

**THE HEALTH HUMP**

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**Abstract.** In this paper, I suggest a novel explanation for a hump-shaped age-consumption profile, based on human aging. The model integrates health in the utility function and utilizes recent estimates on the effects of health on the marginal utility of consumption. The parsimonious model has a closed-form solution for the age of peak consumption and the consumption level at that age relative to initial consumption. A calibration of the model with data from gerontology produces an empirically plausible hump in consumption.

*Keywords:* health, aging, life-cycle consumption.

*JEL:* D91, E21, I10.

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## 1. INTRODUCTION

A salient phenomenon of consumer behavior is a hump-shaped age-consumption profile with consumption peaking at around age 45 to 55 at a level of about 1.1 relative to consumption when entering the work force (e.g. Gourinchas and Parker, 2002). The hump could be regarded as a puzzle for mainstream economics since standard life-cycle theory, based on consumption smoothing, predicts a monotonous (constant, increasing, or declining) age-consumption profile (Attanasio and Weber, 2010). Meanwhile, the literature has proposed several (complementing) mechanisms explaining the consumption puzzle, based on precautionary savings (Carroll and Summers, 1991), changing preferences (Attanasio et al., 1999) or changing household size (Browning and Ejrnaes, 2009), on consumption and labor supply complementarity (Heckman, 1974; Bullard and Feigenbaum, 2005), consumer durables (Fernandez and Krueger, 2007), short-term planning (Caliendo and Huang), and overconfidence (Caliendo and Huang, 2007). Here, I present another mechanism, based on human aging. Compared to the existing proposals the health channel is minimal-invasive since it requires only a modest modification of the standard model, namely to acknowledge that the utility derived from consumption depends on the state of health.

A necessary assumption for the health hump to occur is a negative influence of deteriorating health on utility. This is supported by a recent study by Finkelstein et al. (2013) estimating that the marginal utility from consumption declines by 11% when the number of health deficits increases by one standard deviation. However, there are also earlier studies estimating a higher or lower effect (see Finkelstein et al., 2013, for an overview).

As a stylized fact, humans develop about 3 to 4 percent more health deficits from one birthday to the next. This insight can be gained from a series of studies based on the so-called frailty index (e.g., Mitnitski et al, 2001; 2002a,b; 2005; Rockwood and Mitnitski, 2007). The frailty index measures the proportion of the total potential deficits that an individual has, at a given age. The list of potential deficits ranges from mild nuisances (e.g., reduced vision) to serious ones (e.g., strokes). According to Rockwood and Mitnitski (2007), the exact choice of deficits does not effect the results, provided that sufficiently many indicators (40 or more) are present in the index. Mitnitski et al. (2002a) show that the relative number of health deficits  $D(t)$

increases exponentially with age  $t$  in the following exponential way

$$D(t) = E + Be^{\mu t}. \quad (1)$$

This “law of deficit accumulation” explains around 95 percent of the variation in the data, and its parameters are estimated with great precision. Mitnitski et al., using a data set of 19 to 79 years old Canadians, estimate that for men,  $E = 0.02$  (0.001),  $\log(B) = -5.77$  (0.06), and  $\mu = 0.043$ (0.001), with standard errors in parenthesis. Rockwood and Mitnitski (2007) show that Australians, Swedes, and U.S. Americans accumulate deficits in an exponential way very similar to Canadians. Harttgen et al. (2013) find similar results for European countries. For most nations, the force of aging  $\mu$  is found to be around 3 to 4 percent.

Conceptually, the index of health deficits is very useful for economists since it provides the state of health as one number, which is easy to observe and estimate with great precision. Compared to health capital (Grossman, 1972), which is a latent variable and hard to capture empirically (see e.g. Wagstaff, 1986), the health deficit index allows for an easy transfer of knowledge from medicine and gerontology to economics (and vice versa) and facilitates a straightforward calibration of economic models with health data. It has recently been applied to shed new light on the Preston curve (Dalgaard and Strulik, 2014), the education gradient (Strulik, 2013); and the long-run evolution of retirement (Dalgaard and Strulik, 2012). Here, I use it to show that the life cycle theory of consumption augmented by a negative dependence of utility on health can account for an empirically plausible hump-shaped age-consumption profile.

## 2. THE MODEL

Individuals derive instantaneous utility from consumption and from their health. Health is a shifter of the “ordinary” isoelastic utility function. The state of health is measured by the index of accumulated health deficits  $D$  such that the utility function reads:

$$u(c, D) = D^{-\alpha} \cdot \tilde{u}(c), \quad \text{with } \tilde{u}(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} & \text{for } \sigma \neq 1 \\ \log(c) & \text{for } \sigma = 1. \end{cases} \quad (2)$$

The flow-form of the law of health deficit accumulation (1) is given by

$$\dot{D} = \mu(D - E) \quad \Rightarrow \quad \frac{\dot{D}}{D} = \mu \left( 1 - \frac{E}{D} \right), \quad D > E. \quad (3)$$

Individuals may save for consumption later in life and face a given wage  $w$  and a given interest rate  $r$  such that their budget constraint reads

$$\dot{k} = rk + w - c. \quad (4)$$

We could have alternatively assumed that individuals retire at some age and that labor income is first increasing and then declining with age. Since individuals base their decisions on life time income, these extensions would not change any of the results. In order to demonstrate that the theory does not essentially depend on these features – in contrast to some of the studies cited in the Introduction – we consider a constant stream of labor income.

