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DYNAMICS OF YARDSTICK REGULATION: HISTORICAL COST DATA AND THE RATCHET EFFECT

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Dynamics of Yardstick Regulation: Historical Cost Data and the Ratchet Effect*

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Abstract

Real life applications of yardstick regulation frequently refer to historical cost data. While yardstick regulation cuts the link between firms' own costs and prices firms may charge in a static setting, it does not do so in a dynamic setting where historical cost data is used. A firm *can* influence the price it will be allowed to charge in the future if its behavior today can affect future behavior of other firms that determines the price this firm will be able to charge later on. This paper shows that, assuming that slack, inflation of costs, is beneficial to firms, a trade-off between short term profit through abstinence from slack and the benefit of slack in (infinitely) many periods arises. A ratchet effect that yardstick regulation was meant to overcome can occur and firms can realize positive rents *because of* the use of historical cost data, even if firms are identical. Equilibria with positive slack can exist without any collusion between firms or threat. Moreover, this problem is more severe if the firm with the lowest costs of all other firms instead of the average firm is the yardstick.

Keywords: yardstick regulation, yardstick competition, ratchet effect, historical cost data
JEL classification: L51, L98, C73

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1 Introduction

Natural monopolies are frequently subject to regulation. As 'natural' competition does not force prices towards a perfect competition outcome, often regulatory agencies jump in to 'regulate' profit, prices or revenue. Under traditional rate of return regulation, allowed profit of a firm is linked to capital employed. The well known result of Averch and Johnson (1962) is that this regulation provides incentives for the firms to employ an inefficient input mix and not to engage in cost minimizing behavior – in other words, to produce with some slack. Incentive regulation is meant to address this issue. Price cap regulation, originally suggested by Littlechild (1983), decouples costs incurred and prices allowed to be charged by fixing or capping prices, no matter what costs are. Thus, the firm becomes the residual claimant of all costs not incurred and so, has a strong incentive to produce without slack if profit is worth more to the firm than slack is. Necessarily, the question of how the price cap should be defined arises. If the regulator takes into account profits made and costs incurred, the incentive structure is much less clear cut, as e.g. Train (1991) points out. The basic idea of yardstick regulation, as described by Shleifer (1985), solves this problem by using information on costs of other comparable firms to define prices a firm is allowed to charge. In a static world and in every period prices and costs for each individual firm are, as a consequence, completely independent of each other. In the absence of collusion, yardstick regulation fosters efficient production, especially if firms and circumstances of production are very similar. Tangerås (2002, p. 232) summarizes: "the regulator is able to extract *all* surplus from firms and reach full efficiency if technologies are perfectly correlated." This paper shows that this property does not carry over into a dynamic setting if historical cost data is used. A firm can influence the price it is allowed to charge in the future via its effect on the behavior of other firms. Consequently, without any collusion a ratchet effect can occur under yardstick regulation using historical cost data as a result of individual and independent decision making of firms.

The contribution of this paper is twofold: In a simple dynamic model with three firms, we show that every firm can affect the price it is allowed to charge if this price is a function of the costs of the other two firms in the period before. By this we highlight a feature of real world applications of yardstick regulation that has not received much attention both in academic literature and in regulatory practice: historical costs are used to define constraints. Furthermore, we compare two variants of yardstick regulation: either the firm with the lowest costs of all firms but the evaluated one, or the average of the other firms can be used as the yardstick. Intuitively, orientation at the best seems to be the tougher form. However, we show that choosing this scheme might lead to higher slack and a worse situation for society.

A well cited example of the use of average performance for regulation is the Prospective Payment System of Medicare (originally Shleifer, 1985), whereas e.g. the German regulation for electricity networks follows a best practice/frontier approach. Yardstick mechanisms are also used in the regulation of, for instance, the water industry in the UK (Cowan, 2006) or railway services in Japan (Mizutani et al., 2009).[1] Real life examples of yardstick regulation usually have in common that the price of a service offered is set and known before customers use the service. For instance, at the start of a regulatory period prices or constraints are defined based on observations of costs from the regulatory period before.

Aspects of yardstick regulation that are subject to debate or known drawbacks are collusion among firms (e.g. Tangerås, 2002; Potters et al., 2004), investment behavior (e.g. Dalen, 1998; Sobel, 1999) and the potential inability of a regulator to commit to a regulatory scheme for the future (Faure-Grimaud and Reiche, 2005). Moreover, quality might be adversely affected under incentive regulation in general, which makes additional quality regulation necessary (see Sappington, 2005, for a survey). Firms may also lack comparability necessary for the implementation of yardstick regulation (e.g. Laffont and Tirole, 1993). In this paper, we abstract from these issues and show that the desired outcome, i.e. efficient production, might still not be reached.

We derive our results in a dynamic model with three firms, an infinite horizon and discrete time. As we are interested in the long run effects of the use of historical cost data under yardstick regulation, we focus on the analysis of resulting steady state equilibria. In order to formalize the absence of collusion and Folk Theorem arguments in our result, we define punishment-free Markov-perfect steady state equilibria: these are Markov-perfect steady state Nash equilibria such that firms do not (coordinatedly) choose a uniform slack that is individually optimal for every firm only because of other firms choosing this slack. We show that such equilibria with positive slack, i.e. inefficient production and positive rents for firms, can exist. Furthermore, we show that the highest slack that can exist in such a steady state equilibrium is higher if the firm with the lowest costs of all other firms instead of the average of the other firms is used as the yardstick.

In this paper, the modelling of slack, i.e. lack of costly effort, differs from a major part of the contributions to the debate on incentive regulation, represented especially by Laffont and Tirole (1993) or Laffont (1994): in these models costly effort reduces

[1] While this paper focuses on yardstick regulation of firms, in particular natural monopolies, relative performance measures can be used in a broader range of settings where asymmetric information structures are present. For instance, voters may judge incumbent politicians relative to the performance of other politicians in other jurisdictions (Besley and Case, 1995) or workers might be paid based on their ordinal position of performance among their colleagues (Lazear and Rosen, 1981).

costs of production. In our model, inefficiency costs, slack, are added to real, necessary costs of efficient production and producing with slack offers nonmonetary benefit to the firms. The instantaneous payoff function used is very similar to the one in Blackmon (1994). This is done as it allows for straightforward interpretation of the results and explicitly models the idea that yardstick regulation is meant to solve the inefficiency problem of traditional rate of return regulation. However, this is not a substantive difference but only a different way of presentation.

A key structural element of our model is the time horizon used. In models considering only two periods, the effect driving our results is not present: under yardstick regulation using historical cost data current choices of a firm do not affect the price this firm can charge in the current and the next period. The direct effect is only visible from the next but one period onward. Like Meran and Hirschhausen (2009) we use dynamic programming techniques to account for long run effects of the decisions of firms. However, we come to differing conclusions. The main difference between their model, which is expanding the model of Shleifer (1985), and our approach leading to these differing results is that Meran and Hirschhausen (2009) do not allow the firms to benefit from slack and consequently firms do not gain from keeping costs high.

The remainder of this paper is structured as follows: Section 2 explains the model setup. In Section 3 all possible punishment-free Markov-perfect steady state equilibria are characterized, existence is proven and the two regulatory schemes are compared with respect to equilibrium outcomes. Section 4 concludes.

2 Description of the model

2.1 Firms

There are three firms, labelled $j = i, o, x$, each producing a homogeneous output normalized to one. The output is bought by the consumers. For example, one could think of demand for electricity which is very inelastic with respect to price or demand for some crucial medical treatment. These firms could be thought of as catering three comparable regions with electricity grids as local monopolists. The only way they interact in 'competition' is via the regulation imposed on them. In every period, the regulator assigns a price to each of the firms. Each firm must not charge more than this price for its output, so the regulator defines a price cap which is equivalent to a revenue cap under the assumption of completely inelastic demand. As demand does not react to price in this setting, all firms always charge the maximum price they are allowed to.

Whereas the firms' output is directly observable the underlying cost structure is unknown to the regulator. Each firm verifiably reports its costs to the regulator who cannot distinguish between 'real' necessary cost, $C > 0$, and slack, $S_t^j \geq 0$, defined as additional costs due to inefficient use of resources, and only observes the sum of both. C does not change over time and is the same for all firms. This is equivalent to assuming that the regulator correctly and completely accounts for all heterogeneity between firms and (exogenous) circumstances of production.[2] Each firm chooses its slack and may choose different slacks in different periods. For instance, slack can be interpreted as a lack of (costly) effort from managers, oversized offices or all kinds of 'unnecessary' costs that might occur under rate of return regulation. As slack is inefficient production by definition, the regulator maximizing the utility of society desires to avoid all slack without explicit consideration of a target function.

If firm j chooses a positive slack in period t, it realizes a nonmonetary utility denoted by $B(S_t^j)$. B is twice continuously differentiable with $B(0) = 0$, $1 > B' > 0$ and $B'' < 0$. Accordingly $B(S_t^j) < S_t^j$ for all $S_t^j > 0$. If the sum of necessary costs and slack is smaller than the price the firm is allowed to charge, it additionally realizes a profit. The marginal benefit from an additional unit of profit is constant and normalized to 1. Increasing profit and decreasing slack are two sides of the same coin as they add up to a constant: the price a firm charges less necessary costs. Hence, it is sufficient to explicitly consider just one of the two as the other one emerges as the residual. The instant payoff function of firm j is in every period given by

$$F_t^j = P_t^j - C - S_t^j + B(S_t^j). \tag{1}$$

Firms care about profit and slack only. They discount next period's utility with δ, $0 < \delta < 1$, and maximize their intertemporal utility:

$$\sum_{t=0}^{\infty} \delta^t F_t^j \tag{2}$$

Firms need to break even at all times, so that $C + S_t^j \leq P_t^j$. Slack is 'expensive' not only from the perspective of the regulator or society: one marginal unit of additional profit always results in higher instantaneous utility for the firm than an additional marginal unit of slack would. The only reason why $S_t^j > 0$ could be an optimal choice of j is that it can affect the price j is allowed to charge in later periods.

