

**MINIMUM QUALITY STANDARDS AND
NON-COMPLIANCE**

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Abstract

This paper studies the effect of non-compliance with a minimum quality standard on prices, quality, and welfare in a vertical differentiation model. Non-compliance with a minimum quality standard by a low-quality firm reduces quality levels of both firms and shifts demand from the low-quality to the high-quality firm. Under non-compliance, an increase in the standard increases the quality of both products and shifts demand from the high-quality product to the low-quality product. Stricter government enforcement decreases the quality level of the low-quality firm and shifts demand from the low-quality firm to the high-quality firm. Non-compliance of the low-quality firm increases profits for both firms, reduces consumer surplus, and increases or decreases welfare depending on the market size, the detection probability, surveillance cost, and the minimum quality level.

JEL Classification: K42, L13, L50

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1 Introduction

This paper studies the effect of non-compliance with a minimum quality standard on prices, quality levels, and welfare in a vertical differentiation model. Also, it explores the effect of an increase in the minimum quality standard and the effect of a higher level of government enforcement effort on prices, quality levels, and welfare under non-compliance.

In the European Union firms' investments in product quality are not only driven by consumer preferences, but also by mandatory minimum quality standards that are applied in order to limit external effects such as harmful emissions or risks to consumers. However, non-compliance with these minimum quality standards seems to be not just an exception, but it appears to be the rule in many cases. A significant number of household electrical products imported from outside of the EU does not comply with the respective minimum quality standards. The European Commission reports that only 5% of the mobile household lights tested fully comply with the respective

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administrative or technical requirements. Only about 17% of all cord extension sets meet all requirements, while 58% of the cord extension sets tested are considered unsafe. Other examples are energy saving lamps (23% technical non-compliance), consumer entertainment electronic products (50% technical non-compliance), imported toys (55% technical non-compliance), and Christmas lightning (European Commission, 2013b, Annex 7).

Consumers often might not be aware of products not fulfilling all safety requirements and by products that do not meet all requirements. One reason could be that consumers trust in the competent market surveillance authorities. Since the question of fulfilling the standards is a complex issue, many consumers may simply have to rely on market surveillance authorities, because they are unable to observe the quality of a product in all its dimensions and/or lack sufficient knowledge of the respective standards. Alternatively fragmented surveillance rules could explain consumers' unawareness. If national market authorities treat the same products differently within the single market, consumers lack a clear signal of confidence. Also, consumers might expect that a product not meeting all respective requirements completely will not automatically be unsafe or cause environmental damage. Another explanation for the fact that consumers buy products that do not meet the relevant standards would be a lower preference for safety or environmental issues by consumers compared to the standard-setting authority.

Effective market surveillance aims at identifying unsafe or environmentally harmful products, which are then to be taken off the market. Market surveillance is carried out by the authorities of the member states (European Commission, 2013a). The internal market with products circulating freely within the European Union poses a particular challenge to market surveillance. Cross-border coordination of member states' activities is vital for effective consumer protection in the internal market. Unsafe products not fulfilling product safety requirements may enter the EU market via third countries, if national activities are not coordinated sufficiently. Market surveillance is incomplete today. The market surveillance rules are fragmented and different legal bases apply (Regulation 765/2008 and the General Product Safety Directive 2001/95/EC, or sector-specific European Union harmonization legislation). This may create confusion among national market authorities, consumers, and firms. Product safety requirements determining whether a product is safe and may be marketed are not always clear and consistent. (European Commission, 2013a).

In 2013, the European Commission has proposed the so called "Product Safety and Market Surveillance Package" with the aim to improve consumer product safety and to strengthen market surveillance of products circulating on the internal market by better coordinating member states' activities and streamlining the various legal bases (European Commission 2013a). The main idea of the "Product Safety and Market Surveillance Package" is to increase the probability that products that do not fulfill all requirements are detected. So safety is the main focus of the proposed package. The proposed amendments by the European Parliament focus also on the surveillance of environmental and energy efficiency standards (European Parliament, 2014).

An alternative measure for strengthening the market surveillance is the increase of applicable

minimum quality standards. This leads to a higher quality level of compliant products and might potentially increase the quality level of non-compliant products. So governments are able to choose between both instruments. Since both instruments might be accompanied by different side-effects, the governments have to choose carefully which instrument fits best in which situation. Increasing the safety of consumers or third parties might require a different instrument than reducing external effects caused by a group of products.

This paper relates to the literature of minimum quality standards in several ways. Like the majority of papers on minimum quality standards (e.g. Ronnen, 1991 and Crampes & Hollander, 1995), we consider duopolistic markets, where single product firms face minimum quality standards as exogenous constraints. We assume that quality improvements result in higher variable costs as in Crampes & Hollander (1995), Ecchia & Lamberti (1997), and Petropoulou (2013). This assumption may be appropriate for many household appliances, toys etc., where quality improvements stem (partly) from using high-quality materials or more complex production processes.

The literature on minimum quality standards has stressed that quality choices of oligopolistic firms differ from socially optimal levels (Scarpa, 1998). The literature typically finds that quality levels of products increase with the level of the minimum quality standard (Ronnen 1991, Motta & Thisse, 1993) as long as the quality standard does not reduce the number of firms in the market (Motta & Thisse, 1993).

Recently, several papers have analyzed the effects of non-compliance with a minimum quality standard. Faure, Schleich & Schlomann (2013) test non-compliance with the EU Energy Labeling Directive in a sample of 100,000 appliances from 1,400 retail stores in 27 EU member states. They show that perceived costs and benefits, normative motives, and social influence may explain retailer compliance with the EU energy labeling program. Other papers analyze mainly the effect of non-compliance with environmental standards. For instance, Hatcher (2007) compares emission level standards and standards expressed in terms of emissions per unit of output (ratio standards) under non-compliance. He shows that emission level standards and ratio standards lead to different results with respect to emissions and output. Arguedas, Camacho & Zofio (2010) analyze firm's incentives to adopt abatement technologies, if non-compliance occurs. They find that under certain assumptions imperfect compliance increases firms' incentives to invest in abatement technologies, if emission standards are applied. Arguedas, Cabo & Martín-Herrán (2014) analyze the dynamic interaction of more stringent pollution limits and fines overtime in a dynamic setting. Fine discounts in exchange for firm's environmentally-friendly investment in capital help firms to comply with more stringent standards and are socially desirable. Arguedas (2013) studies optimal fines for exceeding pollution standards. She shows that under non-compliance an optimal fine should decrease with investment effort and positive social costs of sanctioning, whereas under full-compliance the fine should be independent of investment efforts. Chen & Serfes (2012) analyze the effect of a minimum quality standard in a vertical differentiation model, when the government receives only a noisy signal of quality and imposes

a fine on firms that do not comply with the minimum quality standard. They show that in their setting a minimum quality standard may reduce social welfare. Our paper focuses on the comparison of two other policy instruments: an increase of the minimum quality standard and an increase of the surveillance activities under non-compliance.¹ In addition, we show that strategic behavior of the low-quality firm resulting in non-compliance can increase social welfare, even if the minimum quality standard does not decrease social welfare per se.

In this paper, we study the effect of non-compliance with a minimum quality standard on prices, quality levels, and welfare in a vertical differentiation model following Ecchia & Lamberini (1997). We endogenize quality and assume variable cost of quality improvements. Consumers are heterogeneous with respect to their preference for quality. The introduction of an exogenous minimum standard may be motivated by external effects such as environmental harmful pollution or risks to consumers and third parties.

We assume a level of the minimum quality standard that is “tough”, i.e. a minimum quality level that is binding for the high-quality firm. Exceeding the highest quality level available on the market does not imply that this quality level is technically infeasible. It only implies that this level is not profit-maximizing for any firm without regulation.²³

Non-compliance with a minimum quality standard may increase the low-quality firm’s profit. This behavior reduces both quality levels, it increases the price for the high-quality product, decreases the price for the low-quality product, and shifts demand from the low-quality to the high-quality firm. Under non-compliance, an increase in the standard increases the quality difference, increases the price difference, and shifts demand from the high-quality firm to the low-quality firm. A higher level of government enforcement decreases the quality level of the low-quality firm, but shifts demand from the low-quality firm to the high-quality firm. Non-compliance reduces consumer surplus, but increases producer surplus, and increases or decreases welfare, depending on the market size, the effect of quality levels of the externality, the detection probability, surveillance costs, or the minimum quality level. Although overall consumer surplus declines, a subgroup of consumers might gain from non-compliance.

