Commons: Adjustment Paths



Parameters: $r = 0.04$; $\kappa = 10$; $\eta = 2$ and $n = 2$ players. Blue (solid) lines: discount rate from Weitzman (2001): $\mu = 0.04$, $\sigma = 0.03$ (i.e. $\alpha = 0.0225$, $\beta = 1.777$). Red (dashed) lines: higher initial discount rate $\alpha = 0.06$ (implying unsustainability for exponential discounting at $\bar{\rho} = \alpha(\beta - 1)$). Green (dash-dotted) lines: high discount rate ($\alpha = 0.06$) and $n = 10$ players. The lower right panel shows the consumption strategy $c(x)$.

and Becker (1982). In order to minimize changes with regard to the previous section we treat environmental quality as a renewable resource (as in Xepapadeas, 2005):

$$\dot{x} = g(x) - p, \tag{22}$$

in which $g(x)$ is an environmental regeneration function and p is pollution. For simplicity, we model pollution as an input in production such that aggregate output is given by $y = f(k, p)$, in which k is the aggregate capital stock.

The economy is populated by a large number of individuals (normalized to one) who derive utility from consumption and a clean environment. Their lifetime utility at decision time t_0 is given by $W(t_0) = \int_{t_0}^{\infty} D(t_0, t)u(c, x)\mathrm{d}t$. The social planner maximizes lifetime utility of the representative individual taking into account (22) and the accumulation of new capital by net investment:

$$\dot{k} = f(k, p) - c - \delta k, \tag{23}$$

in which δ is the depreciation rate. For exponential discounting it has been shown that this type of problem can lead to a steady state where a higher discount rate improves environmental quality (Smulders, 2007; Asheim, 2008). The reason is that higher impatience increases consumption and reduces investment such that the economy converges towards a lower steady-state capital stock, implying less pollution and higher environmental quality.

The Hamiltonian associated with the maximization problem is given by $H = D(t_0, t)u(c, x) + \lambda [g(x) - p] + \mu [f(k, p) - \delta k - c]$. To further simplify the exposition, I follow Smulders (2007) and impose iso-elastic functions for production and utility, $f(k, p) = k^{\gamma} p^{\epsilon}$, $u(c, x) = (cx^{\phi})^{1-\eta}/(1-\eta)$. Then, as shown in the Appendix, the solution leads to an augmented Ramsey rule for consumption (24) and an equation of motion for optimal pollution (25):

$$\frac{\dot{c}}{c} = \frac{1}{\eta}\left[\gamma k^{\gamma-1} p^{\epsilon} - \delta - \rho(t) + (1-\eta)\phi\frac{\dot{x}}{x}\right] \tag{24}$$

$$\frac{\dot{p}}{p} = \frac{1}{\epsilon+1}\left\{\frac{\phi c p}{\epsilon y x} + g_x - \left[\gamma k^{\gamma-1} p^{\epsilon} - \delta\right] - \gamma\frac{\dot{k}}{k}\right\}. \tag{25}$$

Proposition 5. *Hyperbolic discounting establishes the Golden Rule of capital accumulation and the modified Green Golden Rule as a steady-state outcomes. Exponential discounting is neither consistent with the Golden Rule nor with the modified Green Golden Rule.*

For the proof first note that, at the steady state, $\dot{c} = \dot{x} = 0$ and $\rho(t) = 0$. Then, (24) reduces to $\gamma y/k - \delta = 0$. Let s denote the savings rate such that $c = (1-s)y$ and (23), evaluated at the steady state, becomes $\dot{k} = y - (1-s)y - \delta k = 0$, implying $sy/k - \delta = 0$. Hence, $s = \gamma$. The result that the savings rate equals the capital share in production is known as the Golden Rule of capital accumulation. That it cannot be fulfilled given exponential discounting is textbook knowledge (see, e.g., Barro and Sala-i-Martin, 2004).