We consider an infinite number of periods in order to avoid unrealistic effects

[2]In Shleifer's (1985) one-period model accounting completely and correctly for heterogeneity leads to the efficient equilibrium.

of last rounds in which all slack is zero.[3] Every period there is only one choice per firm to be taken: the slack the firm chooses. The regulatory rule and break even condition are common knowledge, and so are the prices of the current period. Using this knowledge, firms can anticipate how their choice of slack will affect future behavior of the other firms. Accordingly all three current prices are state variables for all j.

Strategies are anonymous, so if firms o and x initially do the same, firm i reacts to a change in behavior of o with constant behavior of x just as it would react to a change vice versa. Simple renaming o into x and x into o does not affect the behavior of i. Firms simultaneously choose their slack every period without observing the current choice of the other firms.

Only Markov-perfect strategies[4] are considered, so firms react to the state variables they observe and do not care about the history of states. We exclude collusion between firms as well as arguments based on Folk Theorems,[5] which can be seen as a form of collusive behavior, from the analysis as yardstick regulation is obviously highly vulnerable to collusion. This paper does not strive to offer solutions for this issue but proceeds to show that even if all collusive behavior can be avoided, uncoordinated individual utility maximization by firms can result in equilibria with positive slack. Therefore we restrict our attention to strategies that are not based on collusion or coordination and exclude that firms coordinatedly choose a uniform slack that is otherwise not an optimal choice for any individual firm.

2.2 Regulatory rules

The price a firm is allowed to charge is derived from costs realized by the other two firms in the previous period. We separately look at two regulatory schemes: average yardstick regulation under which average costs of the other firms are used as the yardstick, and frontier yardstick regulation or best practice regulation under which only the costs of the best performing firm of all others, i.e. the firm with the lowest costs, are the yardstick. For example, the price that firm i is allowed to charge in period $t+1$ is accordingly a function of the slack o and x are choosing in t in both cases:

$$P^i_{t+1} = R^i(S^o_t, S^x_t). \tag{3}$$

[3] It is easy to show that a finite horizon and the corresponding backward solution will result in zero slack starting in the very first period.

[4] The corresponding concept of Markov-perfect equilibria goes back to Maskin and Tirole (1988 a and b).

[5] See e.g. Osborne and Rubinstein (1994) for a description of trigger strategies and Folk Theorems.

Under frontier yardstick regulation the price is given by

$$P^i_{t+1} = \min(C + S^o_t, C + S^x_t) = C + \min(S^o_t, S^x_t) \tag{4}$$

and under average yardstick regulation by

$$P^i_{t+1} = \frac{1}{2}\sum_{j\neq i}\left(C + S^j_t\right) = C + \frac{1}{2}\sum_{j\neq i} S^j_t. \tag{5}$$

Regulatory rules for the other firms and periods are defined analogously. Since necessary costs are constant, C can be factored out under both regulatory regimes and can be normalized to zero. This is equivalent to interpreting P^i_{t+1} as the amount by which the price i may charge in $t+1$ is greater than necessary costs C.[6] In the first period of yardstick regulation, prices are exogenously given: they could be derived from some regulatory rule that was in place before yardstick regulation was implemented.

Lemma 1. *Under both regulatory rules, slacks and prices converge to a steady state in which all firms choose the same slack and realize zero profits due to regulatory mechanics. This slack may be zero.*

Proof. See appendix.

As long as not all firms choose the same slack and this slack is equal to the price they are allowed to charge (C is normalized to zero), the highest slack chosen in t cannot be chosen by any firm anymore in $t+2$ at the latest. Accordingly, there is a downward drift of the highest slack, whenever firms choose differing slacks. As slack cannot become negative, convergence is assured.

3 Equilibrium analysis

It is easy to show that equilibria with very high slack could exist, given initial prices are sufficiently high, if firms punish other firms' uncooperative behavior. Unilateral punishment conditioned on other firms' past behavior is precluded, by restricting our attention to Markov-perfect strategies. However, firms could follow a Markov-perfect strategy which includes extreme slacks, e.g. zero slack, if they observe a specific vector of prices. From the proof of Lemma 1, it directly follows that under frontier yardstick regulation, every firm can force all firms into a steady state equilibrium with zero slack by choosing zero slack once. This is the worst possible steady state

[6]Necessary costs C remain, however, unknown to the regulator.

from the perspective of all firms. Therefore, if at least one firm chooses zero slack, all other firms can choose zero slack, and thereby the highest feasible instantaneous profit without adversely affecting future payoffs. Consequently, the best response to other firms choosing an extreme slack could be choosing the same extreme slack. In the spirit of the Folk Theorem (a threat of) 'joint mutual punishment', i.e. firms each choosing an extreme slack because of other firms choosing this slack, could be used to implement equilibria with very high slack. Such equilibria would involve aspects of a coordination game.

The analysis of corresponding equilibria does not offer much additional insight as yardstick regulation is known to be highly vulnerable to collusion. Joint mutual punishment, that no firm would do unilaterally, can be seen as a form of collusion. To this end, we explicitly exclude all sorts of joint mutual punishment, collusion or coordination from our analysis and show that steady state equilibria with positive slack that are 'punishment-free' can still exist. Therefore, we restrict our attention to the subset of Markov-perfect equilibria satisfying the following definition: Let $f^i(\mathbf{P_t}), f^o(\mathbf{P_t})$ and $f^x(\mathbf{P_t})$ be the Markov-perfect strategies of firms i, o and x respectively, and $\mathbf{P_t}$ be the vector of prices valid for firms i, o and x in period t. Consider a Markov-perfect equilibrium $\left(f^i(\mathbf{P_t}), f^o(\mathbf{P_t}), f^x(\mathbf{P_t})\right)$. It is called punishment-free if for all $\mathbf{P_t}$ where $f^j(\mathbf{P_t}) = f^k(\mathbf{P_t})$, with $j, k = i, o, x$ and $j \neq k$, at least one strategy $f^l(\mathbf{P_t}) = f^j(\mathbf{P_t})$, with $l = i, o, x$, is also the best response to all $m = i, o, x$, with $m \neq l$, choosing $S_t^m > f^m(\mathbf{P_t})$.

In a punishment-free equilibrium, firms do not choose a uniform slack that is optimal if and only if other firms also choose this slack and that is not an optimal choice of slack for any firm if all other firms choose higher slacks. Thus, whenever firms choose the same slack, for at least one firm, this slack must remain optimal if all but this firm choose higher slacks instead. In other words, we exclude that firms choose a uniform slack that is for each firm optimal only because of other firms doing so.

Below we implicitly define an optimal value of slack each, denoted by S^*, that maximizes intertemporal utility given current prices under the respective regulatory regime that can characterize a steady state equilibrium. Furthermore, we derive a unique level of slack, S^{M*} under frontier yardstick regulation and S^{A*} under average yardstick regulation, which offers the highest intertemporal utility for the firms under the respective regime and that can exist in a punishment-free Markov-perfect steady state equilibrium, given that prices are sufficiently high. As will be shown, S^{M*} is implicitly defined by

$$B' = 1 - \delta^2 \tag{6}$$

and S^{A*} by

$$B' = 1 - \frac{\frac{1}{2}\delta^2}{1 - \frac{1}{2}\delta}. \tag{7}$$

Equations (6) and (7) summarize the respective tradeoff between the marginal benefit of reducing slack in the current period and the corresponding marginal costs from adversely affecting future payoff each firm faces every period under both regulatory schemes.

We show that every $S^* \epsilon [0, S^{M*}]$ and $S^* \epsilon [0, S^{A*}]$, under frontier yardstick regulation and average yardstick regulation respectively, can occur in a punishment-free Markov-perfect steady state equilibrium, provided that the initial prices are high enough. Conversely, no other slack is possible in such an equilibrium.

3.1 Optimal slack

Assume there exists a steady state equilibrium consistent with the triple of punishment-free Markov-perfect strategies of firms i, o and x, denoted by $f^i(\mathbf{P_t})$, $f^o(\mathbf{P_t})$ and $f^x(\mathbf{P_t})$ respectively. By definition, strategies need to be optimal in equilibrium. Firms decide on their slack considering their discounted utility in all periods to come given they decide optimally in all future periods given future states. The Principle of Optimality[7] is used to find the resulting optimal level of slack for firm i. So firm i solves the following maximization problem:

$$J^i(\mathbf{P_t}) = J^i(P_t^i, P_t^o, P_t^x) = \max_{S_t^i \leq P_t^i} [F(P_t^i, S_t^i) + \delta J^i(\mathbf{P_{t+1}})], \tag{8}$$

where J^i denotes the value function of firm i and $\mathbf{P_{t+1}}$ is the vector of prices in $t+1$. By Theorem 6.4 and the relaxed Assumption 6.3, i.e. (weak) concavity of the instant payoff function, of Acemoglu (2009) the value function is (weakly) concave in the state variables. Thus, the problem is well-behaved. While the state in t is given, the state in $t+1$ is determined by the regulatory rule. Plugging the general form of this rule in leads to

$$J^i(\mathbf{P_t}) = \max_{S_t^i \leq P_t^i} [F(P_t^i, S_t^i) + \delta J^i\Big(R^i(S_t^o, S_t^x), R^o(S_t^i, S_t^x), R^x(S_t^i, S_t^o)\Big)]. \tag{9}$$

Just as firm i, firms o and x maximize their intertemporal utility given the state variables they observe. So

$$S_t^o = f^o(\mathbf{P_t}) \tag{10}$$

[7]See e.g. Acemoglu (2009) or Stokey, Lucas with Prescott (1989) for a detailed description.

and

$$S_t^x = f^x(\mathbf{P_t}) \tag{11}$$

describe the optimal slack of o and x given $\mathbf{P_t}$, i.e. S_t^o and S_t^x satisfy the respective versions of (9).