The rest of the paper is organized as follows. In the next section, the vertical differentiation model is presented and the case of no government intervention, the case of full compliance with the minimum quality standard, and the case of non-compliance by the low-quality firm with the standard are analyzed. Section 3 analyzes the effect of enhancing the minimum quality standard

¹Chen & Serfes (2012) also analyze the effect of an increase in the fine.

²Referring to the maximum quality level on the market in standard-setting corresponds to the common approach by the European Commissions of referring to “best available techniques”. The Directive on industrial emissions defines “available techniques” as “those developed on a scale which allows implementation in the relevant industrial sector, under economically and technically viable conditions, taking into consideration the costs and advantages, whether or not the techniques are used or produced inside the Member State in question, as long as they are reasonably accessible to the operator”. It is not necessary that this technique is used in the market under consideration.

³A mild minimum quality standard would lead to qualitative similar results. There are also methodological reasons for our assumption: If the level of the minimum quality standard would be lower than the quality level of the high-quality firm without regulation, non-compliance of the low-quality firm would be equivalent to the case of no regulation.

as well as stricter enforcement of a given standard and discusses the choice of policy instruments. Section 4 studies welfare. Section 5 concludes.

2 The Model

Following Ecchia & Lambertini (1997), consider a duopolistic market with vertical product differentiation. Assume that a product is supplied in two quality levels, s_H and s_L , with $s_H > s_L$, and that each firm supplies only one quality level.

The production technology is characterized by variable cost, which are convex in quality and linear in quantity:

$$C_i = s_i^2 q_i. \quad (1)$$

Firms use higher quality materials or more complex production processes to enhance the quality level of their products. This may be an appropriate assumption for many household appliances like vacuum cleaners, for consumer entertainment products or for toys. For these products, non-compliance is a frequent phenomenon (European Commission, 2013b, Annex 7).

Consumers are heterogeneous with respect to their preference for quality, as in Mussa & Rosen (1978). They are characterized by a preference parameter θ , which is uniformly distributed on the interval $[a, b]$ with $b = a + 1^4$. Consumer value all product characteristics including safety and environmental aspects. Each consumer buys at most one unit of the most preferred good. The utility derived from no purchase is zero, while a consumer who buys one unit of the good obtains a net utility of

$$U = \theta s_i - p_i, \quad i = H, L. \quad (2)$$

The consumer heterogeneity can be interpreted as differences in income or as difference in consumption patterns. Frequent usage may be accompanied by a higher willingness to pay for quality. Note that θ can also be interpreted as the marginal rate of substitution between income and quality (Tirole, 1988).

The marginal consumer indifferent between purchasing the high-quality good and the low-quality good is given by $\theta^* = \frac{p_H - p_L}{s_H - s_L}$. Hence, demand for the high-quality good and the low-quality good respectively is given as

$$q_H = b - \frac{p_H - p_L}{s_H - s_L}, \quad q_L = \frac{p_H - p_L}{s_H - s_L} - a. \quad (3)$$

Firms' profits are given as

$$\pi_i = (p_i - s_i^2) q_i. \quad (4)$$

Competition follows a three-stage game: In the first stage, the government chooses a minimum quality level at an exogenous intensity and the intensity of market surveillance. In the second

⁴Assume $b > b^{\min} = \frac{5}{4}$ to guarantee equilibrium existence (Ecchia & Lambertini, 1997). In equilibrium $\theta^* \in [a, b]$.

stage, firms choose quality levels. In the third stage, firms compete in prices.

2.1 No Regulation

Consider first a system with no government intervention. Prices and quality levels can be found in the Appendix. Firms are free to choose quality levels. Both quality levels increase in the maximum willingness to pay b . The difference between quality levels $\Delta s = s_H - s_L$ is independent of b .

Both equilibrium prices and the price differential $\Delta p = p_H - p_L$ increase in b .

The duopoly is symmetric, quantities are $q_H = q_L = \frac{1}{2}$.

2.2 Minimum Quality Standard and Compliance

Now assume the introduction of a minimum quality standard, with which both firms comply. We assume a level of the minimum quality standard that is “tough”, i.e. a minimum quality level that is set equal to the highest quality level available in the market or even exceeds this quality level ($S > s_H$). Exceeding the highest quality level available on the market does not imply that this quality level is technically infeasible. It only implies that this level is not profit-maximizing for any firm without regulation. A mild minimum quality standard would lead to qualitative similar results. Also assume $S \leq S^{\max} = \frac{b+1}{2}$ to guarantee that no firm exits the market. Prices, quality levels, and quantities can be found in the Appendix.

The low-quality firm sets the quality level to the required minimum quality level. The high-quality firm’s optimal response is to raise its quality level to sustain product differentiation.⁵ The introduction of the minimum quality standard increases both quality levels ($s_H^C > s_H$, $s_L^C > s_L$).

Both quality levels increase in the minimum quality standard, with the increase in the quality level of the low-quality firm exceeding that of the high-quality firm ($0 < \frac{\partial s_H^C}{\partial S} < \frac{\partial s_L^C}{\partial S}$). Thus, an increase in the standard decreases the quality difference.

The introduction of the minimum quality standard increases both prices ($p_H^C > p_H$, $p_L^C > p_L$). Firms incur higher variable costs. The quality level of both products increases. The willingness to pay for the increased quality increases also.⁶ Both prices increase in the minimum quality standard. An increase in the standard decreases the price differential ($\frac{\partial p_L^C}{\partial S} > \frac{\partial p_H^C}{\partial S} > 0$).

The introduction of the minimum quality standard shifts demand from the high-quality firm to the low-quality firm, due to the increase in quality levels ($q_H^C < q_H$, $q_L^C > q_L$). An increase in the standard enhances this demand-shifting effect ($\frac{\partial q_H^C}{\partial S} < 0$, $\frac{\partial q_L^C}{\partial S} > 0$). Proposition 1 summarizes the effect of a minimum quality standard for both firms meeting the standard.

Proposition 1 *Suppose a “tough” minimum quality standard is introduced and both firms comply with the standard. Then the standard i) increases both quality levels, ii) increases both prices,*

⁵Homogeneous products would result in Bertrand price competition with marginal cost pricing and zero profits.

⁶Note that quality levels are not profit maximizing anymore.

and iii) shifts demand from the high-quality to the low-quality firm. An increase in the standard i) decreases the quality difference, ii) decreases the price difference, and iii) enhances the demand-shifting effect.

2.3 Minimum Quality Standard and Non-Compliance

Now assume that the government cannot monitor compliance with the standard perfectly. Violations against the standard are detected with probability ϕ . Government incurs surveillance costs $F(\phi)$ ⁷ (Rousseau & Proost, 2005). If discovered by the government, products not complying with the standard are confiscated.⁸ The introduction of a fine for non-compliance has no effect on prices and quality levels, because this it would only constitute an upfront payment.⁹ Prices, quality levels, and quantities can be found in the Appendix. The low-quality firm does not comply with the standard. This is profitable ($\pi_L^{NC} > \pi_L^C$) for many combinations of market size b , detection probability ϕ , and level of the minimum quality standard S (see Figures 6-9 in the Appendix). So in this paper non-compliance is not a mere assumption, but an endogenous result of profit-maximizing behavior.

Consumers may distinguish products of a higher quality level from products of a lower quality level, but cannot observe whether the standard is fulfilled. Alternatively, they do not base their purchase decisions on standard fulfillment, because they either have a lower preference for safety or environmental issues than the government or because they do not expect that a product not meeting all respective requirements completely will be unsafe or cause environmental damages. Both firms know the quality level of both products.

