GOING FROM BAD TO WORSE: ADAPTATION TO POOR HEALTH, HEALTH SPENDING, LONGEVITY, AND THE VALUE OF LIFE

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Going from Bad to Worse: Adaptation to Poor Health, Health Spending, Longevity, and the Value of Life

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Abstract. Aging humans adapt to their worsening state of health and old people are usually happier than estimated by young individuals. In this paper we investigate how adaptation to a deteriorating state of health affects health spending, life expectancy, and the value of life. We set up a life cycle model in which individuals are subject to physiological aging, calibrate it with data from gerontology, and compare behavior and outcomes of adapting and non-adapting individuals. While adaptation generally increases the value of life (by about 2 to 5 percent), its impact on health behavior and longevity depends crucially on whether individuals are aware of their adaptive behavior.

Keywords: Health, Adaption, Aging, Longevity, Health Care Demand.

JEL: D91, J17, J26, I12.

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1. INTRODUCTION

From the gerontological viewpoint, the human life cycle can be characterized as the continuous deterioration of physiological fitness. Most human functions and capabilities are in decline from early adulthood onwards (Case and Deaton, 2005; Skirbekk, 2004; Nair, 2005). Human aging, understood as “the intrinsic, cumulative, progressive, and deleterious loss of function that eventually culminates in death” (Arking, 2006), has a deep foundation in evolutionary biology (Fries, 1980; Gavrilov and Gavrilova, 1991; Robson and Kaplan, 2007) and, at the current state of medical technology, it can at best be delayed, but not avoided. So it seems to be fortunate that aging humans are capable to adapt to this sad state of affairs. However, at closer inspection, doubts may arise. Couldn’t it be that quick adaptation to worsening health induces us to invest less in health maintenance and repair and thus to live a shorter and perhaps overall unhappier life than we could without adaptation?

Assessing the impact of adaptation on health behavior, longevity, and happiness with the means of lab or field experiments, is potentially hard if not impossible because of the missing counterfactual. With the help of economic theory and the design of an appropriate computational experiment an assessment is relatively straightforward. In this paper we propose such a computational experiment. We set up a life cycle model of human aging, in which deliberate health investments reduce the speed of aging and thus the age of death, calibrate the model with gerontological data, and compare behavior and outcomes for adapting and non-adapting individuals. We distinguish between individuals who are aware of their adaptive behavior (so called sophisticated types) and those who are not (naive types). This labeling of types follows Strotz (1955), see also Rabin (1998). We find, perhaps surprisingly, that naive adaptation is conducive to a healthier and longer life. Sophisticated types, on the other hand, spend less on health and live shorter than otherwise identical non-adapting types. We use these results and compute the implied value of life. We find, again perhaps surprisingly, that both naive and sophisticated types experience about the same value of life and that both types experience a significantly higher value of life than non-adapting types. We explain the economic intuition behind these results.

Since the seminal study of Brickman et al. (1978), comparing happiness of paraplegics and lottery winners, the medical and economics literature has provided ample evidence that humans adapt to health problems and rate their happiness or quality of life much higher than predicted
by unaffected persons anticipating negative health events (e.g. Wu, 2001; Albrecht and Devlieger, 1999; Riis et al., 2005). This seems to be true for (mild) nuisances like acne (Baron et al. 2003) as well as for severe disability (Oswald and Powdthavee, 2008). Adaptation after a severe health shock is gradual and perhaps complete. Oswald and Powdthavee (2008) estimate approximately 30 percent (50 percent) hedonic adaptation 3 years after the onset of severe (modest) disability and they could not reject the hypothesis of complete adaptation after 6 years. Using a very large German panel of individuals observed from 1984 to 2006, Pagan-Rodriguez (2010) finds also gradual adaptation to disability and cannot reject the hypothesis of complete adaptation after 7 years.

The observations that healthy persons underestimate the happiness of sick persons and that sick persons believe they would be happier if they had never been sick (Boyd et al., 1990; Riis et al., 2005) indicates that people are not fully aware of their adaptive behavior. It indicates naive rather than sophisticated adaptation. The available evidence suggests also that adaptation to bad health is “genuine” and not driven by an overoptimistic assessment of one’s health and survival probabilities (Wu, 2001) and that the misprediction of healthy people of their adaptive capabilities is hard to explain by focussing illusion (Ubel et al., 2001; Baron et al., 2003).

While most studies focus on adaptation after severe health shocks, we are here mostly (but not exclusively) interested in the gradual and progressive decline of health that comes with age. While many studies document that aggregate measures of happiness or wellbeing do not decline (by much) over the life cycle (e.g. Costa et al., 1987; Diener and Suh, 1998; Deaton, 2007), a particular interesting study in the present context is provided by Lacey et al. (2006) who asked 30 and 70 years old persons to rate their own happiness as well as to estimate happiness of average 30 and 70 years old persons. They found that both, young and old individuals, estimated happiness of the young group to be higher while actually it was the other way round. The authors conclude that people display a “remarkable inability to recognize their own adaptation”.