Assuming that o and x follow $f^o(\mathbf{P_t})$ and $f^x(\mathbf{P_t})$, respectively, we obtain with (9)

$$\begin{aligned} J^i(\mathbf{P_t}) = \max_{S_t^i \le P_t^i} \Big[& F(P_t^i, S_t^i) \\ & + \delta J^i \Big(R^i(f^o(\mathbf{P_t}), f^x(\mathbf{P_t})), R^o(S_t^i, f^x(\mathbf{P_t})), R^x(S_t^i, f^o(\mathbf{P_t})) \Big) \Big]. \end{aligned} \tag{12}$$

As this is a constrained maximization problem, we rewrite (12) as

$$\begin{aligned} J^i(\mathbf{P_t}) = \max_{S_t^i} \Big[& F(P_t^i, S_t^i) \\ & + \delta J^i \Big(R^i(f^o(\mathbf{P_t}), f^x(\mathbf{P_t})), R^o(S_t^i, f^x(\mathbf{P_t})), R^x(S_t^i, f^o(\mathbf{P_t})) \Big) \\ & + \lambda_t^i (P_t^i - S_t^i) \Big] \end{aligned} \tag{13}$$

with the complementary slackness conditions

$$\lambda_t^i \ge 0 \text{ and } \lambda_t^i (P_t^i - S_t^i) = 0. \tag{14}$$

The corresponding first order condition (FOC) for the maximum problem is given by:

$$F_2(P_t^i, S_t^i) + \delta J_2^i(\mathbf{P_{t+1}}) \cdot R_1^o(S_t^i, S_t^x) + \delta J_3^i(\mathbf{P_{t+1}}) \cdot R_1^x(S_t^i, S_t^o) - \lambda_t^i = 0. \tag{15}$$

Accordingly numbers as the lower index mark derivatives and the number describes the argument with respect to which the derivative is taken. The upper index describes the function from which the derivative is taken. If the lower index includes a 't', it is a time index. So, $R_1^o(S_t^i, S_t^x)$ describes how the price o may charge in $t+1$ reacts to a marginal change of the slack of i in t. We only need to look at derivatives to the left, i.e. reductions of slack, as starting from a steady state no firm can increase its slack without violating the break even constraint. Accordingly, throughout this paper, all derivatives are to be understood as left hand side derivatives, i.e. reductions of the respective variable. The corresponding derivatives of the regulatory rules are given in the appendix.

Now let

$$S_t^i = f^i(\mathbf{P_t}) \tag{16}$$

describe the optimal slack of firm i given $\mathbf{P_t}$, i.e. $f^i(\mathbf{P_t})$ is the solution to (15). Inserting this into (13) leads to:

$$
\begin{aligned}
J^i(\mathbf{P_t}) = {} & F(P_t^i, f^i(\mathbf{P_t})) \\
& + \delta J^i\Big(R^i(f^o(\mathbf{P_t}), f^x(\mathbf{P_t})), R^o(f^i(\mathbf{P_t}), f^x(\mathbf{P_t})), R^x(f^i(\mathbf{P_t}), f^o(\mathbf{P_t}))\Big) \\
& + \lambda_t^i(P_t^i - f^i(\mathbf{P_t})).
\end{aligned} \tag{17}
$$

Taking the derivative to the left with respect P_t^i we find with Envelope Theorem:

$$
\begin{aligned}
J_1^i(\mathbf{P_t}) = {} & F_1(P_t^i, S_t^i) \\
& + \delta J_1^i(\mathbf{P_{t+1}}) \cdot R_1^i(S_t^o, S_t^x) \cdot f_1^o(\mathbf{P_t}) + \delta J_1^i(\mathbf{P_{t+1}}) \cdot R_2^i(S_t^o, S_t^x) \cdot f_1^x(\mathbf{P_t}) \\
& + \delta J_2^i(\mathbf{P_{t+1}}) \cdot R_2^o(S_t^i, S_t^x) \cdot f_1^x(\mathbf{P_t}) + \delta J_3^i(\mathbf{P_{t+1}}) \cdot R_2^x(S_t^i, S_t^o) \cdot f_1^o(\mathbf{P_t}) \\
& + \lambda_t^i.
\end{aligned} \tag{18}
$$

Analogously we find

$$
\begin{aligned}
J_2^i(\mathbf{P_t}) = {} & \delta J_1^i(\mathbf{P_{t+1}}) \cdot R_1^i(S_t^o, S_t^x) \cdot f_2^o(\mathbf{P_t}) + \delta J_1^i(\mathbf{P_{t+1}}) \cdot R_2^i(S_t^o, S_t^x) \cdot f_2^x(\mathbf{P_t}) \\
& + \delta J_2^i(\mathbf{P_{t+1}}) \cdot R_2^o(S_t^i, S_t^x) \cdot f_2^x(\mathbf{P_t}) + \delta J_3^i(\mathbf{P_{t+1}}) \cdot R_2^x(S_t^i, S_t^o) \cdot f_2^o(\mathbf{P_t})
\end{aligned} \tag{19}
$$

and

$$
\begin{aligned}
J_3^i(\mathbf{P_t}) = {} & \delta J_1^i(\mathbf{P_{t+1}}) \cdot R_1^i(S_t^o, S_t^x) \cdot f_3^o(\mathbf{P_t}) + \delta J_1^i(\mathbf{P_{t+1}}) \cdot R_2^i(S_t^o, S_t^x) \cdot f_3^x(\mathbf{P_t}) \\
& + \delta J_2^i(\mathbf{P_{t+1}}) \cdot R_2^o(S_t^i, S_t^x) \cdot f_3^x(\mathbf{P_t}) + \delta J_3^i(\mathbf{P_{t+1}}) \cdot R_2^x(S_t^i, S_t^o) \cdot f_3^o(\mathbf{P_t}).
\end{aligned} \tag{20}
$$

Updating (19) and (20) by one period yields

$$
\begin{aligned}
J_2^i(\mathbf{P_{t+1}}) = {} & \delta J_1^i(\mathbf{P_{t+2}}) \cdot R_1^i(S_{t+1}^o, S_{t+1}^x) \cdot f_2^o(\mathbf{P_{t+1}}) \\
& + \delta J_1^i(\mathbf{P_{t+2}}) \cdot R_2^i(S_{t+1}^o, S_{t+1}^x) \cdot f_2^x(\mathbf{P_{t+1}}) \\
& + \delta J_2^i(\mathbf{P_{t+2}}) \cdot R_2^o(S_{t+1}^i, S_{t+1}^x) \cdot f_2^x(\mathbf{P_{t+1}}) \\
& + \delta J_3^i(\mathbf{P_{t+2}}) \cdot R_2^x(S_{t+1}^i, S_{t+1}^o) \cdot f_2^o(\mathbf{P_{t+1}})
\end{aligned} \tag{21}
$$

and

$$
\begin{aligned}
J_3^i(\mathbf{P_{t+1}}) = {} & \delta J_1^i(\mathbf{P_{t+2}}) \cdot R_1^i(S_{t+1}^o, S_{t+1}^x) \cdot f_3^o(\mathbf{P_{t+1}}) \\
& + \delta J_1^i(\mathbf{P_{t+2}}) \cdot R_2^i(S_{t+1}^o, S_{t+1}^x) \cdot f_3^x(\mathbf{P_{t+1}}) \\
& + \delta J_2^i(\mathbf{P_{t+2}}) \cdot R_2^o(S_{t+1}^i, S_{t+1}^x) \cdot f_3^x(\mathbf{P_{t+1}}) \\
& + \delta J_3^i(\mathbf{P_{t+2}}) \cdot R_2^x(S_{t+1}^i, S_{t+1}^o) \cdot f_3^o(\mathbf{P_{t+1}}).
\end{aligned} \tag{22}
$$

Plugging (21) and (22) into the FOC (15) leads to

$$
\begin{aligned}
0 = {} & F_2(P_t^i, S_t^i) \\
& + \delta R_1^o(S_t^i, S_t^x) \cdot \Big(\delta J_1^i(\mathbf{P_{t+2}}) \cdot R_1^i(S_{t+1}^o, S_{t+1}^x) \cdot f_2^o(\mathbf{P_{t+1}}) \\
& \qquad + \delta J_1^i(\mathbf{P_{t+2}}) \cdot R_2^i(S_{t+1}^o, S_{t+1}^x) \cdot f_2^x(\mathbf{P_{t+1}}) \\
& \qquad + \delta J_2^i(\mathbf{P_{t+2}}) \cdot R_2^o(S_{t+1}^i, S_{t+1}^x) \cdot f_2^x(\mathbf{P_{t+1}}) \\
& \qquad + \delta J_3^i(\mathbf{P_{t+2}}) \cdot R_2^x(S_{t+1}^i, S_{t+1}^o) \cdot f_2^o(\mathbf{P_{t+1}}) \Big) \\
& + \delta R_1^x(S_t^i, S_t^o) \cdot \Big(\delta J_1^i(\mathbf{P_{t+2}}) \cdot R_1^i(S_{t+1}^o, S_{t+1}^x) \cdot f_3^o(\mathbf{P_{t+1}}) \\
& \qquad + \delta J_1^i(\mathbf{P_{t+2}}) \cdot R_2^i(S_{t+1}^o, S_{t+1}^x) \cdot f_3^x(\mathbf{P_{t+1}}) \\
& \qquad + \delta J_2^i(\mathbf{P_{t+2}}) \cdot R_2^o(S_{t+1}^i, S_{t+1}^x) \cdot f_3^x(\mathbf{P_{t+1}}) \\
& \qquad + \delta J_3^i(\mathbf{P_{t+2}}) \cdot R_2^x(S_{t+1}^i, S_{t+1}^o) \cdot f_3^o(\mathbf{P_{t+1}}) \Big) \\
& - \lambda_t^i.
\end{aligned}
\tag{23}
$$

In equation (23), we clearly see the consequence of the use of historical cost data under yardstick regulation: The price that firm i can charge in the future is influenced by its behavior today. The choice of slack of i in t does not only define its instantaneous payoff, implicitly represented by $F_2(P_t^i, S_t^i)$, but also affects the prices o and x can charge in $t+1$ via the regulatory rule, $R^o(S_t^i, S_t^x)$ and $R^x(S_t^i, S_t^o)$ respectively. Firms o and x choose their slack in $t+1$ based on the state they observe and under the restriction that they have to break even according to their strategies, $f^o(\mathbf{P_{t+1}})$ and $f^x(\mathbf{P_{t+1}})$. The slacks o and x choose in $t+1$, via the regulatory rule, then affect P_{t+2}^o and P_{t+2}^x and *determine* the price i is allowed to charge in $t+2$, P_{t+2}^i. These three prices are the arguments of the value function of i and in period t, firm i discounts the effects in $t+2$ with δ^2.