Firms' profits are given as

$$\begin{aligned}\pi_H^{NC} &= \left(p_H^{NC} - s_H^{NC^2}\right) q_H^{NC}, \\ \pi_L^{NC} &= \left((1 - \phi)p_L^{NC} - s_L^{NC^2}\right) q_L^{NC}.\end{aligned}\tag{5}$$

In response to the low-quality firm not complying with the standard, the high-quality firm lowers its quality level to the required minimum quality level. The choice of a higher quality level is not optimal for the high-quality firm. So in this setting the minimum quality standard imposed by the government defines in fact a maximum quality level available on the market. Quality levels are strategic complements. So non-compliance with the standard of the low-quality firm reduces both quality levels ($s_H^{NC} < s_H^C$, $s_L^{NC} < s_L^C$), the average quality level and the maximum quality available in the market.

An increase in the minimum quality requirement under non-compliance increases both quality levels, with the quality level of the low-quality firm increasing by less than the quality level of

⁷ Assume $\frac{\partial F}{\partial \phi} > 0$ and $\frac{\partial^2 F}{\partial \phi^2} > 0$.

⁸ In the EU, this is one of the common options for action by the member states' customs authorities, if a product presents a serious risk.

⁹ A sufficiently high fine would result in no non-compliance at all. We assume that this is non-feasible.

the high-quality firm ($0 < \frac{\partial s_L^{NC}}{\partial S} < \frac{\partial s_H^{NC}}{\partial S}$). This is, in contrast to the case of full compliance, the quality difference increases in the standard ($\frac{\partial \Delta s^{NC}}{\partial S} > 0$).

Non-compliance of the low-quality firm increases the price for the high-quality product, if S or ϕ are sufficiently high and decreases the price for the low quality product ($p_H^{NC} > p_H^C$ if $S > \tilde{S} \vee (S < \tilde{S} \wedge \phi > \phi^*)$, $p_L^{NC} < p_L^C$). Both prices and the price difference increase in the standard ($\frac{\partial p_H^{NC}}{\partial S} > \frac{\partial p_L^{NC}}{\partial S}$).

Non-compliance of the low quality firm induces a lower price for low-quality products because consumers are willing to pay less for lower quality and lower cost. The price for the high quality product p_H depends positively on the quality difference Δs , since a higher relative quality level leads to a competitive advantage of the high-quality firm, whereas p_L depends negatively on Δs . The increase of the price difference induced by non-compliance is lower than the induced increase of the quality difference.

Non-compliance of the low-quality firm increases the quantity of the high-quality product and decreases the quantity of the low-quality good ($q_H^{NC} > q_H^C$, $q_L^{NC} < q_L^C$). Thus, non-compliance has the opposite effect as compared to the introduction of a minimum quality standard: It shifts demand from the low-quality firm to the high-quality firm. An increase in the standard, however, again shifts demand from the high-quality firm to the low-quality firm ($\frac{\partial q_H^{NC}}{\partial S} < 0$, $\frac{\partial q_L^{NC}}{\partial S} > 0$).

Proposition 2 summarizes the effect of non-compliance with the minimum quality standard by the low-quality firm.

Proposition 2 *Suppose the low-quality firm does not comply with the minimum standard. Then non-compliance i) reduces both quality levels, ii) causes the high-quality firm to set its quality level according to the minimum quality level, iii) increases the price for the high-quality product, if the minimum quality standard or the detection probability are sufficiently high, and decreases the price for the low-quality product, and iv) shifts demand from the low-quality firm to the high-quality firm. An increase in the standard i) increases the quality difference, ii) increases the price difference, and iii) shifts demand from the high-quality to the low-quality firm.*

3 Government Policies

This section compares the consequences of two government policies: Raising the minimum quality standard and stricter enforcement of an existing minimum quality standard. Prices, quality levels, and quantities can be found in the Appendix.

3.1 Raising the Minimum Quality Standard

Quality levels of both types of the product, s_H^{NC} and s_L^{NC} , are strategic complements. An increase in the minimum quality standard S leads to an increase of both quality levels s_H and s_L under compliance and under non-compliance. It causes both prices to rise in the case of compliance as

well as in the case of non-compliance. An increase in the standard decreases the quantity of the high-quality product.

Under non-compliance, an increase in S has a direct effect on s_H^{NC} , because the H -firm meets exactly the minimum quality standard ($\frac{\partial s_H^{NC}}{\partial S} > 0$). Via best response of the L -firm this causes an increase of s_L^{NC} ($\frac{\partial s_L^{NC}}{\partial S} > 0$). An increase in S causes an increase in the difference in quality levels, because the high-quality firm increases quality more than the low-quality firm ($\frac{\partial \Delta s^{NC}}{\partial S} > 0$). The low quality firm faces the risk that a share of its production is confiscated. This leads to a lower incentive to invest in quality.

Both prices increase in S ($\frac{\partial p_H^{NC}}{\partial S} > 0$, $\frac{\partial p_L^{NC}}{\partial S} > 0$). The price difference between high-quality products and low-quality products increases, since the low quality firm invests in quality by less ($\frac{\partial \Delta p^{NC}}{\partial S} > 0$). In addition, the ability to increase prices following an increase in costs due to higher quality (but not fulfilling the minimum quality standard) is dampened by the risk of non-complying products being detected and confiscated by the market authorities. An increase in S shifts demand from the high-quality firm to the low-quality firm ($\frac{\partial q_H^{NC}}{\partial S} < 0$, $\frac{\partial q_L^{NC}}{\partial S} > 0$).

Proposition 3 summarizes the main results

Proposition 3 *Under non-compliance, an increase in the minimum quality level i) leads to an increase of both quality levels and both prices, ii) decreases the quantity of the high-quality product, iii) increases the quantity of the low-quality product, and iv) increases the difference in quality levels and v) increases the price difference between the high-quality product and the low-quality product.*

3.2 Stricter Enforcement

The regulatory authority may also spend more resources to increase the detection probability ϕ in order to reduce the share of non-compliant products and increase the average quality level of products that are available on the market. This is one of the main ideas of the ‘‘Product Safety and Market Surveillance Package’’ proposed by the European Commission (European Commission, 2013a).

An increase in government enforcement decreases the quality level of the low-quality firm: $\frac{\partial s_L^{NC}}{\partial \phi} < 0$. The reason is that an increase of the detection probability ϕ makes it less profitable for the low quality firm to invest in quality, as a larger share of products is confiscated. The quality level of the high-quality firm remains unchanged due to stricter enforcement ($\frac{\partial s_H^{NC}}{\partial \phi} = 0$). The difference between both quality level increases due to stricter enforcement ($\frac{\partial \Delta s^{NC}}{\partial \phi} > 0$).

A higher level of government enforcement leads to a price increase of the high-quality product ($\frac{\partial p_H^{NC}}{\partial \phi} > 0$) and a price decrease of the low-quality product ($\frac{\partial p_L^{NC}}{\partial \phi} < 0$), if the maximum willingness to pay for quality b is sufficiently high¹⁰. It shifts demand from the low-quality firm to the high quality firm ($\frac{\partial q_H^{NC}}{\partial \phi} > 0$, $\frac{\partial q_L^{NC}}{\partial \phi} < 0$).

¹⁰ $\frac{\partial p_L^{NC}}{\partial \phi} < 0$ if $b \approx 1.7$.

Proposition 4 summarizes the effect of increased government enforcement.

Proposition 4 *Suppose the low-quality firm does not comply with the minimum standard. An increase in government enforcement i) decreases the quality level of the low-quality product, ii) increases the price for the high-quality product and decreases the price for the low-quality product, if the maximum willingness to pay for quality is sufficiently high, and iii) shifts demand from the low-quality firm to the high-quality firm.*

3.3 Choice of Policy Instrument

An increase of S and an increase of ϕ may be seen as policy substitutes under non-compliance, both with the aim to increase the quality of products on the market. But both instruments have different consequences for the regulatory authority, the quality of non-compliant products, prices, and consumption of compliant and non-compliant products.

An increase in the minimum quality standard is free of cost from the government's point of view. It increases the average quality and increases the lowest quality level. An increase in S increases prices. In addition, it increases consumption of low-quality products.

An increase in the detection probability ϕ , in contrast, causes an increase in government spending for market surveillance. It decreases the average quality level and the lowest quality level available on the market, while the quality of the high quality product remains unchanged. It increases prices for the high-quality goods and decreases prices for the low-quality good. It increases consumption of high quality products.