The model that we set up below in order to discuss the effects of adaptation to deteriorating health is particularly suitable for this purpose since it is based on the notion of aging as progressive health deficit accumulation. It is easy to see that the alternative paradigm, the Grossman (1972) model, is less suitable. It is based on health capital accumulation and the assumption that health capital depreciates at a given (potential age-specific) rate \(d(t)\) such that individuals with health capital \(H(t)\) lose health \(d(t)H(t)\) through health depreciation. It thus
assumes, counterfactually, that of two persons of the same age $t$ the one in better health, i.e. with more health capital $H(t)$, loses more health in the next period. These counterfactual assumptions entail some counterfactual predictions. For example, without further amendments, the Grossman model predicts eternal life (Case and Deaton, 2005; Strulik, 2015a) and when death is enforced by design, the Grossman model usually predicts, counterfactually, that health investments decline in old age and near death (Wagstaff, 1986; Zweifel and Breyer, 1997; Strulik, 2015a; Dalgaard and Strulik, 2014b). Most importantly, health capital is a latent variable, unknown to doctors and medical scientists, a fact that confounds any serious calibration of the model. The health deficit model developed by Dalgaard and Strulik (2014a), in contrast, has a foundation in gerontology and can be calibrated straightforwardly using the so-called frailty index (Mitnitski et al., 2002a,b). Since the calibration provides no degrees of freedom, the model can be used to assess health issues quantitatively. In the present context it will be used to assess the impact of adaptation on health investment, aging, wellbeing, longevity, and the value of life.¹

The paper is organized as follows. Section 2 presents a simple deterministic model of health deficit accumulation for 3 different types of individuals: non-adapting, naive, and sophisticated. In this simple model the only life cycle decision is how to spend a given income stream on consumption and health care. We calibrate the model for a reference U.S. American (a white 20 year old men in the year 2000) and evaluate how adaptive behavior affects health expenditure, longevity, and the value of life. We discuss the robustness of these results regarding a larger weight of health in utility and a higher speed of adaptation. We also discuss how results change with improving medical technology and discuss the adaptation process after a severe health shock. In Section 3 we extend the model to uncertain survival and a savings decision and show that all results from the basic model are preserved qualitatively with only minor quantitative changes. Section 4 concludes.

¹Earlier quantitative studies using the health deficit model were concerned with the Preston curve (Dalgaard and Strulik, 2014a), the education gradient (Strulik, 2015), and the long-term evolution of the age at retirement (Dalgaard and Strulik, 2012). Gjerde et al. (2005) applied the Grossman model on health adaptation. Facing the difficulties entailed by the health capital approach concerning calibration and predictive quality, they presented their study as a first attempt to formalize adaption processes in a health context. The present paper confirms their conclusion that sophisticated adaptation leads, ceteris paribus, to less health investment and a shorter life. The (empirically more relevant) case of naive adaption was not addressed by Gjerde et al. (2005).
2. The Basic Model

2.1. Setup. Consider an individual who derives utility from consumption and from being in good health. The actual (objective) state of health is measured by the accumulated health deficits $D$. By subjectively evaluating the state of health the individual compares actual health with a reference state of health $R$. Utility declines in the number of accumulated health deficits and rises in the state of reference health. For non-adapting individuals the reference state of health is a given constant (the state of best health). For adapting individuals, the reference state of health adjusts to the actual state of health according to

\[
\frac{dR}{dt} \equiv \dot{R} = \theta(D - R),
\]

in which $t$ is age and $\theta$ controls the speed of adaptation.

We normalize initial age to zero (which will be age 20 in the calibration). In order to flesh out the basic mechanism as clearly as possible, we assume for the benchmark model that survival is deterministic. Following Dalgaard and Strulik (2014a), individuals accumulate health deficits as they age in the following way:

\[
\dot{D} = \mu(D - Ah^\gamma - a),
\]

in which $\mu$ is the “natural” rate of aging. Health deficit accumulation can be slowed down by health expenditure $h$. The parameters $A$ and $\gamma$ control the state of medical technology $A > 0$, $0 < \gamma < 1$, and $a$ captures environmental influences. As shown in Dalgaard and Strulik (2014a) the law of health deficit accumulation has a deep foundation in gerontology and its parameters can be calibrated using the so called frailty index (Mitnitski et al., 2002a).

Following Finkelstein et al. (2013) we consider the health state as a shifter of the utility function of consumption $\tilde{u}(c)$ such that both utility and marginal utility of consumption are negatively affected by bad health. Specifically we assume that instantaneous utility is given by

\[
u(c, D, R) = \left(\frac{R}{D}\right)^{\frac{\alpha}{\gamma}} \cdot \tilde{u}(c), \quad \text{with } \tilde{u}(c) = \begin{cases} 
\frac{c^{1-\sigma}-1}{1-\sigma} & \text{for } \sigma \neq 1 \\
\log(c) & \text{for } \sigma = 1.
\end{cases}
\]
The parameter $\alpha$ controls by how much an additional health deficit shifts the utility function down. The variable $R$ captures the effect of adaption. In case of spontaneous and perfect adaption, $R = D$ at all ages and utility remains unaffected by deteriorating health.

By allocating expenditure for consumption $c$ and health care $h$ the individual maximizes utility over his or her remaining life-time. For simplicity we consider a constant flow of income over the life time. This simple setup is helpful for an understanding how health adaptation affects behavior. In Section 3 we introduce capital income and a savings decision. The budget constraint is given by

$$y = c + ph,$$

in which $y$ is the flow of income and $p$ is the relative price of health care. The individual takes income and prices parametrically. Summarizing, the individual maximizes life-time utility

$$\int_0^T e^{-\rho t} u(c, D, R) \, dt,$$

subject to (1)–(4), the initial conditions $D(0) = D_0$, $R(0) = R_0$ and terminal health deficits $D(T) = \bar{D}$. The parameter $\rho$ is the discount rate of future utility and $T$ is the age of death. This is a deterministic free terminal value problem. Individuals control through their expenditure plan the accumulation of health deficits and therewith their age of death, which occurs when $\bar{D}$ health deficits have been accumulated. The main question here is whether adapting individuals spend more or less on health and thus expire sooner or later. In Section 3 below we extend the model towards imperfect control by assuming that the stock of accumulated health deficits affects “only” the probability of survival.