From Lemma 1, we know that in every steady state all firms choose the same slack. Thus, starting from a steady state unilateral reduction of the slack of i affects the price o and x may charge in the following period the same way so that

$R_1^o(S_t^i, S_t^x) = R_1^x(S_t^i, S_t^o)$.[8] This reduces (23) to

$$
\begin{aligned}
0 = {} & F_2(P_t^i, S_t^i) \\
& + \delta R_1^o(S_t^i, S_t^x) \cdot \Big(\delta J_1^i(\mathbf{P_{t+2}}) \cdot R_1^i(S_{t+1}^o, S_{t+1}^x) \cdot [f_2^o(\mathbf{P_{t+1}}) + f_3^o(\mathbf{P_{t+1}})] \\
& \qquad + \delta J_1^i(\mathbf{P_{t+2}}) \cdot R_2^i(S_{t+1}^o, S_{t+1}^x) \cdot [f_2^x(\mathbf{P_{t+1}}) + f_3^x(\mathbf{P_{t+1}})] \\
& \qquad + \delta J_2^i(\mathbf{P_{t+2}}) \cdot R_2^o(S_{t+1}^i, S_{t+1}^x) \cdot [f_2^x(\mathbf{P_{t+1}}) + f_3^x(\mathbf{P_{t+1}})] \\
& \qquad + \delta J_3^i(\mathbf{P_{t+2}}) \cdot R_2^x(S_{t+1}^i, S_{t+1}^o) \cdot [f_2^o(\mathbf{P_{t+1}}) + f_3^o(\mathbf{P_{t+1}})] \Big) \\
& - \lambda_t^i.
\end{aligned}
\tag{24}
$$

From Lemma 1 it also follows that, due to regulatory mechanics, in all steady states all firms realize zero profits, i.e. all firms choose the slack that is equal to the maximum price that each firm may charge. So, starting from a steady state a marginal unilateral reduction of the slack of i in t leads to $P_{t+1}^o = P_{t+1}^x < P_{t+1}^i$. Following a punishment-free strategy, the two other firms, o and x, will under both regulatory schemes reduce their slack the next period by exactly the resulting marginal reduction of their respective price, given the price they face is not higher than the unique optimal slack S^{M*} and S^{A*}, respectively. We formalize this in the following Lemma, considering reductions of slack only for both regulatory regimes:

Lemma 2.
(i) Frontier yardstick regulation:
If $P_{t+1}^o = P_{t+1}^x \leq P_{t+1}^i$ *and* $P_{t+1}^o = P_{t+1}^x \leq S^{M*}$,
then $f_2^o(\mathbf{P_{t+1}}) + f_3^o(\mathbf{P_{t+1}}) = f_2^x(\mathbf{P_{t+1}}) + f_3^x(\mathbf{P_{t+1}}) = 1$.
(ii) Average yardstick regulation:
If $P_{t+1}^o = P_{t+1}^x \leq P_{t+1}^i$ *and* $P_{t+1}^o = P_{t+1}^x \leq S^{A*}$,
then $f_2^o(\mathbf{P_{t+1}}) + f_3^o(\mathbf{P_{t+1}}) = f_2^x(\mathbf{P_{t+1}}) + f_3^x(\mathbf{P_{t+1}}) = 1$.

Proof. See appendix.

Intuitively, Lemma 2 means the following: Starting from a steady state, a firm has to reduce its slack if the price that this firm can charge is reduced as it needs to break even. Given that the firm would not voluntarily unilaterally deviate from the steady state equilibrium, it cannot increase its intertemporal payoff by deviating even more than necessary. The fact that another firm also has to reduce its slack by the same amount does not cause additional effects in this case.

[8] We extensively deal with the derivatives of the regulatory rules in the appendix.

With Lemma 2, equation (24) reduces to

$$
\begin{aligned}
0 = {} & F_2(P_t^i, S_t^i) \\
& + \delta R_1^o(S_t^i, S_t^x) \cdot \Big(\delta J_1^i(\mathbf{P_{t+2}}) \cdot [R_1^i(S_{t+1}^o, S_{t+1}^x) + R_2^i(S_{t+1}^o, S_{t+1}^x)] \\
& \qquad + \delta J_2^i(\mathbf{P_{t+2}}) \cdot R_2^o(S_{t+1}^i, S_{t+1}^x) \\
& \qquad + \delta J_3^i(\mathbf{P_{t+2}}) \cdot R_2^x(S_{t+1}^i, S_{t+1}^o)\Big) \\
& - \lambda_t^i.
\end{aligned} \tag{25}
$$

To show how the solutions to this equation differ under both regulatory schemes we need to look at them separately.

Frontier yardstick regulation. From Lemma 1 it followed that in all steady state equilibria firms choose the same slack and the slack is equal to each firm's price due to regulatory mechanics. Therefore, in such a steady state i will choose the same slack every period, i.e. $S_{t+1}^i = S_t^i = S^*$. Every period i could deviate by reducing its slack.[9] So, S^* must solve the FOC in every period. Now assume i marginally reduces its slack in t. From the FOC, it directly follows that it cannot be optimal for i to choose a higher slack in $t+1$ than in t. With $S_t^i < S_{t+1}^i$, the slacks of o and x would have to be smaller than the one i chooses in $t+1$ from the regulatory rule and the break even constraint. Accordingly, in $t+1$ the left hand side derivatives of the regulatory rule with respect to the slack of i drop to zero if $S_t^i < S_{t+1}^i$. It follows that $S_t^i < S_{t+1}^i$ cannot describe an optimal strategy of i: the FOC would not hold in $t+1$ as $F_2(P_t^i, S_t^i) = B' - 1$ is smaller than zero and λ_{t+1}^i is nonnegative from the complementary slackness conditions. We conclude that i marginally reduces its slack in periods t and $t+1$, so that $S_t^i = S_{t+1}^i < S_t^o = S_t^x$. From the regulatory rule, equation (4), the prices o and x may charge in $t+1$ decrease to $P_{t+1}^o = P_{t+1}^x = S_t^i$ and given $S_t^i = S_{t+1}^i$, there is no additional effect on $P_{t+2}^o = P_{t+2}^x$ from the forced change in the behavior of o and x in $t+1$: The prices o and x may charge in $t+2$ are given by $P_{t+2}^o = \min(S_{t+1}^i, S_{t+1}^x)$ and $P_{t+2}^x = \min(S_{t+1}^i, S_{t+1}^o)$. So, if o and x decrease their slack in $t+1$ to $S_t^i = S_{t+1}^i$, they neither change P_{t+2}^o nor P_{t+2}^x. Consequently, in this situation the left hand side derivatives of the regulatory rule are given by $R_2^o(S_{t+1}^i, S_{t+1}^x) = R_2^x(S_{t+1}^i, S_{t+1}^o) = 0$.[10]

Intuitively, i decides about its slack in t, knowing that its slack in $t+1$ will be the same as in t. Hence, deciding about slack in t and $t+1$, firm i knows that P_{t+2}^o and P_{t+2}^x are equal to S_{t+1}^i for *all* $S_{t+1}^x \geq S_{t+1}^i$ and $S_{t+1}^o \geq S_{t+1}^i$ respectively. Accordingly, the only price in $t+2$ that is changed as a consequence of the induced reduction of the slack of o and x to $S_{t+1}^o = S_{t+1}^x = S_{t+1}^i = S_t^i$ is the price that firm i

[9] No firm can increase its slack in a steady state because of the break even constraint.

[10] Derivatives would be greater than zero for *further* decreases of their slack though.

itself can charge in $t+2$, P^i_{t+2}. Further, $R^i_1(S^o_{t+1}, S^x_{t+1}) + R^i_2(S^o_{t+1}, S^x_{t+1}) = 1$ is always true under frontier yardstick regulation (see appendix) and therefore equation (25) reduces to

$$0 = F_2(P^i_t, S^i_t) + \delta R^o_1(S^i_t, S^x_t) \cdot \delta J^i_1(\mathbf{P_{t+2}}) - \lambda^i_t. \tag{26}$$

We consider unilateral reductions of the slack of i starting from a steady state so that $R^o_1(S^i_t, S^x_t) = 1$. Furthermore, with $J^i_1(\mathbf{P_{t+2}}) = 1 + \lambda^i_{t+2}$ (Lemma 4 in the appendix) and $F_2(P^i_t, S^i_t) = B' - 1$, it follows:

$$0 = B' - 1 + \delta^2(1 + \lambda^i_{t+2}) - \lambda^i_t. \tag{27}$$

As the optimization problem is the same in every period in a steady state equilibrium, $\lambda^i_t = \lambda^i_{t+2} = \lambda$. Solving for B' yields the implicit solution for S^*:

$$B' = 1 - \delta^2 + (1 - \delta^2)\lambda. \tag{28}$$

This condition summarizes the tradeoff between marginal benefits and marginal costs of decreasing slack. The less patient firm i is, so the more weight it puts on instantaneous payoff, i.e. the smaller δ is, the greater is B' and with $B'' < 0$ the smaller is the slack i chooses. Therefore, if δ decreases, the firm cares less about slack in the future but grasps profit today. A more detailed intuition based on an infinite geometric series is given in the appendix. If $\lambda > 0$, the constraint must be binding from the complementary slackness conditions. For equation (28) to hold, the greater λ is, the greater must be B' and, as $B'' < 0$, the smaller must be the slack. If the constraint is binding, firm i has to choose a smaller slack than it would otherwise do. Conversely, if λ is zero, the solution to the constrained maximization problem is equal to the solution to the unconstrained maximization problem, i.e. the slack S^{M*} that firm i chooses in equilibrium if all prices are sufficiently high. Consequently, the implicit definition for S^{M*} is given by

$$B' = 1 - \delta^2. \tag{29}$$

Average yardstick regulation. Under this regulatory regime, all relevant derivatives of the regulatory rule are always $\frac{1}{2}$ as each price is the average of two slacks (see appendix). As furthermore in all steady state equilibria, the FOC must hold in every period, we can update the FOC, equation (15), by one period and plug it into (25) to find

$$0 = F_2(P^i_t, S^i_t) + \delta\frac{1}{2} \cdot \left(\delta J^i_1(\mathbf{P_{t+2}}) - F_2(P^i_{t+1}, S^i_{t+1}) + \lambda^i_{t+1}\right) - \lambda^i_t. \tag{30}$$