If the intention of the government is to guarantee a minimum quality level of all products sold on the market, an increase in S seems to be favorable to an increase in ϕ , because it increases both the average and the lowest quality level. The last effect is important, if a low quality level is associated with a risk for consumers, because it makes low-quality products more safe. If, however, the intention of the government is protection of high-quality firms, an increase of ϕ is the better instrument. If consumers buying the low-quality products are at risk or pollute the environment, while there is no such risk or pollution effect associated with high-quality products, increasing ϕ might be preferable, because it increases consumption of high quality products, although it makes low-quality products even worse and more money is spent. Of course, this does not hold when it is the increased detection probability that induces a decrease of s_L below a critical level and thereby causes a new risk. Product characteristics may determine which instrument is preferable. A problem arises if products both are potentially hazardous and produce externalities (energy saving lamps may be an example). The total effect with respect to quality levels and consumption is ambiguous, if both instruments are applied.

In addition, higher level of government enforcement may misguide consumers to have more confidence in all products available on the market. Therefore a higher level of government enforcement should be accompanied by an additional program that raises the awareness for safety problems of consumer products.

4 Welfare Analysis

Assume the regulator's payoff is $R = -(\eta - \alpha q_H s_H^2 - \alpha q_L s_L^2)$, where η denotes the externality and α denotes the effect of the product quality on reducing the externality from the regulator's perspective.^{11,12} Welfare is given as the sum of profits, consumer surplus, and the regulator's payoff minus surveillance cost ($W = \pi_H + \pi_L + CS + R - F$). Profits, consumer surplus, the regulator's payoff, and welfare can be found in the Appendix.

If both firms comply with the minimum quality standard, the minimum quality standard lowers profits for both firms ($\pi_H^C < \pi_H$, $\pi_L^C < \pi_L$), increases consumer surplus ($CS^C > CS$), if $S^{\min} < S^* < S^{\max}$ ¹³, and decreases consumer surplus, if $S > S^*$ ($CS^C < CS$). The introduction of the minimum quality standard increases the regulator's payoff ($R^C > R$). The minimum quality standard increases (decreases) welfare ($W^C \lessgtr W$), if the effect of increased quality levels on the externality α is sufficiently high ($\alpha > \alpha^*$) (low ($\alpha < \alpha^*$)). An increase in the minimum quality standard decreases profits of both firms ($\frac{\partial \pi_H^C}{\partial S} < 0$, $\frac{\partial \pi_L^C}{\partial S} < 0$), decreases consumer surplus ($\frac{\partial CS^C}{\partial S} < 0$), but increases the regulator's payoff ($\frac{\partial R^C}{\partial S} > 0$). It increases welfare ($\frac{\partial W^C}{\partial S} > 0$) (see numerical simulations in the Appendix).

Compared to the case of full compliance, non-compliance of the low-quality firm increases the H -firm's profits ($\pi_H^{NC} > \pi_H^C$). Depending on the detection probability ϕ , non-compliance may also increase profits for the low-quality firm – otherwise, it would comply (see figures 6-9 in the Appendix). Non-compliance reduces consumer surplus ($CS^{NC} < CS^C$).

Non-compliance also decreases the regulator's payoff, if the market size b is sufficiently small. But for a sufficiently large market size and a sufficiently large detection probability ϕ , the regulator's payoff under non-compliance may exceed the payoff under compliance.

If the market size b is sufficiently small and if S is sufficiently high, welfare under non-compliance may be higher than welfare under compliance ($W^{NC} \lessgtr W^C$). If the effect of the product quality on the externality (α) is sufficiently high and S and ϕ are sufficiently low, welfare is lower under non-compliance than under compliance. Welfare increases in market size b , because the regulator's payoff increases in market size.

There is a dynamic effect of an increase in the standard: A higher standard increases the incentive for the low-quality firm for non-complying behavior for a given detection probability ϕ . So it may be the increase in the minimum quality standard that may cause the switch of the low-quality firm from compliance to non-compliance. This switch also changes the behavior of the high-quality firm, which may also lower its product quality to the level of the minimum quality standard.

Consumers benefit from compliance with a minimum quality standard. The group of consumers as a whole loses due to non-compliance. But subgroups of consumers are affected differently by non-complying behavior. Three subgroups can be identified: The first group consists

¹¹ Similar results would hold, if α was the weight of the regulator's payoff in the social welfare function.

¹² See Pottier, Espagne & Dumas (2015) for the quadratic form of the payoff function.

¹³ $S^* = \frac{166 + 9\sqrt{265} - 137}{2}$.

of consumers, who buy high-quality products under compliance and buy high-quality products under non-compliance. They receive a lower quality product, but pay a higher price. In the second group, consumers buy low-quality products under compliance and buy high-quality products under non-compliance. They obtain the same quality level ($s_L^C = S = s_H^{NC}$) but pay a higher price. The third group buys low-quality products under compliance and still buys low-quality products under non-compliance. They obtain a lower quality ($s_L^C < s_L^{NC}$), but also pay a lower price. This is the only group of consumers that might gain under non-compliance, provided that the low quality does not imply damaging effects for consumers.

Proposition 5 summarizes the main results.

Proposition 5 *If both firms comply with the minimum quality standard, its introduction i) lowers profits for both firms, ii) increases consumer surplus, if $S < S^*$, and decreases consumer surplus, if $S > S^*$, iii) increases the regulator's payoff, and iv) increases (decreases) total welfare, if α is sufficiently high (low). Non-compliance of the low quality firm i) increases profits of the high quality firm and the low quality firm, iii) reduces consumer surplus and iv) reduces (increases) the regulator's payoff, if the market size is sufficiently small (if the market size and the detection probability are sufficiently high), and increases (decreases) welfare, if the market size b is sufficiently small (large) and α is sufficiently low (high) and ϕ and S are sufficiently high (low).*

The effect of an increase in the standard on profits, consumer surplus, the regulator's payoff, and welfare can be found in the Appendix.

An increase in the standard under non-compliance decreases profits of both firms ($\frac{\partial \pi_H^{NC}}{\partial S} < 0$, $\frac{\partial \pi_L^{NC}}{\partial S} < 0$) and decreases consumer surplus ($\frac{\partial CS^{NC}}{\partial S} < 0$). It increases the regulator's payoff ($\frac{\partial R^{NC}}{\partial S} > 0$). An increase in the standard increases (decreases) welfare ($\frac{\partial W^{NC}}{\partial S} > 0$), if the market size is sufficiently large (small), and if surveillance cost F , the minimum quality standard, and the detection probability are sufficiently low (high).

Under non-compliance, profits of the high-quality firm increase in the detection probability ϕ ($\frac{\partial \pi_H^{NC}}{\partial \phi} > 0$), profits of the low-quality firm decrease ($\frac{\partial \pi_L^{NC}}{\partial \phi} < 0$). Consumer surplus decreases in the detection probability ϕ ($\frac{\partial CS^{NC}}{\partial \phi} < 0$), the regulator's payoff increases ($\frac{\partial R^{NC}}{\partial \phi} > 0$). Under non-compliance, welfare increases in the detection probability ϕ , if the market size is sufficiently large.

Proposition 6 summarizes the main results.

Proposition 6 *An increase in the standard under non-compliance i) decreases profits of both firms, ii) decreases consumer surplus, iii) increases the regulator's payoff, and iv) increases (decreases) welfare, if the market size is sufficiently large (small), and if surveillance cost, the minimum quality standard, and the detection probability are sufficiently low (high). An increase in the detection probability i) increases profits of the high-quality firm, ii) decreases profits of the low-quality firm, iii) decreases consumer surplus, and iv) increases the regulator's payoff. It increases (decreases) welfare, if the market size is sufficiently large (small).*

5 Conclusion

In this paper, we have studied the effect of non-compliance with a minimum quality standard on prices and quality levels in a vertical differentiation model. Since in many markets non-compliance with quality standards frequently occurs, our results offer some insight in the effectiveness of standard-setting and/or increasing the detection probability. Non-complying behavior is an endogenous result of our model.

Non-compliance by the low quality firm also increases profits of the high quality firm, affects the quality level of the high-quality product negatively, reduces consumer surplus, and increases or decreases welfare, depending on the market size, the effect of quality levels of the externality, the detection probability, and the minimum quality level. Consumers are not affected equally by non-compliance. While on average consumers lose due to non-compliance, a subgroup of consumers might benefit. Hence, non-compliance should be considered seriously by policy makers.