The Hamiltonian associated with problem (1)–(5) reads

$$H = u(y - ph, D, R) + \lambda_D(D - Ah^\gamma - a) + \lambda_R(D - R),$$

in which $\lambda_D$ and $\lambda_R$ are the co-state variables (shadow prices) of health deficits and reference health. We distinguish 3 types of individuals:

- non-adapting (benchmark): $\theta = 0$
- naive: $\theta > 0$, $\lambda_R = 0$
- sophisticated: $\theta > 0$, $\lambda_R \geq 0$. 

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For non-adapting types, the reference stock does not change with changing health. For naive types, the reference stock changes but individuals fail to take this into account in their calculus. Sophisticated types realize their adaptive behavior and take it into account in their consumption and health spending decision.

The optimal solution fulfills the first order conditions

\[ \rho \left( \frac{R}{D} \right) c^{-\sigma} = -\lambda_D \mu A \gamma h^{\gamma - 1} \]  
\[ -\alpha \left( \frac{R}{D} \right) \frac{\bar{u}(c)}{D} + \lambda_D \mu + \lambda_R \theta = \lambda_D \rho - \dot{\lambda}_D \]  
\[ \alpha \left( \frac{R}{D} \right) \frac{\bar{u}(c)}{R} - \lambda_R \theta = \lambda_R \rho - \dot{\lambda}_R. \]

Condition (6) requires that the marginal benefit from an additional unit of health expenditure on the right hand side equals the marginal costs in terms of foregone utility from consumption on the left hand side. To see that, notice that health deficits are a “bad” such that the associated shadow price \( \lambda_D \) is negative. An additional unit of health expenditure reduces health deficits by \( \mu A \gamma h^{\gamma - 1} \) (compared to laissez faire), the utility of which is evaluated by \( \lambda_D \), i.e. the contribution of an additional health deficit to the objective function. Condition (7) and (8) require that the shadow prices of deficits and reference stock change according to the contribution of an additional unit of \( D \) and \( R \) to the objective function. Condition (8) applies only to sophisticated types. Additionally, the optimal solution fulfills the terminal condition \( H(T) = 0 \) and the solution for sophisticated types fulfills the terminal condition \( \lambda_R(T) = 0 \).

2.2. Calibration. The solution solves the dynamic system (1), (2), (7), and (8), taken into account the static equations (4) and (6) and the initial and terminal conditions. In order to obtain the numerical solution we specify the parameters of the model. Since the biological parameters are estimated with high precision, there are relatively no degrees of freedom. We take the estimate \( \mu = 0.043 \) from Mitnitski et al. (2002a), implying that aging individuals develop 4.3 percent more health deficits per year. From Mitnitski et al.’s (2002a) regression analysis we back out \( D(0) = 0.0274 \) as the relevant initial value for a 20 year old and \( \dot{D} = 0.1 \) 55.5 years later; the average life-expectancy of a 20 year old U.S. American in the year 2000 was 55.5 years (i.e. death at 75.5; NVSS, 2012). Following Dalgaard and Strulik (2014a) we identify \( a \) by assuming that before 1900 the role of technology in the repair of health deficits of adults was virtually zero. Matching the life expectancy of a 20 year old U.S. American in 1900 (which
was 42 years; NCHS, 1980) we estimate \( a = 0.013 \). Secondly, we set \( \gamma = 0.19 \) as estimated by Dalgaard and Strulik (2014a), normalize \( p = 1 \), set \( y \) to 35320 (the average annual pay for U.S. workers in the year 2000; BLS, 2011), and adjust \( A \) such that the reference individual (a non-adapting U.S. American) expires at age 75.5. This provides the estimate \( A = 0.0014 \).

Turning towards the utility function we begin with a modest impact of health on utility by setting \( \alpha = 0.1 \). This means that an unexpected increase of health deficits from \( D_0 \) by one standard deviation reduces the marginal utility from consumption by 5.4 percent.\(^2\) This value is below the mean estimate of Finkelstein et al. (2013) who find that a one-standard deviation increase of chronic diseases is associated with a 11% decline in the marginal utility of consumption (with a 95% confidence band from 2.7% to 16.8%). We later consider a larger impact of health on marginal utility. Finally we set \( \sigma \) such that all types of individuals on average spend 13.1 percent of their income on health. This value matches the health expenditure share of GDP in the U.S. in the year 2000 (World Bank, 2015). It leads to the estimate \( \sigma = 1.05 \), a value that accords well with recent estimates of the intertemporal elasticity of substitution, suggesting that the “true” value of \( \sigma \) is probably close to unity (e.g. Chetty et al., 2006).

For the benchmark run we set the speed of adjustment of the health reference stock \( \theta \) to 0.3. This means that about 60% of an initial gap between actual health and reference health are closed after 3 years and 82 percent are closed after 7 years. This value is a compromise between the values suggested by empirical estimates on the speed of health adaptation (Wu, 2001; Oswald and Powdthavee, 2008; Pagan-Rodriguez, 2010). We consider faster adjustment in the sensitivity analysis. For the benchmark run we set the initial reference stock \( R_0 \) to \( D_0 \), implying the normalizing assumption that utility from consumption for all three types of individuals is unaffected by health at the initial state of best health.

2.3. Results. Figure 1 shows results for the age trajectories of health expenditure, health deficits, and instantaneous utility for the basic run.\(^3\) Blue (solid) lines represent the non-adapting types, red (dashed) lines represent naive types, and green (dash-dotted) lines represent sophisticated types. Instantaneous utility is measured relative to initial utility of non-adapting individuals. Health expenditure is increasing with age and highest for the oldest individuals.