Applying the same reasoning as above with $S^i_{t+1} = S^i_t = S^*$, $J^i_1(\mathbf{P_{t+2}}) = 1 + \lambda^i_{t+2}$, $F_2(P^i_t, S^i_t) = B' - 1$ and $\lambda = \lambda_t = \lambda^i_{t+1} = \lambda^i_{t+2}$ we find

$$0 = B'(1 - \frac{1}{2}\delta) + 1(\frac{1}{2}\delta^2 + \frac{1}{2}\delta - 1) + \lambda(\frac{1}{2}\delta^2 + \frac{1}{2}\delta - 1), \tag{31}$$

and solving for B', it follows the implicit solution for S^*:

$$B' = \frac{(1 - \frac{1}{2}\delta^2 - \frac{1}{2}\delta) + \lambda(1 - \frac{1}{2}\delta^2 - \frac{1}{2}\delta)}{1 - \frac{1}{2}\delta}. \tag{32}$$

Under average yardstick regulation, B' also decreases in δ and hence the slack i chooses increases in the weight the firm puts on future payoff. Again B' increases in λ so the slack chosen if the constraint is binding is smaller than the slack chosen if all prices are sufficiently high. The solution to the corresponding unconstrained maximization problem, i.e. the slack S^{A*} firm i chooses in equilibrium if all prices are sufficiently high, does not include λ. So, S^{A*} is implicitly defined by

$$B' = 1 - \frac{\frac{1}{2}\delta^2}{1 - \frac{1}{2}\delta}. \tag{33}$$

Inspection of equations (28) and (32) reveals that no slack higher than S^{M*} and S^{A*} can exist in a steady state under the respective regulatory regime. As $B'' < 0$ and λ is nonnegative from the complementary slackness conditions, neither (28) nor (32) could hold in any steady state with slack greater than S^{M*} and S^{A*} respectively. In such a steady state, marginal benefits of unilaterally reducing slack would be greater than marginal costs of doing so. Consequently, firm i would unilaterally deviate by reducing its slack, which contradicts the existence of such punishment-free Markov-perfect steady state equilibria. This leads to the following proposition that is directly derived from the analysis above:

Proposition 1.
(i) In any punishment-free Markov-perfect steady state equilibrium under frontier yardstick regulation the slack is between 0 and S^{M*}, $S^* \epsilon [0, S^{M*}]$.
(ii) In any punishment-free Markov-perfect steady state equilibrium under average yardstick regulation the slack is between 0 and S^{A*}, $S^* \epsilon [0, S^{A*}]$.

3.2 Steady state equilibria

From Lemma 1, it followed that there cannot exist any asymmetric steady state equilibrium. It is straightforward that the analysis above can analogously be done for firms o and x. Taking the optimal strategies of firms o and x as given, we show that it is optimal for i to follow the same strategy. By doing this, we prove the existence of the equilibria characterized above.

Assume optimal Markov-perfect strategies of firms o and x under frontier yardstick regulation are given by

$$f^o(\mathbf{P_t}) = f^x(\mathbf{P_t}) = S^* = \min(S^{M*}, P_t^i, P_t^o, P_t^x). \tag{34}$$

So, firms o and x choose S^{M*} or at least one firm $j = i, o, x$ cannot choose any higher slack without violating the break even constraint given $\mathbf{P_t}$. In the latter case, this firm's choice of slack would remain optimal if the other firms chose higher slacks instead.[11] Furthermore, in all steady states with slack greater than S^{M*}, firms would unilaterally deviate by reducing slack. Consequently, if the above strategies constitute an equilibrium, it is punishment-free.

Given the above strategies, it cannot be optimal for firm i to choose any slack greater than S^{M*} as it could reduce its slack to S^{M*} without affecting any price in $t+1$. As $F_2(P_t^i, S_t^i) < 0$, this would result in higher instantaneous and intertemporal payoff. The same is true for any $S_t^i > S_t^o = S_t^x$ as i's slack does not affect future prices if $S_t^i > S_t^o = S_t^x$ from the regulatory rule. Accordingly the FOC cannot hold for $S_t^i > S_t^o = S_t^x$ as $F_2(P_t^i, S_t^i) < 0$ and λ_t^i is nonnegative from the complementary slackness conditions. Thus, it is never optimal for i to choose a slack higher than o and x under frontier yardstick regulation, and the optimal strategy of i given $\mathbf{P_t}$ and the strategies of o and x must satisfy $f^i(\mathbf{P_t}) \leq \min(S^{M*}, P_t^i, P_t^o, P_t^x)$.

We now show that this inequality holds with equality: As the value function is concave in the state variables, F is strictly concave in slack and the left hand side derivative of the regulatory rule with respect to the slack of i must be equal to one in all steady states with $S^* > 0$, $\lambda > 0$ in all steady states with $S^* < S^{M*}$. Accordingly the steady state described by S^{M*} is strictly preferred by firm i over all other steady states with lower slack. (Obviously all steady states with positive slack are preferred by i over the one with zero slack.)

From the concavity of the value function and the strict concavity of F concerning slack, it also follows that λ decreases in the steady state value of slack for all $S^* < S^{M*}$. As a consequence, firm i never unilaterally deviates by reducing slack from a situation where all firms choose the same slack, given $S_t^i \leq S^{M*}$: If firm i unilaterally reduces its slack starting from such a situation in t, the constraint is not binding in that period, so λ_t^i needs to be zero from the complementary slackness conditions. With the concavity of the value function and strict concavity of F with respect to slack this cannot be optimal as the FOC could not hold. Then

[11] In the notation of the definition of punishment-free Markov-perfect equilibria, p. 7, this firm facing the lowest price is labelled firm l.

$f^i(\mathbf{P_t}) = S^* = \min(S^{M*}, P^i_t, P^o_t, P^x_t)$ is the optimal strategy given the strategies of o and x. As $\min(S^{M*}, P^i_t, P^o_t, P^x_t)$ can take on every value between zero and S^{M*} depending on initial prices, existence of a punishment-free Markov-perfect steady state equilibrium under frontier yardstick regulation is established for every slack $S^* \epsilon [0, S^{M*}]$.

Assume further that optimal strategies of firms o and x under average yardstick regulation are given by

$$f^o(\mathbf{P_t}) = f^x(\mathbf{P_t}) = S^* = \min(S^{A*}, P^i_t, P^o_t, P^x_t). \tag{35}$$

With the same reasoning as above, it follows that if these strategies constitute an equilibrium, it is punishment-free. The strategies given by equations (34) and (35), differ only by the unique optimal value of slack, given prices are sufficiently high. Accordingly the corresponding proof for average yardstick regulation is very similar to the one above. It is not optimal for firm i to choose a slack higher than the one o and x choose given their above strategies: First, note that under average yardstick regulation, all relevant derivatives of the regulatory rule are equal to $\frac{1}{2}$ as every price is the average of the slacks of the other two firms of the period before. Now consider $S^i_t > S^o_t = S^x_t$: neither o nor x would choose a higher slack in $t+1$ than in t as then $\min(P^i_{t+1}, P^o_{t+1}, P^x_{t+1}) = P^i_{t+1} = S^o_t = S^x_t$. It follows that the highest possible slack from $t+2$ on would not be greater than $S^o_t = S^x_t$ for all slacks $S^i_t > S^o_t = S^x_t$. As $F_2(P^i_t, S^i_t) < 0$, i could increase its instantaneous and intertemporal payoff by decreasing its slack and choosing $S^i_t = S^o_t = S^x_t$. The rest of the proof is a straightforward repetition of the arguments above using S^{A*} and the corresponding derivatives of the regulatory rule.

We summarize these findings in the following proposition:

Proposition 2.
(i)
Under frontier yardstick regulation, the triple of strategies $\left(f^i(\mathbf{P_t}), f^o(\mathbf{P_t}), f^x(\mathbf{P_t})\right)$ *with* $f^j(\mathbf{P_t}) = S^* = \min(S^{M*}, P^i_t, P^o_t, P^x_t)$, $j = i, o, x$, *constitutes a punishment-free Markov-perfect steady state equilibrium. Every slack* $S^* \epsilon [0, S^{M*}]$ *can exist in equilibrium and* S^{M*} *offers the highest intertemporal payoff for firms.*

(ii)
Under average yardstick regulation, the triple of strategies $\left(f^i(\mathbf{P_t}), f^o(\mathbf{P_t}), f^x(\mathbf{P_t})\right)$ *with* $f^j(\mathbf{P_t}) = S^* = \min(S^{A*}, P^i_t, P^o_t, P^x_t)$, $j = i, o, x$, *constitutes a punishment-free Markov-perfect steady state equilibrium. Every slack* $S^* \epsilon [0, S^{A*}]$ *can exist in equilibrium and* S^{A*} *offers the highest intertemporal payoff for firms.*

It is important to note that the regulator cannot induce the zero slack steady state by simply setting all prices to zero. In our analysis, necessary costs have been normalized to zero. However, the reason why regulatory schemes like yardstick regulation exist essentially is that the regulator does not know how large necessary costs of production are. Otherwise, she could directly mandate optimal prices without applying yardstick regulation. By exogenously setting too low prices in the first regulatory period, the regulator risks firms going bankrupt, as they cannot break even anymore. While it is not explicitly modeled in this paper, it seems reasonable that it is crucial to the regulator that firms subject to regulation, producing without slack, can cover their real and necessary costs. One could think of a large welfare loss outside of the model that is associated with firms, that provide essential services, not being able to cover their real and necessary costs.