Under non-compliance, an enhancement of the minimum quality standard induces an increase of the quality level of low-quality products, increases the average quality level but also increases the quality difference. A higher level of government enforcement, however, lowers the quality of the low quality firm, but shifts demand from the low-quality firm to the high-quality firm.

The main idea of the “Product Safety and Market Surveillance Package” proposed by the European Commission is to increase the probability of detecting non-compliant products. Our results, however, show that a higher level of government enforcement has a negative effect on the quality level of low-quality products. It also increases the probability, that non-complying behavior is welfare-increasing. Therefore maybe it should be accompanied by an additional program that raises the awareness of consumer to problems of non-compliant products. If the government is interested in the overall safety of products, an increase of the minimum quality standard might be preferable to an increase of the detection probability. So the European Commission should maybe not focus only on the detection probability, but mention also an increase of the minimum quality standard in the light of non-compliance.

If consumers buying the low-quality products are at risk or pollute the environment (and would be also at risk or pollute the environment under an enhanced minimum quality standard under non-compliance), while there is no such risk or pollution effect associated with high-quality products, increasing the detection probability might be preferable, because it shifts demand from the low-quality firm to the high quality firm. For those products, the amendment by the European Parliament, focusing also on the environmental characteristics of products, might be the preferable strategy.

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Appendix

No Regulation

$$\begin{aligned}
 s_H &= \frac{4b+1}{8}, s_L = \frac{4b-5}{8} \\
 p_H &= \frac{25+8b+16b^2}{64}, p_L = \frac{49-40b+16b^2}{64}. \\
 \Delta s &= s_H - s_L = \frac{3}{4} \\
 \Delta p &= p_H - p_L = \frac{6b+3}{8} \\
 q_H &= q_L = \frac{1}{2}
 \end{aligned}$$

Minimum Quality Standard and Compliance

Introduction of a Minimum Standard

$$\begin{aligned}
 s_H &= \frac{4b+1}{8} \leq S \leq S^{\max} = \frac{b+1}{2} \\
 s_H^C &= \frac{1+b+S}{3}, s_L^C = S. \\
 p_H^C &= \frac{5(b+1)^2 - 2S(b+1) + 11S^2}{27} \\
 p_L^C &= \frac{(b+1)(7-2b) + 2S(4b-5) + 19S^2}{27} \\
 q_H^C &= \frac{2(b+1) - 4S}{9}, q_L^C = \frac{7-2b+4S}{9}. \\
 \Delta s^C &= s_H^C - s_L^C = \frac{b+1-2S}{3} \\
 p_H^C - p_H &= \frac{(8S-4b+5)(88S+28b-71)}{1728} > 0 \\
 p_L^C - p_L &= \frac{(8S-4b+5)(152S+140b-175)}{1728} > 0 \\
 \Delta p^C &= p_H^C - p_L^C = \frac{(b+1-2S)(4S+7b-2)}{27} \\
 q_H^C - q_H &= -\frac{8S-4b+5}{18} < 0 \\
 q_L^C - q_L &= \frac{8S-4b+5}{18} > 0
 \end{aligned}$$

Increase in Minimum Standard

$$\begin{aligned}
 \frac{\partial s_H^C}{\partial S} &= \frac{1}{3} > 0 \\
 \frac{\partial p_H^C}{\partial S} &= \frac{2(11S-b-1)}{27} > 0 \\
 \frac{\partial p_L^C}{\partial S} &= \frac{2(19S+4b-5)}{27} > 0 \\
 \frac{\partial (p_H^C - p_L^C)}{\partial S} &= -\frac{2(8S+5b-4)}{27} < 0 \\
 \frac{\partial q_H^C}{\partial S} &= -\frac{4}{9} < 0 \\
 \frac{\partial q_L^C}{\partial S} &= \frac{4}{9} > 0
 \end{aligned}$$

Minimum Quality Standard and Non-Compliance

Compliance vs. Non-Compliance

$$\begin{aligned}
 s_H^{NC} &= S, s_L^{NC} = \frac{4S - \Omega - (1-\phi)(2-b)}{6}, \\
 \text{with } \Omega &= \sqrt{(1-\phi)^2(b-2)^2 - 4S(1-\phi)(b-2) + 4S^2(3\phi+1)} \\
 p_H^{NC} &= \frac{(1-\phi)(2b+5)\Omega - 4S\Omega + 2S(1-\phi)(4b+1) + 2S^2(23-15\phi) - (1-\phi)^2(2b+5)(b-2)}{54(1-\phi)},
 \end{aligned}$$

$$\begin{aligned}
p_L^{NC} &= \frac{5(1-\phi)^2(b-2)^2 - 5(1-\phi)(b-2)\Omega - 8S\Omega - 2S(1-\phi)(b-2) + 2S^2(19-3\phi)}{54(1-\phi)} \\
q_H^{NC} &= \frac{(1-\phi)(4b+1) - 8S + 2\Omega}{9(1-\phi)}, \\
q_L^{NC} &= \frac{8S - 2\Omega - 4(1-\phi)(b-2)}{9(1-\phi)} \\
\frac{\partial \Omega}{\partial S} &= \frac{4S(3\phi+1) - 2(1-\phi)(b-2)}{\Omega} > 0, \text{ if } S > S^{\min} \\
(S^{\min} = \frac{4b+1}{8}) &= \Omega^{\min} = \frac{1}{4} \sqrt{3(27 - 47\phi) + 48b\phi(b+2) + 16\phi^2(b-2)^2} > 0 \\
\Omega(S^{\max} = \frac{b+1}{2}) &= \Omega^{\max} = \sqrt{9(1-\phi) + 3b\phi(b+4) + \phi^2(b-2)^2} > 0 \\
\Delta_S^{NC} - \Delta_S^C &= \frac{(6S + \Omega - 3b + \phi(b-2))}{6} > 0 \\
p_H^{NC} - p_H^C &= \frac{\Delta_1 + \Delta_2 + \Delta_3}{54(1-\phi)}, \text{ with} \\
\Delta_1 &= (1-\phi)(\phi(2b+5)(b-2) - 3b(4b+7)), \\
\Delta_2 &= 2S(3(1-\phi)(2b+1) + 4S(3-\phi)), \\
\Delta_3 &= \Omega((1-\phi)(2b+5) - 4S). \\
p_H^{NC} &> p_H^C, \text{ if } S > \tilde{S} \vee \phi > \phi^* \\
\Delta_1 &= (1-\phi)(\phi(2b+5)(b-2) - 3b(4b+7)) < 0, \\
\Delta_2 &= 2S(3(1-\phi)(2b+1) + 4S(3-\phi)) > 0, \frac{\partial \Delta_2}{\partial S} > 0 \\
\Delta_3 &= \Omega((1-\phi)(2b+5) - 4S) > 0, \text{ if } S < S^* = \frac{(1-\phi)(2b+5)}{4}, \\
\frac{\partial \Delta_3}{\partial S} &> 0, \text{ if } S < S^* = \frac{(1-\phi)(2b+5)}{4},
\end{aligned}$$

case I. $S = S^{\min} = \frac{4b+1}{8}$, $\min(\Delta_2)$ and $\min(\Delta_3)$ for S^{\min} , as $\frac{\partial \Delta_2}{\partial S} > 0$ and $\frac{\partial \Delta_3}{\partial S} > 0$

$$\begin{aligned}
&\Delta_1 + \Delta_2 + \Delta_3 \\
&= \frac{3(3-29\phi) + 12b(11\phi + 4b\phi - 9) - 8\phi^2(2b+5)(b-2) - 4(10\phi + 4b\phi - 9)\Omega^{\min}}{8} > 0, \\
\text{if } \phi > \phi^* &= \frac{3(-640b + 736b^2 + 384b^3 - 665) + 15\sqrt{4480b + 4928b^2 + 768b^3 + 1049}}{8(2b+5)(-70b + 48b^2 + 32b^3 - 65)}, \\
\phi^* &> 0 \text{ for } b > \frac{5}{6}.
\end{aligned}$$

case II. $S = S^{\max} = \frac{b+1}{2}$, $\max(\Delta_2)$ and $\max(\Delta_3)$ for S^{\min} , as $\frac{\partial \Delta_2}{\partial S} > 0$ and $\frac{\partial \Delta_3}{\partial S} > 0$

$$\begin{aligned}
&\Delta_1 + \Delta_2 + \Delta_3 \\
&= 3(3 - 5\phi) + 3b\phi(2b+3) - \phi^2(2b+5)(b-2) - (5\phi + 2b\phi - 3)\Omega^{\max} > 0 \\
\frac{\partial \Delta_2}{\partial S} &= 6(1-\phi)(2b+1) + 16S(3-\phi) > 0 \\
\frac{\partial \Delta_3}{\partial S} &= -\frac{2(4b+1)(b-2)(1-2\phi) + 2\phi^2(4b+1)(b-2) - 4S(1-\phi)(8b+15\phi+6b\phi-7) + 32S^2(3\phi+1)}{\Omega} < 0, \\
\text{if } S > S^* &= \frac{(1-\phi)(2b+5)}{4}, \frac{\partial \Delta_3}{\partial S} < 0 \text{ for } S > S^{\min}.
\end{aligned}$$

$$p_H^{NC} - p_H^C = \frac{\Delta_1 + \Delta_2 + \Delta_3}{54(1-\phi)} > 0, \text{ if } S > \tilde{S}$$