\(^2\) According to Mitnitski et al. (2001) the standard deviation of most health deficits in the frailty index is around \( 0.4/\hat{\mu} \), in which \( \hat{\mu} \) is the mean of the particular deficit. The mean frailty index from (1) for individuals between 19 and 79 years is about 0.05 with a standard deviation of about 0.02.

\(^3\) A description of the solution method can be found in the Appendix.
in line with the empirical observation (e.g. Meara et al., 2004) and in contrast to the predictions by the Grossman model with or without adaptive behavior (Grossman, 1972; Gjerde et al., 2005). The perhaps most surprising result is that naive types spend the most on health, live the healthiest life, and die latest. Total discounted life time spending on health of naive types is almost 8 percent above that of non-adaptive types. They die at age 75.8, i.e. about 4 months later than non-adaptive types. The reason is, that naive types live a happier life than non-adapting types. With aging and health deterioration, utility of non-adaptive types declines relatively quickly whereas adaptive types manage to live a happier life by adjusting their health ambitions to the deteriorating health level. Since life is more worthwhile at any point of time, naive types spend more on health in order to live longer and outlive non-adapting types.

Figure 1: Health Adaptation: Benchmark Run

![Figure 1: Health Adaptation: Benchmark Run](image)

Blue (solid) lines: non-adaptive type. Red (dashed) lines: naive type. Green (dash-dotted) lines: sophisticated type. Utility is instantaneous utility relative to initial utility of a non-adapting individual.

Sophisticated types understand that they will adapt to deteriorating health and thus spend less on health during most of their life. Total discounted life time spending on health of sophisticated types is more than 18 percent below that of non-adaptive types. Consequently their health declines fastest and they live the shortest life, dying at age 74.8, i.e. a year before naive types and 7 months before non-adaptive types. Yet, sophisticated types enjoy life more. Spending less on health allows them to spend more on consumption such that instantaneous utility is above the trajectory of non-adaptive types and, except of the time near death, also above the utility of naive types. In order to assess this better we compute the value of life as a monetary expression of aggregate utility experienced during life until its end, that is period utility is converted by the unit value of an “util”, \( \text{VOL} = \int_0^T e^{-\rho \tau} u[c(\tau), D(\tau), R(\tau)]d\tau/u_0[c(0), D(0), R(0)] \). The benchmark calibration predicts a VOL at age 20 of about $6.5 million for the non-adapting type, which is somewhat below Murphy and Topel’s (2006, Fig. 3) estimate of the VOL at age
25 of $7 million. The value of life of sophisticated types exceeds that of non-adaptive types by 2 percent and that of naive types by 0.05 percent. Perhaps surprisingly, it makes little difference for the value of life whether adapting individuals understand their adaptive behavior or not, although it affects health behavior quite strongly.

Table 1: Sensitivity Analysis

<table>
<thead>
<tr>
<th>case</th>
<th>life time $h$</th>
<th>life expect.</th>
<th>value of life</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>naive sophis</td>
<td>naive sophis</td>
<td>naive sophis</td>
</tr>
<tr>
<td>1) benchmark</td>
<td>7.67 -18.49</td>
<td>0.53 -1.30</td>
<td>2.09 2.14</td>
</tr>
<tr>
<td>2) $\alpha$ = 0.2</td>
<td>12.17 -28.22</td>
<td>0.88 -2.13</td>
<td>3.79 3.94</td>
</tr>
<tr>
<td>3) $\theta$ = 0.5</td>
<td>7.87 -20.32</td>
<td>0.54 -1.44</td>
<td>2.26 2.32</td>
</tr>
<tr>
<td>4) $\Delta w$ = 50%</td>
<td>7.31 -19.26</td>
<td>0.61 -1.62</td>
<td>1.96 2.02</td>
</tr>
<tr>
<td>5) $\Delta A$ = 50%</td>
<td>5.34 -24.78</td>
<td>1.35 -5.52</td>
<td>1.18 1.33</td>
</tr>
<tr>
<td>6) $\Delta D_0$ = 50%</td>
<td>6.11 -3.33</td>
<td>0.21 -0.12</td>
<td>5.96 5.97</td>
</tr>
</tbody>
</table>

All values as deviation in percent from the optimal solution for a non-adapting individual; life time $h$ is the discounted life time expenditure on health; life expect. is life expectancy; $A = 0.0014$, $\sigma = 1.05$.

We next analyze in Table 1 the sensitivity of results. These experiments can also be seen as numerical results on the comparative statics of the model. The table shows for naive and sophisticated types the deviation of the solution from non-adaptive types in percent. Results are shown for discounted life time health expenditure, the length of life, and the value of life. The first row re-iterates results from the benchmark run. The second row considers a higher weight of health in utility. A value of $\alpha = 0.2$ means that marginal utility from consumption declines by 10.5 percent when health deficits increase by one standard deviation, a value close to Finkelstein et al.’s point estimate. Naturally, the behavior and outcomes for adaptive types differs more strongly from that for non-adaptive types when health matters more. Naive types spend 12 percent more on health and live almost one percent longer than non-adaptive types while sophisticated types spend 28 percent less on health and their live expectancy declines by more than 2 percent. Again the value of life for both adapting types is similar and about 4 percent larger than for non-adaptive types.

The third row of Table 1 considers a higher speed of adjustment for reference health. When $\theta = 0.5$, 78 percent of the initial gap between health and reference health are closed after 3 years (97 percent after 7 years). Despite this (implausibly) high speed, results deviate only slightly from the benchmark run. Apparently, results are relatively insensitive to the speed of adjustment. It is the feature of adjustment as such and not so much the speed at which it happens
that matters. Case 4) demonstrates that results are largely insensitive to income variation. Of course, everybody lives substantially longer given the higher income, but relatively speaking, results differ only marginally from the benchmark run.