Given this restriction and that no slack higher than S^{M*} and S^{A*} under the respective regime can exist in a steady state, it seems reasonable that the regulator initially sets prices which are relatively high. Therefore, comparing the upper ends of the intervals of feasible steady state slacks seems particularly relevant.

3.3 Comparative dynamics

From Propositions 1 and 2, we know that every slack between 0 and S^{M*} under frontier yardstick regulation and between 0 and S^{A*} under average yardstick regulation can describe a steady state equilibrium. Furthermore, we know that there cannot exist punishment-free Markov-perfect steady state equilibria with higher slack under the respective regulatory regime. By comparing the implicit solutions for S^{M*} and S^{A*}, we find that all punishment-free Markov-perfect steady state equilibria under average yardstick regulation can be equilibria under frontier yardstick regulation while the reverse is not true. This leads to the following proposition:

Proposition 3. *The highest slack that can be realized in a punishment-free Markov-perfect steady state equilibrium is greater under frontier yardstick regulation than under average yardstick regulation.*

Proof. S^{A*} is implicitly defined by (33) and the corresponding value under frontier yardstick regulation, S^{M*}, is implicitly defined by (29). As $B' > 0$ and $B'' < 0$, $S^{M*} > S^{A*}$ if the following inequality holds:

$$1 - \frac{\frac{1}{2}\delta^2}{1 - \frac{1}{2}\delta} > 1 - \delta^2. \tag{36}$$

Rearranging yields

$$1 > \delta.$$

Hence, inequality (36) always holds. □

Intuitively, orientation at the performance of 'the best' of all other firms rather than the average of all other firms to define constraints for a firm under yardstick regulation seems to be the tougher regulation. Incentives to produce efficiently, i.e. without slack, should be strong. Proposition 3 questions this intuition. Using historical cost data of other firms allows each firm to influence the own yardstick. As this influence is greater under frontier yardstick regulation all firms could be less willing to 'push' the other firms because they will have to 'push back' in return.

4 Conclusion

While Shleifer's (1985) version of yardstick regulation uses *current* performance of other firms to find *current* constraints for an evaluated firm, real life applications of yardstick regulation frequently define constraints, e.g. prices allowed to be charged, *ex ante* based on data from the regulatory period(s) before. This use of historical cost data in yardstick regulation enables a firm to affect the price it can charge in the future. Affecting other firms' constraints and thus behavior, the current performance of a firm is directly linked to its own future constraints.

This analysis showed in a simple model framework that inefficient steady state equilibria in which all firms choose positive slack can exist under yardstick regulation without any form of collusion if historical cost data is used. Furthermore, the highest slack that can exist in a punishment-free Markov-perfect steady state equilibrium is higher under frontier yardstick regulation, where the firm with the lowest costs of all but the evaluated firm defines the yardstick, than if the average of all other firms is used. This challenges the perception that incentives to produce efficiently are strongest if the best of all other firms is the yardstick in a yardstick regulation using historical cost data.

Appendix

Proof of Lemma 1

As regulatory rules are anonymous and C is normalized to zero, only 5 relevant different cases can be distinguished, potentially with indices changed and updated though:

(I) $P_t^i = P_t^o = P_t^x = S_t^i = S_t^o = S_t^x$

(II) $S_t^i = S_t^o > S_t^x$

(III) $S_t^i > S_t^o = S_t^x$

(IV) $S_t^i > S_t^o > S_t^x$

(V) $S_t^i = S_t^o = S_t^x < P_t^i = P_t^o = P_t^x$

The reasoning is explained below in detail for case (II) under frontier yardstick regulation and average yardstick regulation, the remaining is then a straightforward application along these lines.

Case (I):
If all three prices and all three slacks are the same in t the regulatory rule does not force any change. Prices in $t+1$ are the same as in t and the same slack as in t is possible for all firms.

Frontier yardstick regulation
Case (II):

$$P_{t+1}^i = P_{t+1}^o = S_t^x < P_{t+1}^x = S_t^i = S_t^o$$

$$\Rightarrow$$

$$S_{t+1}^i \leq S_t^x$$
$$S_{t+1}^o \leq S_t^x$$
$$S_{t+1}^x \leq S_t^i = S_t^o$$

$$\Rightarrow$$

$$S_{t+2}^i \leq P_{t+2}^i \leq S_t^x$$
$$S_{t+2}^o \leq P_{t+2}^o \leq S_t^x$$
$$S_{t+2}^x \leq P_{t+2}^x \leq S_t^x$$

then either case (I) or one of the cases (II)-(V) applies.

Under frontier yardstick regulation, the price that a firm is allowed to charge is the minimum of the slacks the other two firms chose in the period before. Therefore, if firms i and o choose the same slack in t and firm x chooses a smaller one, the price i and o are allowed to charge in $t+1$ is equal to S_t^x while P_{t+1}^x is equal to the slack i and o choose in t. In $t+1$, x may, consequently, choose any slack that is not greater than $P_{t+1}^x = S_t^i = S_t^o$. The slack i and o can choose is not greater than $P_{t+1}^i = P_{t+1}^o = S_t^x$ and hence smaller than $P_{t+1}^x = S_t^i = S_t^o$. In $t+2$, the price i, o and x may charge is not greater than the smallest slack in t, i.e. S_t^x. Only one firm, x, can choose a higher slack than this in $t+1$. But even if it does so, the smaller one of any two slacks chosen in $t+1$ cannot be greater than S_t^x. In $t+2$, either all three firms choose the same slack and this slack is equal to the price they may charge or one of the cases (II) to (V) applies.

Case (III):

$$S_{t+1}^i \leq P_{t+1}^i = S_t^x = S_t^o$$
$$S_{t+1}^o \leq P_{t+1}^o = S_t^x = S_t^o$$
$$S_{t+1}^x \leq P_{t+1}^x = S_t^o = S_t^x$$

then either case (I) or one of the cases (II)-(V) applies.

Case (IV):

$$P_{t+1}^i = S_t^x$$
$$P_{t+1}^o = S_t^x$$
$$P_{t+1}^x = S_t^o$$

$$\Rightarrow$$

$$S_{t+1}^i \leq S_t^x$$
$$S_{t+1}^o \leq S_t^x$$
$$S_{t+1}^x \leq S_t^o$$

$$\Rightarrow$$

$$S_{t+2}^i \leq P_{t+2}^i \leq S_t^x$$
$$S_{t+2}^o \leq P_{t+2}^o \leq S_t^x$$
$$S_{t+2}^x \leq P_{t+2}^x \leq S_t^x$$

then either case (I) or one of the cases (II)-(V) applies.

Case (V):

$$P^i_{t+1} = P^o_{t+1} = P^x_{t+1} = S^i_t = S^o_t = S^x_t$$

then either case (I) or one of the cases (II)-(V) applies.

Average yardstick regulation
Case (II):

$$\begin{aligned}
P^i_{t+1} &= \frac{S^o_t + S^x_t}{2} \\
P^o_{t+1} &= \frac{S^i_t + S^x_t}{2} \\
P^x_{t+1} &= \frac{S^i_t + S^o_t}{2} = S^i_t = S^o_t
\end{aligned}$$

$$\Rightarrow$$

$$\begin{aligned}
S^i_{t+1} &\leq \frac{S^o_t + S^x_t}{2} < S^i_t \\
S^o_{t+1} &\leq \frac{S^i_t + S^x_t}{2} < S^o_t \\
S^x_{t+1} &\leq \frac{S^i_t + S^o_t}{2} = S^i_t = S^o_t \text{ [and } S^x_{t+1} \gtreqless S^x_t]
\end{aligned}$$

$$\Rightarrow$$

$$\begin{aligned}
S^i_{t+2} \leq P^i_{t+2} &\leq \frac{S^i_t + S^x_t + S^i_t + S^o_t}{4} < S^i_t = S^o_t \\
S^o_{t+2} \leq P^o_{t+2} &\leq \frac{S^o_t + S^x_t + S^i_t + S^o_t}{4} < S^o_t = S^i_t \\
S^x_{t+2} \leq P^x_{t+2} &\leq \frac{S^o_t + S^x_t + S^i_t + S^x_t}{4} < S^i_t = S^o_t
\end{aligned}$$

So the highest slack chosen in t cannot be chosen by anyone in $t + 2$. Then either case (I) or one of the cases (II)-(V) applies.

Under average yardstick regulation, the price that a firm may charge is equal to the average of the slacks that the other two firms chose in the period before. So, if firms i and o choose the same slack in t and firm x chooses a smaller slack, the price i and o are allowed to charge in $t+1$ is smaller than the one x may charge and smaller than the slack i and o choose in t. Accordingly both have to choose a smaller slack in $t+1$. In $t+1$, x may choose a slack that is greater than S_t^x but not greater than the slack i and o choose in t. In $t+2$, all prices are smaller than the greatest slack in t so that this slack cannot be chosen anymore. Then either all three firms choose the same slack and this slack is equal to the price they may charge or one of the cases (II) to (V) applies.

Case (III):

$$P_{t+1}^i = \frac{S_t^o + S_t^x}{2} = S_t^o = S_t^x$$
$$P_{t+1}^o = \frac{S_t^i + S_t^x}{2}$$
$$P_{t+1}^x = \frac{S_t^i + S_t^o}{2}$$

$$\Rightarrow$$

$$S_{t+1}^i \leq \frac{S_t^o + S_t^x}{2} = S_t^o = S_t^x < S_t^i$$
$$S_{t+1}^o \leq \frac{S_t^i + S_t^x}{2} < S_t^i \text{ [and } S_{t+1}^o \gtreqless S_t^o\text{]}$$
$$S_{t+1}^x \leq \frac{S_t^i + S_t^o}{2} < S_t^i \text{ [and } S_{t+1}^x \gtreqless S_t^x\text{]}$$

So the highest slack chosen in t cannot be chosen by anyone in $t+1$. Then either case (I) or one of the cases (II)-(V) applies.