Using the information that $\gamma k^{\gamma-1} p^{\epsilon} - \delta = 0$ and noting that, at the steady state, $\dot{p} = \dot{k} = 0$, equation (25) reduces to

$$\frac{\phi c p}{\epsilon y x} + g_x = 0 \quad \Rightarrow \quad \frac{\phi(1-\gamma)p}{\epsilon x} + g_x = 0, \tag{26}$$

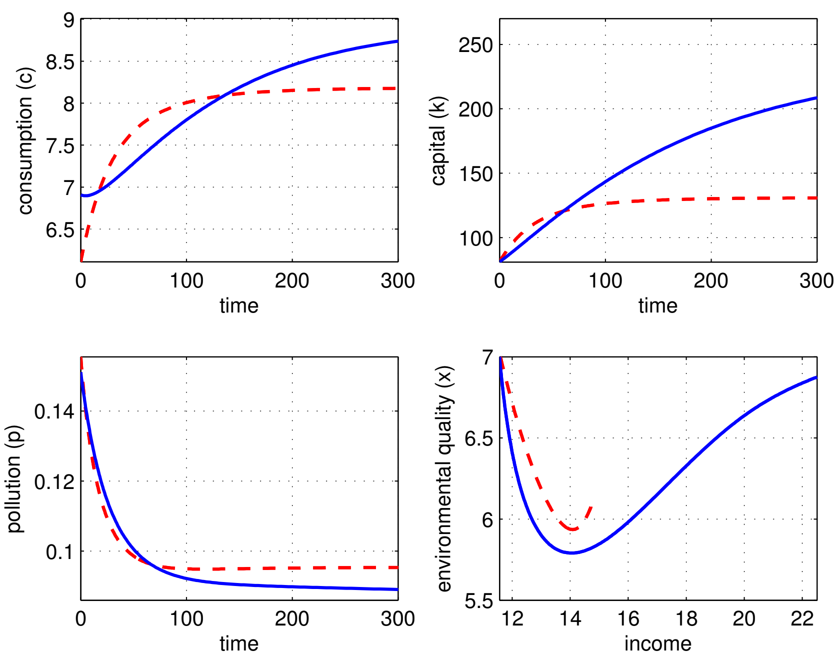
where the last equality follows from $c = (1-s)y$ and $s = \gamma$. As shown in the Appendix, this condition, in conjunction with the Golden Rule, maximizes utility at the steady state. It has therefore been dubbed the modified Green Golden Rule (Smulders, 2007). That exponential discounting cannot meet the Green Golden Rule is evident from the fact that, at the steady state supported by exponential discounting, we have (from (24) with $\rho(t) = \bar{\rho}$) that $\gamma k^{\gamma-1} p^{\epsilon} - \delta = \bar{\rho}$, implying that the right hand side of (26) equals $\bar{\rho} \neq 0$.

Finally, we investigate adjustment dynamics. For simplicity we retain the functional form of $g(x)$ from (6), i.e.

$$\dot{x} = rx\left(1 - \frac{x}{\kappa}\right) - p. \tag{27}$$

The economy is described by the dynamic system (23)-(25) and (27) with $y = f(k, p) = k^{\gamma} p^{\epsilon}$. Since four dimensions cannot be analyzed using phase diagram techniques, we start right away with a numerical analysis. I set $r = 0.04$, $\kappa = 10$, $\eta = 2$ as for the simple model from Section 3. I set the capital share γ to 0.6, capturing a broad definition of capital, and I set the pollution externality to 0.3. In order to start the economy at a relatively early phase of industrialization, I set the initial value for capital at 30 percent of steady-state level and the initial value of environmental quality to 70 percent of the pristine level κ. Adjustment dynamics are shown in Figure 6.

Figure 6: Economic Growth and Hyperbolic Discounting



Parameters: $r = 0.04$; $\kappa = 10$; $\eta = 2$; $\gamma = 0.6$; $\epsilon = 0.1$; $\phi = 0.3$; $k(0) = 0.3k^*$, $x(0) = 0.7x^*$. Blue (solid) lines: discount rate from Weitzman (2001): $\mu = 0.04$, $\sigma = 0.03$ (i.e. $\alpha = 0.0225$, $\beta = 1.777$). Red (dashed) lines: same economy with exponential discounting $\bar{\rho} = \alpha(\beta - 1) = 0.0175$).