In the fifth row of Table 1 we consider the impact of medical advances. An increase of $A$ by 50 percent would be accomplished, for example, if medical technology improves by 1 percent per year for 40 years. The improving health technology has a very powerful impact on the age at death, which raises by 18.6 years to 94.1 for non-adaptive types. For adaptive types, technological progress increases the deviation of life expectancy from that of non-adaptive types but it reduces the deviation of the value of life. The dominating effect appears to be the overall increasing life expectancy and not so much the increasing differences between individuals. Naive types live 1.3 percent longer and sophisticated types 5.5 percent shorter than non-adaptive types but the excess value of life gained from adaptation reduces to somewhat above 1 percent for both adaptive types.

![Figure 2: Adjustment after Bad Health Shock at Age 20](image)

As benchmark but $D_0$ 50 percent higher, without adjustment of $R_0$ (20 years shorter life). Blue (solid) lines: non-adaptive type. Red (dashed) lines: naive type. Green (dash-dotted) lines: sophisticated type.

Finally, we consider in the sixth row of Table 1 the power of adjustments to severe health shocks. For that purpose we assume that $D_0$ increases unexpectedly by 50%, i.e. without adjustment of $R_0$, which is kept at benchmark level. Although the initial deviation of health deficits (of 1.37 percentage points) from benchmark could be regarded as modest, it has severe impact on successive health deficit accumulation such that individuals die more than 20 years earlier (at age 51.2 in the case of non-adaptive types). The experiment thus represents a chronic, gradually disabling, and eventually deadly disease like, for example Huntington disease (Oster et al., 2013). As a response to the diagnosis, all individuals increase health expenditure and in particular sophisticated types, implying that life expectancy of adapting individuals deviates
now relatively little from that of non-adapting individuals. Despite these small differences, adaptive types are able to experience a higher value life, about 6 percent above that of non-adaptive types. The panel on the right hand side of Figure 2 shows why. It depicts the utility of the 3 types relative to initial benchmark utility, i.e. relative to utility before the health shock. While utility drops down initially by about the same amount for all 3 types, utility of adaptive types recovers quickly despite of further deteriorating health and it returns to almost benchmark utility at age 30 (10 years after the health shock).

3. Model Extensions

3.1. Uncertain Survival. We next consider robustness of results with respect to extending the model to uncertain survival. In so doing, we utilize the fraction of health deficits accumulated by a person in order to predict his or her survival probability. This approach captures well the biological approach to aging, which aspires to replace age as a proximate determinant of death by the loss of bodily function as a deep determinant (“Only if we can substitute the operation of the actual physiological mechanism for time we have a firm idea of what we are talking about.”, Arking, 2006, p. 10). Following Strulik (2015b), we assume that the unconditional survival probability at age \( t \), denoted by \( S(t) \), depends on the accumulated health deficits at that age, we then impose a particular parametrization of this function, feed in the estimates from Mitnitski et al. (2002a) on the association between age and health deficits, and predict the association of age and survival, which is confronted with estimates of \( S(t) \) from life tables.

A parsimonious representation of the survival function is given by the logistic:

\[
S(D) = \frac{1 + \omega}{1 + \omega e^{\xi D}}. \tag{9}
\]

The survival probability is unity at the state of best health (\( D = 0 \)) and declines with first increasing and then decreasing rate as more health deficits are accumulated. The panel on the left hand side of Figure 3 shows the association between \( D \) and \( S \) implied by (9) for \( \omega = 0.02 \) and \( \xi = 40 \). The middle panel shows the association between age and accumulated deficits estimated by Mitnitski et al. (2002a) for 19-75 year old Canadian men \( (R^2 = 0.95) \). When we feed these data into the \( S(D(t)) \) function, we get the “reduced form”, \( S(t) \), which shows survival as a function of age. The implied functional relationship is shown on the right hand side of Figure 3. Stars in the panel on the right hand side indicate the survival probability estimated from life
tables for U.S. American men 1975-1999, taken from Strulik and Vollmer (2013). The model’s prediction fits the data reasonably well. The model predicts a life expectancy of 55.3 for a 20 years old (death at 75.3) while it was actually 55.5 for our reference American in the year 2000 (see last section).

Figure 3: Health-Dependent Survival and Survival by Age

\( S(t) \) is the unconditional probability to survive to age \( t \). Left: assumed function \( S(D) \). Middle: Estimated association \( D(t) \) (Mitnitski et al., 2002a). Right: Predicted (line) and estimated (stars) association between age and survival probability (estimate from Strulik and Vollmer (2013). Implied life expectancy at 20: 55.3 years.

Facing uncertain death, rational individuals calculate the expected utility from life-time consumption by multiplying the instantaneous utility experienced at age \( t \) with the probability to survive to age \( t \). Following Kamien and Schwartz (1980, Section 9, Part I), the present value of expected utility experienced over the life cycle can then be represented as

\[
\int_0^T S(D) e^{-\rho t} u(c,D,R) dt,
\]

which replaces (5). This approach implies that individuals assess their health and its implications for survival correctly, the only potential mistake that they make regards life time utility due to failed anticipation of adaptation to deteriorating health. This notion captures in a stylized way the evidence cited in the Introduction, which finds little support for the hypothesis that the high happiness of sick individuals could be explained by their wrong assessment of their actual state of health.