Case (IV):

$$P_{t+1}^i = \frac{S_t^o + S_t^x}{2}$$
$$P_{t+1}^o = \frac{S_t^i + S_t^x}{2}$$
$$P_{t+1}^x = \frac{S_t^i + S_t^o}{2}$$

$$\Rightarrow$$

$$S^i_{t+1} \leq \frac{S^o_t + S^x_t}{2} < S^i_t$$
$$S^o_{t+1} \leq \frac{S^i_t + S^x_t}{2} < S^i_t$$
$$S^x_{t+1} \leq \frac{S^i_t + S^o_t}{2} < S^i_t$$

So the highest slack chosen in t cannot be chosen by anyone in $t+1$. Then either case (I) or one of the cases (II)-(V) applies.

Case (V):

$$P^i_{t+1} = \frac{S^o_t + S^x_t}{2} = P^o_{t+1} = \frac{S^i_t + S^x_t}{2} = P^x_{t+1} = \frac{S^i_t + S^o_t}{2} = S^i_t = S^o_t = S^x_t$$

$$\Rightarrow$$

$$S^i_{t+1} \leq P^i_{t+1} = S^i_t = S^o_t = S^x_t$$
$$S^o_{t+1} \leq P^o_{t+1} = S^o_t = S^i_t = S^x_t$$
$$S^x_{t+1} \leq P^x_{t+1} = S^x_t = S^i_t = S^o_t$$

Then either case (I) or one of the cases (II)-(V) applies.

As long as slacks differ in period t, in $t+2$ at the latest, the highest slack of t cannot be chosen by any firm anymore under both regulatory regimes. (Under frontier yardstick regulation, at the latest in $t+2$, no slack higher than the smallest of t can be chosen.) Consequently, the maximum of the three slacks monotonically decreases, potentially with a delay that is not greater than two periods. Furthermore, all slacks are bounded below at zero. It follows that slacks necessarily have to converge. As the price for each firm is in every period the minimum or the average of the slacks of the other two firms in the period before, prices converge too. Prices and slacks cannot converge to different values so that profits of all firms must be zero in every steady state. □

Derivatives of the regulatory rules

Frontier yardstick regulation
We focus on the example of the price firm i can charge in $t+1$. The corresponding derivatives regarding reductions of slack for the other firms and for all other periods are found analogously. The regulatory rule is given by (4):

$$P^i_{t+1} = C + \min(S^o_t, S^x_t) = R^i(S^o_t, S^x_t).$$

The relevant left hand side derivatives for $S_t^o \neq S_t^x$ are given by

$$R_1^i(S_t^o, S_t^x) = \frac{\partial R^i(S_t^o, S_t^x)}{\partial S_t^o} = \begin{cases} 1 & \text{for } S_t^o < S_t^x \\ 0 & \text{for } S_t^o > S_t^x \end{cases} \tag{37}$$

and

$$R_2^i(S_t^o, S_t^x) = \frac{\partial R^i(S_t^o, S_t^x)}{\partial S_t^x} = \begin{cases} 0 & \text{for } S_t^o < S_t^x \\ 1 & \text{for } S_t^o > S_t^x \end{cases} \tag{38}$$

Starting from $S_t^o = S_t^x$ and for a constant slack of the respective other firm, the left hand side derivative is equal to one for both firms. However, the price firm i is allowed to charge in $t+1$ is reduced by *one* marginal unit if either firm o or firm x or both firms simultaneously reduce their respective slack in t by one marginal unit. In particular, slightly abusing notation, we have

$$R_1^i(S_t^o, S_t^x) + R_2^i(S_t^o, S_t^x) = \frac{\partial R^i(S_t^o, S_t^x)}{\partial S_t^o} + \frac{\partial R^i(S_t^o, S_t^x)}{\partial S_t^x} = 1 \text{ for all } S_t^o, S_t^x. \tag{39}$$

For simultaneous reductions of the slacks of both firms, we are clearly not holding the respective other slack constant. However, as for simultaneous changes in the slacks it is unimportant for our result whether the change of the slack of firm o or of x or of both change the constraint of firm i, we refrain from introducing additional notation that does not provide further insights.

To derive (39) for $S_t^o = S_t^x$ and simultaneous changes of slack of o and x, let

$$\widetilde{S_t^o} = S_t^o + \epsilon$$

and

$$\widetilde{S_t^x} = S_t^x + \epsilon,$$

where $\epsilon \neq 0$.
For $S_t^o = S_t^x$ and $\widetilde{S_t^o} = \widetilde{S_t^x}$ we see that

$$\min(S_t^o, S_t^x) = S_t^o = S_t^x \text{ and } \min(\widetilde{S_t^o}, \widetilde{S_t^x}) = \widetilde{S_t^o} = \widetilde{S_t^x},$$

then

$$\min(\widetilde{S_t^o}, \widetilde{S_t^x}) - \min(S_t^o, S_t^x) = \epsilon.$$

In analogy to the definition of the derivative, we find

$$\lim_{\epsilon \to 0} \frac{\min(S_t^o + \epsilon, S_t^x + \epsilon) - \min(S_t^o, S_t^x)}{\epsilon} = 1.$$

Average yardstick regulation
The regulatory rule is given by (5):

$$P^i_{t+1} = \frac{1}{2}\sum_{j\neq i}\left(C + S^j_t\right) = C + \frac{1}{2}\sum_{j\neq i} S^j_t = R^i(S^o_t, S^x_t).$$

Consequently, all changes in slack of any firm will result in changes in the prices the other two firms may charge in the following period by half of the magnitude of the aforementioned change. Spelled out for the price firm i can charge in $t+1$ this is

$$R^i_1(S^o_t, S^x_t) = R^i_2(S^o_t, S^x_t) = \frac{1}{2}. \tag{40}$$

Under average yardstick regulation, all other derivatives of the regulatory rule with respect to one of the two relevant slacks are equal to $\frac{1}{2}$, too.

Proof of Lemma 2

Recall the FOC, equation (15),

$$F_2(P^i_t, S^i_t) + \delta J^i_2(\mathbf{P_{t+1}}) \cdot R^o_1(S^i_t, S^x_t) + \delta J^i_3(\mathbf{P_{t+1}}) \cdot R^x_1(S^i_t, S^o_t) - \lambda^i_t = 0$$

and complementary slackness conditions (14):

$$\lambda^i_t \geq 0 \text{ and } \lambda^i_t(P^i_t - S^i_t) = 0.$$

Assume firms are in a steady state so that $P^i_t = P^o_t = P^x_t = S^i_t = S^o_t = S^x_t$ and $P^i_t \leq S^{M*}$ under frontier yardstick regulation and $P^i_t \leq S^{A*}$ under average yardstick regulation. If i's choice of slack is optimal, the FOC and complementary slackness conditions must hold.

Now, assume one of the other firms, e.g. firm o, instead chooses a marginally smaller slack in t so that $P^i_{t+1} = P^x_{t+1} < P^o_{t+1}$ and $P^i_{t+1} < S^{M*}$ under frontier yardstick regulation and $P^i_{t+1} < S^{A*}$ under average yardstick regulation. From the break even condition, we know that i has to reduce its slack by at least the marginal change of the price that it may charge in $t+1$ so that the left hand side derivative of $f^i(\mathbf{P_{t+1}})$ with respect to i's own price cannot be smaller than one. Clearly, the sum of the left hand side derivatives $f^i_1(\mathbf{P_{t+1}}) + f^i_3(\mathbf{P_{t+1}}) \geq 1$ too. (Throughout this paper, we are only considering reductions of slack.)

As we require equilibria to be punishment-free, this equation holds with equality and $S^i_{t+1} = P^i_{t+1}$. To show this, we fix the slacks o and x at their respective highest admissible value of slack in $t+1$, $S^o_{t+1} = P^o_{t+1}$ and $S^x_{t+1} = P^x_{t+1}$. Consequently, the respective left hand side derivatives of the regulatory rule for firm i are the same as in a steady state as i cannot choose a slack higher than its price and thus $S^i_{t+1} = \min(S^i_{t+1}, S^o_{t+1}, S^x_{t+1})$. If firm i decreases its slack by even more than the marginal change of its price to any $\underline{S}^i_{t+1} < P^i_{t+1}$, the constraint is not binding in $t+1$. It follows that $\lambda^i_{t+1} = 0$ from the complementary slackness conditions. With $\underline{S}^i_{t+1} < S^i_t$, it also follows that $F_2(P^i_{t+1}, \underline{S}^i_{t+1}) > F_2(P^i_t, S^i_t)$ as F is strictly concave in slack. Besides, the value function is concave in the state variables so that $J^i_2(\mathbf{P_{t+2}})$ and $J^i_3(\mathbf{P_{t+2}})$ are not smaller than the corresponding derivatives in the initial steady state, where the FOC held, as prices are not greater than in that steady state. Hence, the FOC cannot hold in $t+1$ so that $\underline{S}^i_{t+1}$ is not the optimal choice of i. Thus, i does not unilaterally reduce its slack by more than what is forced by the reduction of its price in this setting, i.e. $S^i_{t+1} = P^i_{t+1}$.

We can apply the same reasoning as above for firms o and x to show that no firm unilaterally chooses a slack in $t+1$ that is smaller than $P^i_{t+1} = P^x_{t+1}$ if the other two firms choose their respective highest admissible slack here. As we require equilibria to be punishment-free, firms do not coordinatedly choose a uniform smaller slack because of other firms choosing this uniform slack. It follows that $f^i_1(\mathbf{P_{t+1}}) + f^i_3(\mathbf{P_{t+1}}) = 1$ in this setting.