Let  $\rho$  denote the time preference rate and let the initial age be normalized to zero (later, in the calibration, this will be an actual age of 20). Individuals maximize their life time utility  $\int_0^T e^{-\rho t} u(c, D) dt$ , given initial capital and health deficits and initial capital,  $D(0) = D_0$ ,  $k(0) = k_0$ , and terminal health deficits and terminal capital  $D(T) = \bar{D}$ ,  $k(T) = \bar{k}$ . The individual dies at age  $T$  when  $\bar{D}$  health deficits have been accumulated.

The current-value Hamiltonian of this maximization problem is

$$H = D^{-\alpha} \tilde{u}(c, D) + \lambda_k (w + rk - c) - \lambda_D \mu (D - E), \quad (5)$$

in which  $\lambda_k$  and  $\lambda_D$  denote the co-states. The first order condition for consumption and the co-state equation for capital are

$$D^{-\alpha} c^{-\sigma} - \lambda_k = 0, \quad \text{and} \quad (6)$$

$$\lambda_k = \lambda_k \rho - \dot{\lambda}_k. \quad (7)$$

For the full solution, we would also need to evaluate the costate equation for  $D$  and the Hamiltonian at the time of death  $H(T)$ . Here, we are only interested in consumption relative to initial consumption, which can be obtained independently from the full solution of the model.

### 3. RESULTS

Differentiating (6) with respect to age and inserting  $\dot{\lambda}_k$  and  $\lambda_k$  provides the health-modified Ramsey rule for optimal consumption:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma} \left( r - \rho - \alpha \frac{\dot{D}}{D} \right). \quad (8)$$

The solution is very intuitive. For  $\alpha = 0$ , we obtain the standard solution. For  $\alpha > 0$ , individuals take their aging into account. Since they derive less pleasure from an extra unit of consumption in poor health, i.e. when health deficits  $D$  are large, consumption growth is declining in the speed of health deficit accumulation  $\dot{D}/D$  weighted by the importance of health for utility ( $\alpha$ ).

From inspection of (8) we see that  $r > \rho$ , i.e. the standard assumption in models of economic growth, is necessary but not sufficient for a first positive and then negative rate of consumption growth, i.e. for a hump. Inspection of (3) shows that the rate of health deficit accumulation is initially close to zero (when  $D$  is close to  $E$ ) and increasing with age, i.e. it is higher when many health deficits have been accumulated. This means that the health term dominates when  $D$  becomes sufficiently large, and the growth rate becomes negative. Sufficient for a hump is thus that the health term becomes dominating before the individual dies (in conjunction with  $r > \rho$ ).

If it exists, the age of peak consumption is straightforward to compute in closed-form. Setting  $\dot{c} = 0$  in (8) and using (3), we obtain health deficits at the age of peak consumption:

$$D = D^* \equiv \frac{\alpha\mu E}{\alpha\mu - (r - \rho)}, \quad (9)$$

which are independent from the curvature of consumption utility ( $\sigma$ ). Inserting  $D^*$  into (1) and solving for  $t$  provides the age of peak consumption:

$$t = t^* \equiv \frac{1}{\mu} \log \left( \frac{(r - \rho)E}{\alpha\mu - (r - \rho)B} \right). \quad (10)$$

The age of peak consumption is increasing in the interest rate  $r$  and declining in the time preference rate  $\rho$  as well as in the rate of aging  $\mu$ . Peak consumption is also reached earlier if health is important for utility (if  $\alpha$  is large).

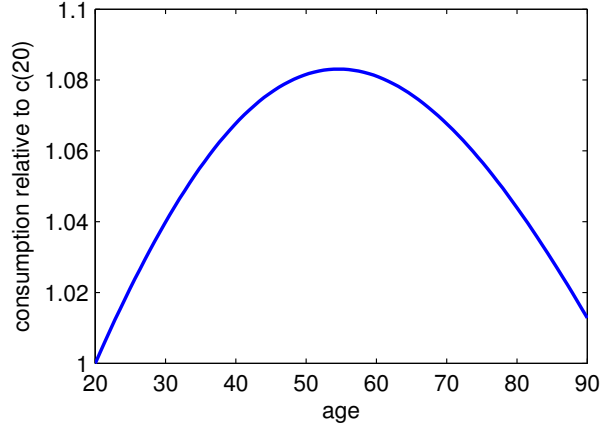
In order to obtain the level of peak consumption, we solve the differential equation (8) given initial consumption  $c_0$ :

$$c(t) = c_0 e^{1/\sigma \int_0^t r - \rho - \alpha\mu[1 - E/D(\tau)]d\tau} = c_0 e^{1/\sigma[(r - \rho)t - \alpha \log(E + B \exp(\mu t) + \alpha \log(E + B))]} \quad (11)$$