For $b = 3$, $\phi = 0.5$, $\tilde{S} \approx 1.6352$.

$$\text{for } S^{\max}: \frac{\partial \Delta_2}{\partial S} + \frac{\partial \Delta_3}{\partial S} = \frac{2\Omega^{\max}(15-7\phi+2b(9-5\phi)) - 6(3-2\phi) - 12b\phi(b+3) - 2\phi^2(14b+10b^2+13)}{\Omega^{\max}} > 0$$

$$p_H^{NC} > p_H^C, \text{ if } S > \tilde{S} \vee \phi > \phi^*$$

$$p_L^{NC} - p_L^C = -\frac{\Delta_1 - \Delta_2 - \Delta_3}{54(1-\phi)}, \text{ with}$$

$$\Delta_1 = \Omega(8S + 5(1-\phi)(b-2)),$$

$$\Delta_2 = 2S(16S\phi - 3(1-\phi)(3b-4)),$$

$$\Delta_3 = (1-\phi)(3(2-10b+3b^2) - 5\phi(b-2)^2).$$

$$p_L^{NC} < p_L^C$$

$$\begin{aligned}
\Delta_1 &= \Omega (8S + 5(1 - \phi)(b - 2)) > 0, \frac{\partial \Delta_1}{\partial S} > 0, \\
\Delta_2 &= 2S(16S\phi - 3(1 - \phi)(3b - 4)) > 0, \text{if } S > S^* = \frac{3(1-\phi)(3b-4)}{16\phi}, \\
\frac{\partial \Delta_2}{\partial S} &> 0 \wedge S > S^{**} = \frac{3(3b-4)(1-\phi)}{32\phi} \\
S^* - S^{\max} &= \frac{3(3b-4) - \phi(17b-4)}{16\phi} > 0 \text{ if } \phi < \frac{3(3b-4)}{(17b-4)} \\
\Delta_3 &= (1 - \phi) \left(3(2 - 10b + 3b^2) - 5\phi(b - 2)^2 \right) > 0, \\
\text{if } b > b^* &= \frac{5(3-2\phi) + 3\sqrt{19-10\phi}}{9-5\phi} \\
\text{case I. } S = S^{\min} &= \frac{4b+1}{8}, \min(\Delta_1) \text{ and } \min(\Delta_2) \text{ for } S^{\min}, \text{ as } \frac{\partial \Delta_1}{\partial S} > 0 \text{ and } \frac{\partial \Delta_2}{\partial S} > 0 \\
\Delta_1 - \Delta_2 - \Delta_3 &= \frac{9(9b-4) - 3\phi(59b+4b^2-38) - 20\phi^2(b-2)^2 + (9(b-1) - 5\phi(b-2))4\Omega^{\min}}{4} > 0, \\
\text{if } \phi < 1 & \\
\text{case II. } S = S^{\max} &= \frac{b+1}{2}, \max(\Delta_1) \text{ and } \max(\Delta_2) \text{ for } S^{\min}, \text{ as } \frac{\partial \Delta_1}{\partial S} > 0 \text{ and } \frac{\partial \Delta_2}{\partial S} > 0 \\
\Delta_1 - \Delta_2 - \Delta_3 &= 9(3b - 2) - 3\phi(21b + b^2 - 10) - 5\phi^2(b - 2)^2 + (3(3b - 2) - 5\phi(b - 2))\Omega^{\max} > 0, \\
\text{if } \phi < 1 & \\
p_L^{NC} &< p_L^C \\
q_H^{NC} - q_H^C &= \frac{\Delta_1 - \Delta_2 + \Delta_3}{9(1-\phi)}, \\
\text{with } \Delta_1 &= (1 - \phi)(2b - 1), \Delta_2 = 4S(\phi + 1), \Delta_3 = 2\Omega. \\
q_H^{NC} &> q_H^C \\
\Delta_1 &= (1 - \phi)(2b - 1) > 0, \\
\Delta_2 &= 4S(\phi + 1) > 0, \frac{\partial \Delta_2}{\partial S} > 0 \\
\Delta_3 &= 2\Omega > 0, \frac{\partial \Delta_3}{\partial S} > 0 \\
S = S^{\min} &= \frac{4b+1}{8}, \min(\Delta_2) \text{ and } \min(\Delta_3) \text{ for } S^{\min}, \text{ as } \frac{\partial \Delta_2}{\partial S} > 0 \text{ and } \frac{\partial \Delta_3}{\partial S} > 0 \\
\Delta_1 - \Delta_2 + \Delta_3 &= \frac{4\Omega^{\min} + \phi - 8b\phi - 3}{2} > 0, \text{ if } \phi < 1 \\
S = S^{\max} &= \frac{b+1}{2}, \max(\Delta_2) \text{ and } \max(\Delta_3) \text{ for } S^{\min}, \text{ as } \frac{\partial \Delta_2}{\partial S} > 0 \text{ and } \frac{\partial \Delta_3}{\partial S} > 0 \\
\Delta_1 - \Delta_2 + \Delta_3 &= 2\Omega^{\max} - 4b\phi - \phi - 3 > 0, \text{ if } \phi < 1 \\
q_L^{NC} - q_L^C &= -\frac{\Delta_1 - \Delta_2 + \Delta_3}{9(1-\phi)}, \\
\text{with } \Delta_1 &= (1 - \phi)(2b - 1), \Delta_2 = 4S(\phi + 1), \Delta_3 = 2\Omega. \\
q_L^{NC} &< q_L^C
\end{aligned}$$

Increase in Minimum Standard under Non-Compliance

$$\begin{aligned}
\frac{\partial s_L^{NC}}{\partial S} &= \frac{\Delta_1 - \Delta_2 + \Delta_3}{3\Omega}, \\
\text{with } \Delta_1 &= (1 - \phi)(b - 2), \Delta_2 = 2S(3\phi + 1), \Delta_3 = 2\Omega. \\
\frac{\partial s_L^{NC}}{\partial S} &> 0 \\
\Delta_1 &= (1 - \phi)(b - 2) > 0, \text{if } b > 2, \\
\Delta_2 &= 2S(3\phi + 1) > 0, \frac{\partial \Delta_2}{\partial S} > 0 \\
\Delta_3 &= 2\Omega > 0, \frac{\partial \Delta_3}{\partial S} > 0
\end{aligned}$$

case I. $S = S^{\min} = \frac{4b+1}{8}$, $\min(\Delta_2)$ and $\min(\Delta_3)$ for S^{\min} , as $\frac{\partial \Delta_2}{\partial S} > 0$ and $\frac{\partial \Delta_3}{\partial S} > 0$

$$\Delta_1 - \Delta_2 + \Delta_3 = \frac{8\Omega^{\min} - 9 + 5\phi - 16b\phi}{4} > 0 \text{ if } \phi < 1.$$

case II. $S = S^{\max} = \frac{b+1}{2}$, $\max(\Delta_2)$ and $\max(\Delta_3)$ for S^{\min} , as $\frac{\partial \Delta_2}{\partial S} > 0$ and $\frac{\partial \Delta_3}{\partial S} > 0$

$$\Delta_1 - \Delta_2 + \Delta_3 = 2\Omega^{\max} - 4b\phi - \phi - 3 > 0 \text{ if } \phi < 1$$

$$\frac{\partial s_L^{NC}}{\partial S} > 0$$

$$\frac{\partial(S - s_L^{NC})}{\partial S} = \frac{\Delta_1 - \Delta_2 + \Delta_3}{3\Omega},$$

with $\Delta_1 = \Omega$, $\Delta_2 = (1 - \phi)(b - 2)$, $\Delta_3 = 2S(3\phi + 1)$.