Everything else is kept from the simple model such that the Hamiltonian associated with the maximization problem becomes

\[
H = S(D) u(y - ph, D, R) + \lambda_D \mu(D - Ah^\gamma - a) + \lambda_R \theta(D - R),
\]

and the former first order conditions (6)–(8) modify to

\[
pS(D) \left( \frac{R}{D} \right)^\alpha c^{-\sigma} = -\lambda_D \mu A \gamma h^{\gamma - 1} \tag{10}
\]

\[
\left( S'(D) - \alpha \frac{S(D)}{D} \right) \left( \frac{R}{D} \right)^\alpha \tilde{u}(c) + \lambda_D \mu + \lambda_R \theta = \lambda_D \rho - \dot{\lambda}_D \tag{11}
\]

\[
\alpha S(D) \left( \frac{R}{D} \right)^\alpha \tilde{a}(c) \left( \frac{R}{c} \right) - \lambda_R \theta = \lambda_R \rho - \dot{\lambda}_R \tag{12}
\]
with $S(D)$ from (9). The boundary conditions are the same as for the simple model.

Matching the data for a 20 years old U.S. American man, as described in Section 2 leads to mild adjustments of the power of medical technology (now $A = 0.00146$) and the curvature of the utility function (now $\sigma = 1.0$). All other parameters are kept from the simple model. Additionally, we have $\omega = 0.02$ and $\xi = 40$, as estimated with the help of Figure 3. Table 2 reports results for health spending, life expectancy, and the value of life in the same style as Table 1 for the simple model. The most salient feature is that all results from the simple model are preserved. The difference between outcomes for adapting and non-adapting types becomes somewhat smaller, for both naive and sophisticated types, and across all 5 computational experiments. Overall we confirm the observation of Strulik (2015b), made in the context of no adaptation, that including uncertain survival adds more realism (and complexity) but changes outcomes and predictions only marginally.

<table>
<thead>
<tr>
<th>case</th>
<th>life time $h$</th>
<th>life expect.</th>
<th>value of life</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) benchmark</td>
<td>6.12</td>
<td>-14.51</td>
<td>0.44</td>
</tr>
<tr>
<td>2) $\alpha = 0.2$</td>
<td>10.19</td>
<td>-22.88</td>
<td>0.77</td>
</tr>
<tr>
<td>3) $\theta = 0.5$</td>
<td>6.30</td>
<td>-15.97</td>
<td>0.45</td>
</tr>
<tr>
<td>4) $\Delta A = 50%$</td>
<td>3.73</td>
<td>-18.91</td>
<td>1.31</td>
</tr>
<tr>
<td>5) $\Delta D_0 = 50%$</td>
<td>5.85</td>
<td>-3.48</td>
<td>0.17</td>
</tr>
</tbody>
</table>

All values as deviation in percent from the optimal solution for a non-adapting individual; life time $h$ is the discounted life time expenditure on health; life expect. is life expectancy; $A = 0.00146$, $\sigma = 1.0$. Other Parameters as for Table 1.

3.2. **Extension: Life Cycle Savings.** We next introduce an endogenous savings decision. This allows individuals to re-allocate funds for consumption or health care over time.\(^4\) The budget constraint then becomes

$$\dot{k} = w + rk - c - ph,$$

in which $r$ is the interest rate and $k$ are assets. Temporarily we return to the deterministic model. For the benchmark run we assume that there is neither inheritance nor bequest, such that $k(0) = k_0$, $k(T) = \bar{k}$. Individuals maximize life time utility (5) subject to (1)–(3) and (13) and boundary conditions for health deficits and assets.

\(^4\)Most importantly, since individuals base their optimization calculus on permanent income, the solution would not change by the introduction of an exogenous age of retirement or a more realistic hump-shaped age-income profile (as long as it provides the same present value of life time income).
From the first order conditions we obtain the modified Ramsey rule:

\[
\frac{\dot{c}}{c} = \frac{r - \rho + \alpha \left( \frac{\dot{h}}{h} - \frac{\dot{D}}{D} \right)}{\sigma} \tag{14}
\]

Notice that the standard Ramsey rule is obtained without health in the utility function (for \(\alpha = 0\)). With health in the utility function but without adaptation we observe a tendency for consumption to decline, in particular in old age when \(\dot{D}/D\) is large. Adaptation tends to neutralize this effect. For the special case of spontaneous adaptation (i.e. \(R = D\)) the standard Ramsey rule re-emerges.

Moreover, we obtain from the first order conditions:

\[
\frac{\dot{h}}{h} = \frac{1}{1 - \gamma} \left( r - \rho + \frac{\dot{\lambda}_D}{\lambda_D} \right) \tag{15}
\]

\[
\frac{\dot{\lambda}_D}{\lambda_D} = \rho - \mu - \frac{\mu \gamma A h^{\gamma - 1} e^\sigma}{p} \left[ \alpha \frac{\tilde{u}(c)}{D} - \lambda_R \theta \left( \frac{R}{D} \right)^{-\alpha} \right] \tag{16}
\]

For the special case when health does not matter for utility (\(\alpha = \theta = 0\)), we obtain the so-called health Euler equation of Dalgaard and Strulik (2014a), according to which the age-profile of health care expenditure is fully determined by the relative size of the interest rate \(r\) and the rate of aging \(\mu\) (\(\dot{\lambda}_D/\lambda_D = \rho - \mu\)). The health Euler equation is very intuitive: if the interest rate exceeds the rate of aging, it is desirable to save for health investments later in life (see Dalgaard and Strulik, 2014a, for details). Taking into account that health matters for instantaneous utility modifies this result. As shown in (16), the growth rate of the shadow price for health deficits declines for non-adapting and naive individuals (for \(\alpha > 0\) and \(\lambda_R = 0\)), implying that health expenditure grows at a lower rate than estimated by Dalgaard and Strulik (2014a). For sophisticated types (for \(\lambda_R > 0\)), on the other hand, the model predicts higher expenditure growth compared to non-adaptive and naive types. These results are intuitively plausible. For non-adapting and naive types, the health complementarity with consumption is all that matters. They understand that consumption will provide less utility in a bad state of health (i.e. in old age) and thus spend already relatively much on health care early in life in order to delay aging. Sophisticated types, in contrast, understand their adaptation to deteriorating health and thus allocate less funds to health when they are young.