It is interesting to inspect first the adjustment dynamics implied by using the equivalent constant discount rate (of 0.0175). They are shown by red (dashed) lines in Figure 6. The upper panels show the concave adjustment paths for consumption and capital stock, a behavior, well known from the standard neoclassical growth model. Pollution is strongest in the early phase of industrialization and declines to almost steady-state level in a century (lower left panel). Consequently, environmental quality x falls initially, overshoots mildly and adjusts from below towards its steady-state level. The lower right panel shows the implied association between income and environmental quality. The economy starts at the North-West corner with low income and high environmental quality and converges with overshooting behavior of environmental quality towards the steady state of high income. This overshooting behavior is known as the environmental Kuznets curve.[5]

Solid (blue) lines in Figure 6 show the adjustment dynamics for hyperbolic discounting. Individuals consume initially more due to the high present-bias (upper left panel). This means that they invest and pollute initially a little less (upper right and lower left panel). The discount rate, however, declines quickly and after less than 20 years, consumption falls short of the level implied by exponential discounting. Investment, in turn, exceeds the level implied by exponential discounting and after about 60 years the capital stock exceeds the level implied by exponential discounting (upper right panel). The economy becomes richer and cleaner at about the same time as pollution falls below the level implied by exponential discounting. After a century, the exponentially discounting economy basically converged to its steady state while the hyperbolically discounting economy keeps growing and becoming cleaner. As a result, the environmental Kuznets curve becomes more pronounced than for exponential discounting (lower right panel).

6. Conclusion

In this paper I proposed a time-consistent method to discount at a declining rate and applied it to three canonical environmental problems. I compared the solutions to those obtained under conventional exponential discounting, applying the normalization that both methods provide the same present value of a constant flow. I showed that hyperbolic discounting outperforms exponential discounting and leads to a socially optimal situation at the steady state (consumption

[5]Brock and Taylor, 2005; Egli and Steger, 2007. The standard environmental Kuznets curve refers to a u-shaped association between income and the pollution stock. Here, this is expressed as an inverted u-shaped association between income and environmental quality.

of maximum yield, implementation of the Golden Rule and the Green Golden Rule). Interestingly, hyperbolic discounting performs also better off the steady state despite the initially high present-bias and discount rates that by far exceed the equivalent constant rate. The reason is that in finite time the declining discount rate falls short of any positive natural growth rate of the resource and extinction in finite time can be ruled out by the optimality conditions. This means that any trajectory irrespective of potentially very high initial discount rates converges eventually to the long-run social optimum. Under exponential discounting, however, asymptotic extinction is a real threat, in particular when property rights are not defined or hard to enforce and many parties exploit an exhaustible resource non-cooperatively.

These results are policy relevant since they resolve the concern of time-inconsistent, suboptimal, and potentially disastrous outcomes when policymakers hyperbolically discount cost and benefits of projects. Multiplicative separability of the discount factor ensures time consistency and socially optimal solutions at the steady state. A discounting rule is relatively easy to implement as an institution and then hard to change. These features are in particular important when long-run decisions are made subject to a slow changing state variable, such as the environment.

Appendix

Derivation of (20). Using the utility function $u = c^{1-\eta}/(1-\eta)$ with $\eta = 2$ and inserting g_x, equation (19) simplifies to

$$\frac{\dot{c}}{c} = \frac{1}{2}\left\{r - \frac{2rx}{\kappa} - (n-1)\frac{\partial c}{\partial x} - \rho(t)\right\}, \tag{A.1}$$

To solve the problem with the method of undetermined coefficients, assume that $c = ax$, implying $\dot{x} = a\dot{x}$ and $\partial c/\partial x = a$. Applying this information, (A.1) becomes

$$\dot{c} = ax\frac{1}{2}\left[r - \frac{2rx}{\kappa} - (n-1)a - \rho(t)\right] = a\dot{x} = a\left[rx\left(1 - \frac{x}{\kappa}\right) - nax\right] \tag{A.2}$$

where the last equality follows from (16). Thus

$$\frac{r}{2} - \frac{rx}{\kappa} - (n-1)\frac{a}{2} - \frac{\rho(t)}{2} = r - \frac{rx}{\kappa} - na.$$

Solving for a provides $a = (r + \rho(t))/(n+1)$ and thus (20) in the main text.