$$\frac{\partial(S - s_L^{NC})}{\partial S} > 0$$

$$\Delta_1 = \Omega > 0, \frac{\partial \Delta_1}{\partial S} > 0,$$

$$\Delta_2 = (1 - \phi)(b - 2) > 0,$$

$$\Delta_3 = 2S(3\phi + 1) > 0, \frac{\partial \Delta_3}{\partial S} > 0$$

$S = S^{\min} = \frac{4b+1}{8}$, $\min(\Delta_1)$ and $\min(\Delta_3)$ for S^{\min} , as $\frac{\partial \Delta_1}{\partial S} > 0$ and $\frac{\partial \Delta_3}{\partial S} > 0$

$$\Delta_1 - \Delta_2 + \Delta_3 = \frac{9 + \phi(16b - 5) + \Omega^{\min}}{4}$$

$$\frac{\partial(S - s_L^{NC})}{\partial S} > 0$$

$$\frac{\partial p_H^{NC}}{\partial S} = \frac{\Delta_1 - \Delta_2 - \Delta_3}{27(1 - \phi)\Omega}, \text{ with}$$

$$\Delta_1 = \Omega(2S(23 - 15\phi) + (1 - \phi)(4b + 1)),$$

$$\Delta_2 = (1 - \phi)^2(4b + 1)(b - 2),$$

$$\Delta_3 = 2S(8S(3\phi + 1) - (1 - \phi)(8b - 7 + 3\phi(2b + 5)))$$

$$\frac{\partial p_H^{NC}}{\partial S} > 0$$

$$\Delta_1 = \Omega(2S(23 - 15\phi) + (1 - \phi)(4b + 1)) > 0, \frac{\partial \Delta_1}{\partial S} > 0$$

$$\Delta_2 = (1 - \phi)^2(4b + 1)(b - 2) > 0, \text{ if } b > 2,$$

$$\Delta_3 = 2S(8S(3\phi + 1) - (1 - \phi)(8b - 7 + 3\phi(2b + 5))) > 0,$$

$$\text{if } S > S^* = \frac{(1 - \phi)(8b - 7 + 3\phi(2b + 5))}{8(3\phi + 1)},$$

$$S^* - S^{\max} = \frac{4b - 11 - 2\phi(7b - 5) - 3\phi^2(2b + 5)}{8(3\phi + 1)} > 0,$$

$$\text{if } \phi < \frac{\sqrt{(73b + 70)(b - 2)} - 7b + 5}{3(2b + 5)},$$

$$\frac{\partial \Delta_3}{\partial S} > 0$$

case I. $S = S^{\min} = \frac{4b+1}{8}$, $\min(\Delta_1)$ and $\min(\Delta_3)$ for S^{\min} , as $\frac{\partial \Delta_1}{\partial S} > 0$ and $\frac{\partial \Delta_3}{\partial S} > 0$

$$\Delta_1 - \Delta_2 - \Delta_3 = \frac{(4b + 1)(4\Omega^{\min}(27 - 19\phi) - 12\phi(2b - 1) - 4\phi^2(10b + 7))}{16} > 0,$$

if $\phi < 1$

case II. $S = S^{\max} = \frac{b+1}{2}$, $\max(\Delta_1)$ and $\max(\Delta_3)$ for S^{\max} , as $\frac{\partial \Delta_1}{\partial S} > 0$ and $\frac{\partial \Delta_3}{\partial S} > 0$

$$\Delta_1 - \Delta_2 - \Delta_3$$

$$= (27b + 24 - \phi(19b + 16))\Omega^{\max} - 3(3 - 2\phi) - 6b\phi(b + 3) - \phi^2(14b + 10b^2 + 13) > 0,$$

if $\phi < 1$

$$\frac{\partial p_H^{NC}}{\partial S} > 0$$

$$\begin{aligned}
& \frac{\partial p_L^{NC}}{\partial S} = \frac{\Delta_1 - \Delta_2 + \Delta_3}{27(1-\phi)\Omega}, \text{ with} \\
\Delta_1 &= (1-\phi)^2 (b-2)^2, \\
\Delta_2 &= 2S(16S(3\phi+1) - (7-15\phi)(1-\phi)(b-2)), \\
\Delta_3 &= \Omega(2S(19-3\phi) - (1-\phi)(b-2)). \\
& \frac{\partial p_L^{NC}}{\partial S} > 0 \\
\Delta_1 &= (1-\phi)^2 (b-2)^2 > 0, \\
\Delta_2 &= 2S(16S(3\phi+1) - (7-15\phi)(1-\phi)(b-2)) > 0, \\
\text{if } S > S^* &= \frac{(7-15\phi)(1-\phi)(b-2)}{16(3\phi+1)}, \\
S^* - S^{\max} &= -\frac{b+22+2\phi(23b-10)-15\phi^2(b-2)}{16(3\phi+1)} < 0, \\
& \frac{\partial \Delta_2}{\partial S} > 0 \\
\Delta_3 &= \Omega(2S(19-3\phi) - (1-\phi)(b-2)) > 0 \\
\text{if } S > S^* &= \frac{(1-\phi)(b-2)}{2(19-3\phi)}, \\
S^* - S^{\max} &= -\frac{21+18b-\phi(2b+5)}{2(19-3\phi)} < 0 \\
& \frac{\partial \Delta_3}{\partial S} > 0 \\
& \text{case I. } S = S^{\min} = \frac{4b+1}{8}, \min(\Delta_2) \text{ and } \min(\Delta_3) \text{ for } S^{\min}, \text{ as } \frac{\partial \Delta_2}{\partial S} > 0 \text{ and } \frac{\partial \Delta_3}{\partial S} > 0 \\
& \Delta_1 - \Delta_2 + \Delta_3 \\
&= \frac{(9(8b+3) - \phi(8b+11))4\Omega^{\min} - 324b - 24\phi(-23b+32b^2-1) + 4\phi^2(64b+7)(b-2)}{16} > 0, \\
& \text{if } \phi < 1 \\
& \text{case II. } S = S^{\max} = \frac{b+1}{2}, \max(\Delta_2) \text{ and } \max(\Delta_3) \text{ for } S^{\max}, \text{ as } \frac{\partial \Delta_2}{\partial S} > 0 \text{ and } \frac{\partial \Delta_3}{\partial S} > 0 \\
& \Delta_1 - \Delta_2 + \Delta_3 \\
&= (3(6b+7) - \phi(2b+5))\Omega^{\max} - 9(3b+2) - 6\phi(3b+8b^2-2) + \phi^2(16b+13)(b-2) > 0, \\
& \text{if } \phi < 1 \\
& \frac{\partial p_L^{NC}}{\partial S} > 0 \\
& \frac{\partial(p_H^{NC} - p_L^{NC})}{\partial S} = \frac{\Delta_1 - \Delta_2 + \Delta_3}{27(1-\phi)\Omega}, \text{ with} \\
\Delta_1 &= \Omega(8S(1-3\phi) + (1-\phi)(5b-1)), \\
\Delta_2 &= (1-\phi)^2(5b-1)(b-2), \\
\Delta_3 &= 2S(8S(3\phi+1) + 7 + b + 2\phi(10b-11) - 3\phi^2(7b-5)) \\
& \frac{\partial(p_H^{NC} - p_L^{NC})}{\partial S} > 0 \\
\Delta_1 &= \Omega(8S(1-3\phi) + (1-\phi)(5b-1)) > 0, \\
& \frac{\partial \Delta_1}{\partial S} > 0 \\
\Delta_2 &= (1-\phi)^2(5b-1)(b-2) > 0, \\
\Delta_3 &= 2S(8S(3\phi+1) + 7 + b + 2\phi(10b-11) - 3\phi^2(7b-5)) > 0, \\
& \frac{\partial \Delta_3}{\partial S} > 0 \\
& S = S^{\min} = \frac{4b+1}{8}, \min(\Delta_1) \text{ and } \min(\Delta_3) \text{ for } S^{\min}, \text{ as } \frac{\partial \Delta_1}{\partial S} > 0 \text{ and } \frac{\partial \Delta_3}{\partial S} > 0 \\
& \Delta_1 - \Delta_2 + \Delta_3 \\
&= \frac{81b+3\phi(-44b+56b^2-1) - \phi^2(13b+1)(8b-7) + 4\Omega^{\min}(9b-\phi(17b+2))}{4} > 0 \\
& \frac{\partial(p_H^{NC} - p_L^{NC})}{\partial S} > 0
\end{aligned}$$