---

5This way deteriorating health may motivate a hump-shaped age-profile of consumption, see Strulik (2015c).
The life cycle trajectories for the three types are shown in Figure 4. The upper-left panel confirms the inference from the first order conditions. Sophisticated types prefer a much steeper age-profile of health expenditure. The lower left panel shows the interesting implications for consumption. For non-adapting types consumption deteriorates continuously during the lifetime due to the health-consumption complementarity. As evidenced in the lower right panel, it is optimal for non-adapting types to go into debt early life in order to finance the desired high level of consumption when in good health.

Adaptation, in contrast, leads to a much more balanced age-consumption profile. It is also interesting to see that sophisticated types prefer a higher consumption level at any given age. Naive types spend too much for health expenditure early in life because they fail to understand their adaptation to deteriorating health.

Figure 4: Health Adaptation: Savings

Table 3 reports results from the sensitivity analysis. The main takeaway here is that the consideration of savings modifies results from the benchmark model only marginally. This
holds true for all cases discussed and despite the relatively large differences with respect to the allocation of funds over life shown in Figure 3.

Table 3: Results: Life Cycle Savings

<table>
<thead>
<tr>
<th>case</th>
<th>life time h</th>
<th>life expect.</th>
<th>value of life</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>naive sophis</td>
<td>naive sophis</td>
<td>naive sophis</td>
</tr>
<tr>
<td>1) benchmark</td>
<td>6.93</td>
<td>-16.53</td>
<td>0.52</td>
</tr>
<tr>
<td>2) (\alpha = 0.2)</td>
<td>10.24</td>
<td>-25.57</td>
<td>0.87</td>
</tr>
<tr>
<td>3) (\theta = 0.5)</td>
<td>7.15</td>
<td>-18.65</td>
<td>0.54</td>
</tr>
<tr>
<td>4) (\Delta A = 50%)</td>
<td>4.50</td>
<td>-21.93</td>
<td>1.29</td>
</tr>
<tr>
<td>5) (\Delta D_0 = 50%)</td>
<td>5.54</td>
<td>-2.58</td>
<td>0.20</td>
</tr>
</tbody>
</table>

All values as deviation in percent from the optimal solution for a non-adapting individual; life time \(h\) is the discounted life time expenditure on health; life expect. is life expectancy; \(A = 0.0013, \sigma = 1.13\). Other Parameters as for Table 1.

3.3. Extension: Life Cycle Savings & Uncertain Survival. Finally we combine the two previous extensions and investigate the case of uncertain survival with endogenous savings. For that purpose we assume perfect annuities such that the interest rate is given by the sum of the rate of return on capital \(r\) plus the instantaneous mortality rate \(m = -\dot{S}/S\). Given the annuity market, individuals inherit no wealth and leave no bequests. Capital left over at death is distributed among the survivors by the annuity supplier. We thus implicitly assume that our “Reference-American” is surrounded by sufficiently many other individuals of the same age. The adjusted budget constraint is given by

\[
\dot{k} = w + (r + m)k - c - ph. \tag{17}
\]

The solution of the associated maximization problem leads to the modified Ramsey rule (14), the shadow price equation (12) and the following equations of motion:

\[
\dot{h} = \frac{1}{1 - \gamma} \left( r + m - \rho + \frac{\dot{\lambda}_D}{\lambda_D} \right) \tag{18}
\]

\[
\dot{\lambda}_D = \rho - \mu - \mu \gamma \frac{A h^{\gamma - 1} \sigma}{p S(D)} \left( \frac{R}{D} \right)^{-\alpha} \left[ \left( \alpha \frac{S(D)}{D} - \frac{\partial S(D)}{\partial D} \right) \left( \frac{R}{D} \right)^{\alpha} \tilde{u}(c) - \lambda R \theta \right], \tag{19}
\]

which generalizes the health Euler equation for the case of uncertain survival.

Results are shown in Table 4. The most salient impression is again that the presence of uncertain survival and endogenous savings modifies the results for the simple case only marginally. Overall, the deviation of outcomes for adapting types from non-adapting types is reduced a bit.
further. The robust conclusion emerges that naive types spend about 6 percent more on health and live about 0.3 percent longer than non-adapting types while sophisticated types spend about 12 percent less on health and live about 0.5 percent shorter. Despite their opposing response to adaptation both naive and sophisticated types experience a value of life that is about 2 percent greater than that of non-adapting types. This difference is predicted to decline further with medical technological progress (case 5). Irrespective of health behavior, the trait of adaptation is helpful to cope with severe health shocks. After an increase of initial health deficits by 50 percent both naive and sophisticated types are predicted to draw about 6 percent more value out of their life than non-adapting types.