The right hand side of (11) follows from inserting  $D(\tau)$  from (1) and solving the integral. Finally, inserting  $t^*$  from (10) into (11) and dividing by  $c(0)$  provides peak consumption relative to initial consumption. Peak consumption is increasing in the interest rate and declining in the

time preference rate and the curvature parameter  $\sigma$ , as well as declining in the rate of aging  $\mu$  and the weight of health in utility  $\alpha$ .

Figure 1: The Health Hump



The Figure shows the solution (11) for  $r - \rho = 0.008$ ,  $\alpha = 0.3$ , and  $\sigma = 1$ ; consumption is expressed relative to initial consumption at age 20.

Since the biological parameters are estimated with high precision, there are relatively few degrees of freedom. Given the estimates of Mitnitski et al. (2002a), cited in the Introduction, the age of peak consumption is solely determined by the difference  $r - \rho$  and the weight of health in utility  $\alpha$ . The size of the hump, additionally, depends on  $\sigma$ . Figure 1 shows the age-consumption profile for  $\sigma = 1$ ,  $\alpha = 0.3$ , and  $r - \rho = 0.008$ , assuming an initial age of 20 years. Table 1 shows results for  $\sigma = 1$  and alternative values for  $r - \rho$  and  $\alpha$  for  $\sigma = 1$ . Recent estimates of the intertemporal elasticity of substitution suggest that the “true” value of  $\sigma$  is probably close to unity (e.g. Chetty et al., 2006). For values of  $\alpha$  around 0.3 and an interest rate differential of .8 percent, the age of peak consumption is in the neighborhood of the empirical estimates and the size of the hump is somewhat smaller than suggested by the literature, leaving room for alternative channels.

As shown in Table 1, one gets a bigger hump from larger values of  $\alpha$  and  $\mu$ . A value of  $\alpha$  much larger than 0.3, however, is perhaps hard to defend against the empirical observation of Finkelstein et al. (2012). This study estimates that an increase of 1 standard deviation of health deficits reduces marginal utility from consumption by about 11%. According to Mitnitski et al. (2001) the standard deviation of most health deficits in the frailty index is around  $0.4/\tilde{\mu}$ , in which  $\tilde{\mu}$  is the mean of the particular deficit. The mean frailty index from (1) for individuals

Table 1: Health Hump – Sensitivity Analysis

		$r - \rho$		
		0.005	0.008	0.011
$\alpha = 0.2$	$t_{max}$	50.85	103.45	–
	$c_{max}$	1.04	1.22	–
$\alpha = 0.3$	$t_{max}$	32.57	54.61	84.05
	$c_{max}$	1.01	1.08	1.25
$\alpha = 0.4$	$t_{max}$	22.46	39.96	56.54
	$c_{max}$	1.01	1.04	1.13

$t_{max}$  is the age of maximum consumption;  $c_{max}$  is the level of maximum consumption relative to consumption at age 20. Numbers are rounded.

between 19 and 79 years is about 0.05 with a standard deviation of about 0.02. The estimate of Finkelstein et al. then suggests that

$$\frac{(0.05 + 0.02)^{-\alpha}}{(0.05)^{-\alpha}} = 1 - 0.11 \quad \Rightarrow \quad \alpha = \frac{\log(0.89)}{\log(5/7)} = 0.34.$$

However, as acknowledged in the Introduction, earlier studies using other methods estimated different effects of health on marginal utility. It should also be noted that Finkelstein et al. use a different concept of health deficits, comprising only seven chronic diseases, i.e. considerably less than what is included in the frailty index, which may cause measurement error in the estimation of  $\alpha$ . A value of around 0.34 should thus not be taken at face value but at as an initial reference point. It implies that an empirically plausible consumption hump is observed when the interest exceeds the time preference rate by about 0.8 to 1.1 percent.

#### 4. FINAL REMARKS

This study has suggested a novel motivation of the hump-shaped age-consumption profile based on a standard life cycle model extended by health-dependent utility and human aging. When calibrated with the estimated law of health deficit accumulation, the model produces an empirically plausible hump. It can be shown that these results are robust against extensions towards uncertain survival and individual health investments. With respect to the impact of health deficits on marginal utility, the model could benefit from future studies corroborating the available estimates. Yet, compared to health capital, a non observable entity, the use of health deficits appears to be in any case one step forward in quantitative life cycle economics.



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