Golden Rule and Modified Green Golden Rule. The Golden Rule is defined as the utility maximizing savings rate at the steady state. Let the savings rate be denoted by s. Inserting $c = (1-s)y$ in (23) and using the Cobb-Douglas production function provides $\dot{k}/k = sk^{\gamma-1}p^{\epsilon} - \delta$. Evaluating the function at the steady state and solving for capital provides $k^* = [(p^{\epsilon}s)/\delta]^{1/(1-\gamma)}$. Furthermore, from (22), $g(x) = p$ at the steady state. Inserting this information and $c = (1-s)y$ into the utility function, we obtain steady-state utility:

$$u = \frac{1}{1-\eta}\left\{(1-s)\left(\frac{s}{\delta}\right)^{\frac{\gamma}{1-\gamma}} g(x)^{\frac{\epsilon}{1-\gamma}} x^{\phi}\right\}^{1-\eta}. \tag{A.3}$$

Maximizing utility with respect to s provides the first order condition

$$-\left(\frac{s}{\delta}\right)^{\frac{\gamma}{1-\gamma}} + (1-s)\left(\frac{s}{\delta}\right)^{\frac{\gamma}{1-\gamma}} \frac{\gamma}{1-\gamma}\frac{1}{s} = 0 \quad \Leftrightarrow \quad \frac{s}{1-s} = \frac{\gamma}{1-\gamma}$$

and thus $s = \gamma$, the Golden Rule of capital accumulation.

Maximizing utility (A.3) with respect to x provides the first order condition

$$g'(x)\frac{\gamma\epsilon}{1-\gamma}g(x)^{\frac{\epsilon}{1-\gamma}}\frac{x^{\phi}}{g(x)} + \phi g(x)^{\frac{\epsilon}{1-\gamma}}\frac{x^{\phi}}{x} = 0.$$

Simplifying, we obtain

$$g'(x) + \frac{(1-\gamma)\phi}{\epsilon}\frac{g(x)}{x} = 0. \tag{A.4}$$

Derivation of (24) and (25). Using the parameterized functional forms for utility and production, the first order conditions and costate equations for a maximum of the Hamiltonian are given by:

$$D(t_0, t)c^{-\eta}x^{\phi(1-\eta)} - \mu = 0 \tag{A.5}$$

$$-\lambda + \mu\frac{\epsilon y}{p} = 0 \tag{A.6}$$

$$\phi D(t_0, t)c^{1-\eta}x^{\phi(1-\eta)-1} - \lambda g'(x) = \dot{\lambda} \tag{A.7}$$

$$\mu\left(\frac{\gamma y}{k}-\delta\right)=-\dot{\mu}. \tag{A.8}$$

Differentiating (A.5) with respect to time and eliminating μ and $\dot{\mu}$ in (A.8) provides

$$-\frac{\dot{\mu}}{\mu}=\frac{\gamma y}{k}-\delta=\eta\frac{\dot{c}}{c}+(1-\eta)\phi\frac{\dot{x}}{x}+\rho(t). \tag{A.9}$$

Solving for $\dot{c}/c$ provides the modified Ramsey rule (24). Inserting (A.5) into (A.7) and eliminating the discount factor provides

$$\phi\frac{c}{x}\frac{1}{\mu}+\lambda g'(x)=\dot{\lambda}. \tag{A.10}$$

Differentiating (A.6) with respect to time and eliminating λ and $\dot{\lambda}$ in (A.10) provides:

$$\frac{\phi cp}{\epsilon yx}+g'(x)=\frac{\dot{p}}{p}+\frac{\dot{y}}{y}-\frac{\dot{\mu}}{\mu}. \tag{A.11}$$

Finally, using (A.8) and $\dot{y}/y=\gamma\dot{k}/k+\epsilon\dot{p}/p$ we obtain (25) in the text.

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