Symmetrically the same reasoning applies for all firms with indices changed. □

From the proof above, we can clearly point out the vulnerability of yardstick regulation against the threat of joint mutual punishment and collusion in general. As shown above, it is not optimal in this case for firm i to unilaterally choose any slack smaller than the price that it can charge in $t+1$. However, for example, if at least one other firm chose a smaller slack than this slack under frontier yardstick regulation, it would be optimal for i to do so too. As this applies for all firms, allowing for coordination like joint mutual punishment could lead to $f^i_1(\mathbf{P_{t+1}}) + f^i_3(\mathbf{P_{t+1}}) > 1$. This would give room to equilibria with much *higher* slack than S^{M*} under frontier yardstick regulation and S^{A*} under average yardstick regulation by increasing costs of reducing slack for all firms.

Lemma 3.
(i) Frontier yardstick regulation:
If $P_t^i = P_t^x < P_t^o$ *and* $P_t^i \leq S^{M*}$, *then* $f_2^i(\mathbf{P_t}) = 0$.
(ii) Average yardstick regulation:
If $P_t^i = P_t^x < P_t^o$ *and* $P_t^i \leq S^{A*}$, *then* $f_2^i(\mathbf{P_t}) = 0$.

Proof. Assume firms are in a steady state so that $P_t^i = P_t^o = P_t^x = S_t^i = S_t^o = S_t^x$ and $P_t^i \leq S^{M*}$ under frontier yardstick regulation and $P_t^i \leq S^{A*}$ under average yardstick regulation. As i's choice of slack is optimal, the FOC, equation (15), and complementary slackness conditions (14) must hold.

Now, assume that instead the price relevant for one of the firms, e.g. firm o, is higher $\overline{P}_t^o > P_t^i = P_t^x$. Applying the same reasoning as in the proof of Lemma 2, it follows that no firm unilaterally chooses a smaller slack than $P_t^i = P_t^x$ in this setting. Furthermore, in a punishment-free equilibrium, firms do not coordinatedly choose a uniform (lower) slack that no firm would choose unilaterally if all other firms were to choose higher slacks. Firm i cannot choose any slack higher than its price because of the break even constraint: this implies that $S_t^i = P_t^i$.

Hence, the optimal slack of firm i is the same for $\overline{P}_t^o > P_t^i = P_t^x$ and $P_t^o = P_t^i = P_t^x$, with $P_t^i \leq S^{M*}$ under frontier yardstick regulation and $P_t^i \leq S^{A*}$ under average yardstick regulation. It directly follows that $f_2^i(\mathbf{P_t}) = 0$ in this setting. □

An intuition for Lemma 3 under frontier yardstick regulation is the following: Firm i knows that the lowest slack in t describes an upper bound for all slacks and prices from $t+2$ onwards. So, as long as the slacks of the other two firms are not smaller than the one i chooses, this upper bound is the same for every slack o and x choose and all prices o and x face. Thus, the marginal benefits and costs of a reduction of slack do not depend on these prices in this setting. Consequently, the decision of i is not affected. Again, the same reasoning applies for $P_t^i = P_t^o < P_t^x$ as well as for firms o and x with changed indices.

Lemma 4.
(i) Frontier yardstick regulation:
If $P^o_{t+2} = P^x_{t+2} < P^i_{t+2}$ *and* $P^o_{t+2} = P^x_{t+2} \leq S^{M*}$*, then* $J^i_1(\mathbf{P^{t+2}}) = 1 + \lambda^i_{t+2}$.
(ii) Average yardstick regulation:
If $P^o_{t+2} = P^x_{t+2} < P^i_{t+2}$ *and* $P^o_{t+2} = P^x_{t+2} \leq S^{A*}$*, then* $J^i_1(\mathbf{P^{t+2}}) = 1 + \lambda^i_{t+2}$.

Proof. Recall equation (18):

$$\begin{aligned} J^i_1(\mathbf{P_t}) = F_1(P^i_t, S^i_t) \\ &+ \delta J^i_1(\mathbf{P_{t+1}}) \cdot R^i_1(S^o_t, S^x_t) \cdot f^o_1(\mathbf{P_t}) + \delta J^i_1(\mathbf{P_{t+1}}) \cdot R^i_2(S^o_t, S^x_t) \cdot f^x_1(\mathbf{P_t}) \\ &+ \delta J^i_2(\mathbf{P_{t+1}}) \cdot R^o_2(S^i_t, S^x_t) \cdot f^x_1(\mathbf{P_t}) + \delta J^i_3(\mathbf{P_{t+1}}) \cdot R^x_2(S^i_t, S^o_t) \cdot f^o_1(\mathbf{P_t}) \\ &+ \lambda^i_t. \end{aligned}$$

The way the value function of i is affected by a change of the price that firm i may charge depends on how the other two firms react to this change. Using Lemma 3 for the reactions of o and x, inserting $F_1(P^i_t, S^i_t) = 1$ and updating (18) by two periods complete the proof. □

Intuitively, Lemma 4 says that if firms are in the steady state equilibrium described by S^{M*}, under frontier yardstick regulation, or S^{A*}, under average yardstick regulation, and $\lambda = 0$, they would not change their slack if their price was higher, but would realize a positive profit that period. Consequently, the discounted sum of the utility of i increases by 1 if the price that firm i is allowed to charge in t increases by one unit. In any steady state equilibrium with a slack smaller than S^{M*} or S^{A*}, respectively, we have $\lambda > 0$. Hence, firms would like to move to a steady state equilibrium with higher slack, but cannot do so because of the (binding) break even constraint. Reductions of the prices firms can charge then have a larger impact on the intertemporal payoff.

Intuition for S^{M*} based on geometric series

When firm i decides on the slack in t, it considers that its slack defines an upper bound for all prices from $t+2$ onwards under frontier yardstick regulation given $S^i_t \leq \min(S^o_t, S^x_t)$. From the proof of Lemma 2, we know that o and x choose the highest slack that they are allowed to, given $P^o_{t+1} = P^x_{t+1} \leq P^i_{t+1}$ and $P^o_{t+1} = P^x_{t+1} \leq S^{M*}$, in their optimal decision. Firm i has to trade off profit in t and $t+1$ against slack in $t, t+1, t+2, ..., \infty$ when it decides about $S^i_t = S^{M*}$. (As the price i may charge in $t+1$ is unaffected by S^i_t, it can 'cash in' the profit from reducing slack twice.) In the steady state equilibrium described by S^{M*}, implicitly defined by (29), marginal costs of reducing slack and marginal benefits of doing so must be equal to each other, so that

$$1 + \delta \cdot 1 = \sum_{z=o}^{\infty} \delta^z \cdot B'.$$

With $\delta < 1$, it follows that

$$1 + \delta \cdot 1 = B' \frac{1}{1-\delta}.$$

Rearranging yields

$$B' = 1 - \delta^2,$$

which replicates the implicit definition of S^{M*} given by equation (29).

References

Acemoglu, D. (2009): Introduction to Modern Economic Growth. Princeton University Press, Princeton, NJ.

Averch, H. and L. L. Johnson (1962): Behavior of the Firm under Regulatory Constraint. *American Economic Review* 52(5), 1052-1069.

Besley, T. and A. Case (1995): Incumbent Behavior: Vote-Seeking, Tax-Setting, and Yardstick Competition. *American Economic Review* 85(1), 25-45.

Blackmon, B. G. (1994): Incentive Regulation and the Regulation of Incentives. Kluwer, Boston, MA.

Cowan, S. (2006): Network Regulation. *Oxford Review of Economic Policy* 22(2), 248-259.

Dalen, D. M. (1998): Yardstick Competition and Investment Incentives. *Journal of Economics & Management Strategy* 7(1), 105-126.

Faure-Grimaud, A. and S. Reiche (2006): Dynamic Yardstick Mechanisms. *Games and Economic Behavior* 54(2), 316-335.

Laffont, J.-J. (1994): The New Economics of Regulation Ten Years After. *Econometrica* 62(3), 507-537.

Laffont, J.-J. and J. Tirole (1993): A Theory of Incentives in Procurement and Regulation. The MIT Press, Cambridge, MA.

Lazear, E. P. and S. Rosen (1981): Rank-Order Tournaments as Optimum Labor Contracts. *Journal of Political Economy* 89(5), 841-864.

Littlechild, S. C. (1983): Regulation of British Telecommunications' Profitability, Report to the Secretary of State, Department of Industry. Her Majesty's Stationery Office, London.

Maskin, E. and J. Tirole (1988a): A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs. *Econometrica* 56(3), 549-569.

Maskin, E. and J. Tirole (1988b): A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles. *Econometrica* 56(3), 571-599.

Meran, G. and C. v. Hirschhausen (2009): A Modified Yardstick Competition Mechanism. *Journal of Regulatory Economics* 35(3), 223-245.

Mizutani, F., H. Kozumi and N. Matsushima (2009): Does Yardstick Regulation Really Work? Empirical Evidence from Japan's Rail Industry. *Journal of Regulatory Economics* 36(3), 308-323.

Osborne, M. J. and A. Rubinstein (1994): A Course in Game Theory. The MIT Press, Cambridge, MA.

Potters, J., B. Rockenbach, A. Sadrieh and E. v. Damme (2004): Collusion under Yardstick Competition: An Experimental Study. *International Journal of Industrial Organization* 22(7), 1017-1038.

Sappington, D. E. M. (2005): Regulating Service Quality: A Survey. *Journal of Regulatory Economics* 27(2), 123-154.

Shleifer, A. (1985): A Theory of Yardstick Competition. *RAND Journal of Economics* 16(3), 319-327.

Sobel, J. (1999): A Reexamination of Yardstick Competition. *Journal of Economics & Management Strategy* 8(1), 33-60.

Stokey, N. L., R. E. Lucas with E. C. Prescott (1989): Recursive Methods in Economic Dynamics. Harvard University Press, Cambridge, MA.

Tangerås, T. P. (2002): Collusion-proof Yardstick Competition. *Journal of Public Economics* 83(2), 231-254.

Train, K. E. (1991): Optimal Regulation. The Economic Theory of Natural Monopoly. The MIT Press, Cambridge, MA.