<table>
<thead>
<tr>
<th>case</th>
<th>life time $h$</th>
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<tr>
<td></td>
<td>naive sophis</td>
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<td>naive sophis</td>
</tr>
<tr>
<td>1) benchmark</td>
<td>5.69 -12.24</td>
<td>0.34 -0.56</td>
<td>1.94 1.98</td>
</tr>
<tr>
<td>2) $\alpha = 0.2$</td>
<td>8.96 -19.81</td>
<td>0.60 -0.92</td>
<td>3.54 3.64</td>
</tr>
<tr>
<td>3) $\theta = 0.5$</td>
<td>5.85 -13.52</td>
<td>0.35 -0.63</td>
<td>2.11 2.15</td>
</tr>
<tr>
<td>4) $\Delta w = 50%$</td>
<td>5.25 -12.55</td>
<td>0.40 -0.70</td>
<td>1.81 1.85</td>
</tr>
<tr>
<td>5) $\Delta A = 50%$</td>
<td>3.23 -15.91</td>
<td>1.02 -2.95</td>
<td>0.96 1.05</td>
</tr>
<tr>
<td>6) $\Delta D_0 = 50%$</td>
<td>5.68 -1.88</td>
<td>0.11 -0.03</td>
<td>5.93 5.94</td>
</tr>
</tbody>
</table>

All values as deviation in percent from the optimal solution for a non-adapting individual; life time $h$ is the discounted life time expenditure on health; life expect. is life expectancy; $A = 0.00135$, $\sigma = 1.085$. Other Parameters as for Table 1.

4. Conclusion

In this paper we developed a life cycle model in which human aging is based on a gerontological foundation of health deficit accumulation and investigated the role of adaptation to deteriorating health for health expenditure, life expectancy, and the value of life. We calibrated the model for a 20 year old average male U.S. American in the year 2000 and contrasted the outcome without adaptation with those for otherwise identical individuals who adapt to deteriorating health in a conscious (sophisticated) or unconscious (naive) way. Perhaps surprisingly, we found that naive types invest the most in their health and live the longest. Their high health expenditure is motivated by two mechanisms. Firstly, they are not aware of their adaptation to poor health, which motivates health expenditure above that of sophisticated types. Secondly, due to their adaptation, they lead a relatively happy life, in particular in old age. They thus have a strong desire to extend it, which motivates health expenditure above that of non-adapting types.
Sophisticated types, in contrast, are predicted to invest the least in health and live the shortest life. Yet, they experience the greatest value of life. Their awareness of adaptation to poor health induces these individuals to put more emphasis on current consumption and to derive more pleasure here and now at the expense of worse future health. While the health behavior of naive and sophisticated types differs drastically and has significant impact on their life expectancy, we find, perhaps surprisingly, only small differences for the experienced value of life. According to our benchmark specification, the VOL of sophisticated types is estimated to be 0.05 percent above that of naive types. The impact of adaptation as such on the VOL is somewhat larger but, again, perhaps smaller than expected. Both naive and sophisticated types experience an about two percent higher VOL compared to non-adapting types (for which the VOL at age 20 is estimated to be $6.5 million). This finding may have some importance for public policy and the problem whose preferences should be used in determining the value of public health projects (Menzel et al., 2002). Should public health projects be evaluated on the basis of utility of the average, potentially non-sick citizen or on the basis of utility of the actual patients who may have largely adapted to their poor health condition? While our study provides no answer to this normative question, our quantitative results suggest that the expected error from using the “wrong” preferences will be small.

Our framework could be extended and applied to investigate other life cycle decisions and health behaviors. One extension could address the impact of health adaptation on retirement. Under the assumption that the retirement decision is, among other things, determined by the subjectively perceived health status, we would expect that adaptation to poor health leads to later retirement for sophisticated agents but not necessarily for naive agents who may actually aim at an early retirement. Through the retirement decision and the length of the working life we would expect that health adaptation also affects the education decision and thus labor income. Another potential extension could consider the role of adaptation for unhealthy consumption and individual investments of time and effort in health promoting activities. In particular, it would be interesting to investigate how individuals change their nutrition behavior when they adapt to increasing body weight in a naive or sophisticated way.
Appendix: Solution Method

The set of first order conditions including the initial and terminal conditions represent a continuous time, two-point boundary value problem with free terminal value. We numerically solve for the optimal path of variables by using the Relaxation procedure (see Trimborn et al., 2008). Since the procedure was designed to solve problems with a fixed endpoint, we modified it such that it can handle problems with a free terminal time. Furthermore, we use a recently developed method to ensure that non-negativity constraints for health investments and consumption hold (see Trimborn, 2013, for details).

The idea of our modification of the Relaxation algorithm is to distinguish between age $t$ of the individual and computational time $\tau$ for which the solution is computed. While the domain of $t$, $[0, T]$, is endogenous, the domain of $\tau$ is exogenous and normalized to the unit interval $[0, 1]$. We assume that $t$ proceeds proportionally to $\tau$, $t = \psi \tau$, with a constant $\psi$. Hence, the variable $\psi$ governs the terminal time by the equation $T = \psi$.

To illustrate our procedure in detail we denote the vector of variables by $x$ and the set of first order conditions represented by differential equations by $dx/dt = f(x)$. Firstly, we have to take into account that the algorithm solves for the path of the variables with respect to $\tau$ instead of $t$. Hence, the set of differential equations modifies to

$$\frac{dx}{d\tau} = \frac{dx}{dt} \frac{dt}{d\tau} = f(x)\psi.$$

Secondly, we augment the set of variables by $t$ and $\psi$, and the two differential equations

$$\frac{dt}{d\tau} = \psi,$$

$$\frac{d\psi}{d\tau} = 0.$$

While the first differential equation establishes that $t$ and $\tau$ are proportional with proportionality factor $\psi$, the second one determines that $\psi$ is a constant. Furthermore, we fix the starting time by augmenting the initial conditions with the equation $t(0) = 0$. The terminal optimality condition for the Hamiltonian requiring $H(T) = 0$ implicitly determines the optimal stopping time and therefore $\psi$.

The solution vector consists of the optimal values of $x$ on a time mesh $\tau \in [0, 1]$ and a corresponding time vector $t$ enabling us to assign the solution $x$ to the individual’s age $t$. 
References