$$\frac{\partial q_H^{NC}}{\partial S} = -4 \frac{\Delta_1 - \Delta_2 + \Delta_3}{9(1-\phi)\Omega},$$

with $\Delta_1 = (1-\phi)(b-2)$, $\Delta_2 = 2S(3\phi+1)$, $\Delta_3 = 2\Omega$

$$\frac{\partial q_H^{NC}}{\partial S} < 0$$

$$\frac{\partial q_L^{NC}}{\partial S} = -\frac{\partial q_H^{NC}}{\partial S}$$

$$\frac{\partial q_L^{NC}}{\partial S} > 0$$

5.0.1 Increase in Government Enforcement

$$\frac{\partial s_L^{NC}}{\partial \phi} = -\frac{\Delta_1 + \Delta_2 - \Delta_3}{6\Omega}$$

with $\Delta_1 = 2S(3S+b-2)$, $\Delta_2 = \Omega(b-2)$, $\Delta_3 = (b-2)^2(1-\phi)$.

$$\frac{\partial s_L^{NC}}{\partial \phi} < 0$$

$$\Delta_1 = 2S(3S+b-2) > 0, \frac{\partial \Delta_1}{\partial S} > 0,$$

$$\Delta_2 = \Omega(b-2) > 0, \frac{\partial \Delta_2}{\partial S} > 0,$$

$$\Delta_3 = (b-2)^2(1-\phi) > 0,$$

$$S = S^{\min} = \frac{4b+1}{8}, \min(\Delta_1) \text{ and } \min(\Delta_2) \text{ for } S^{\min}, \text{ as } \frac{\partial \Delta_1}{\partial S} > 0 \text{ and } \frac{\partial \Delta_2}{\partial S} > 0$$

$$\Delta_1 + \Delta_2 - \Delta_3 = \frac{96b-141-128\phi(b-1)+16b^2(2\phi+3)+32(b-2)\Omega^{\min}}{32} > 0,$$

if $\phi < 1$

$$\frac{\partial s_L^{NC}}{\partial \phi} < 0$$

$$\frac{\partial p_H^{NC}}{\partial \phi} = \frac{\Delta_1 + \Delta_2 + \Delta_3 - \Delta_4}{54(1-\phi)^2\Omega}, \text{ with } \Delta_1 = (1-\phi)^2(2b+5)(b-2)(2S - (1-\phi)(b-2)), \Delta_2 =$$

$$\Omega \left((1-\phi)^2(2b+5)(b-2) + 16S^2 \right), \Delta_3 = 2S^2(1-\phi)(10b+7-3\phi(2b+5)), \Delta_4 = 8S^3(3\phi+5).$$

$$\frac{\partial p_H^{NC}}{\partial \phi} > 0$$

$$\Delta_1 = (1-\phi)^2(2b+5)(b-2)(2S - (1-\phi)(b-2)) > 0, \frac{\partial \Delta_1}{\partial S} > 0$$

$$\Delta_2 = \Omega \left((1-\phi)^2(2b+5)(b-2) + 16S^2 \right) > 0, \frac{\partial \Delta_2}{\partial S} > 0$$

$$\Delta_3 = 2S^2(1-\phi)(10b+7-3\phi(2b+5)) > 0, \text{ if } \phi < \phi^* = \frac{10b+7}{6b+15}, \frac{\partial \Delta_3}{\partial S} > 0$$

$$\Delta_4 = 8S^3(3\phi+5) > 0, \frac{\partial \Delta_4}{\partial S} > 0$$

case I. $S = S^{\min} = \frac{4b+1}{8}$, $\min(\Delta_1)$, $\min(\Delta_2)$, $\min(\Delta_3)$ and $\min(\Delta_4)$ for S^{\min} , as $\frac{\partial \Delta_1}{\partial S} > 0$, $\frac{\partial \Delta_2}{\partial S} > 0$, $\frac{\partial \Delta_3}{\partial S} > 0$ and $\frac{\partial \Delta_4}{\partial S} > 0$

$$\frac{\Delta_1 + \Delta_2 + \Delta_3 - \Delta_4}{4\sqrt{3(27-47\phi)+48b\phi(b+2)+16\phi^2(b-2)^2(3(4b+8b^2-13)-4\phi(2b+5)(b-2)(2-\phi))}}$$

$$+ \frac{27(8b+16b^2-53)-9\phi(164b+208b^2+64b^3-457)}{64}$$

$$+ \frac{2\phi^2(2b+5)(352b-16b^2-397)+64\phi^3(2b+5)(b-2)^2}{64} > 0$$

case II. $S = S^{\max} = \frac{b+1}{2}$, $\max(\Delta_1)$, $\max(\Delta_2)$, $\max(\Delta_3)$ and $\max(\Delta_4)$ for S^{\max} , as $\frac{\partial \Delta_1}{\partial S} > 0$, $\frac{\partial \Delta_2}{\partial S} > 0$, $\frac{\partial \Delta_3}{\partial S} > 0$ and $\frac{\partial \Delta_4}{\partial S} > 0$

$$\Delta_1 + \Delta_2 + \Delta_3 - \Delta_4 =$$

$$\begin{aligned} & \frac{2\sqrt{9(1-\phi)+3b\phi(b+4)+\phi^2(b-2)^2}(3(b+2)(2b-1)-\phi(2b+5)(b-2)(2-\phi))}{+9(b^2-7)-6\phi(19b+17b^2+3b^3-22)} \\ & - \frac{-\phi^2(2b+5)(-28b+b^2+25)+2\phi^3(2b+5)(b-2)^2}{2} > 0 \\ & \frac{\partial p_H^{NC}}{\partial \phi} > 0 \end{aligned}$$

$$\begin{aligned} & \frac{\partial p_L^{NC}}{\partial \phi} = -\frac{\Delta_1 - \Delta_2 + \Delta_3 + \Delta_4}{54(\phi-1)^2\Omega}, \text{ with } \Delta_1 = (1-\phi)^2(b-2)^2(2S - (1-\phi)(b-2)), \\ & \Delta_2 = \Omega(32S^2 - 5(1-\phi)^2(b-2)^2), \Delta_3 = 2S^2(7-15\phi)(1-\phi)(b-2), \Delta_4 = 16S^3(3\phi+5) \\ & \frac{\partial p_L^{NC}}{\partial \phi} < 0 \text{ if } b \gtrsim 1.7 \end{aligned}$$

$$\Delta_1 = (1-\phi)^2(b-2)^2(2S - (1-\phi)(b-2)) > 0, \frac{\partial \Delta_1}{\partial S} > 0$$

$$\Delta_2 = \Omega(32S^2 - 5(1-\phi)^2(b-2)^2) > 0, \frac{\partial \Delta_2}{\partial S} > 0$$

$$\Delta_3 = 2S^2(7-15\phi)(1-\phi)(b-2) > 0, \frac{\partial \Delta_3}{\partial S} > 0$$

$$\Delta_4 = 16S^3(3\phi+5) > 0, \frac{\partial \Delta_4}{\partial S} > 0$$

$$\begin{aligned} & \text{case I. } S = S^{\min} = \frac{4b+1}{8}, \min(\Delta_1), \min(\Delta_2), \min(\Delta_3) \text{ and } \min(\Delta_4) \text{ for } S^{\min}, \text{ as } \frac{\partial \Delta_1}{\partial S} > 0, \\ & \frac{\partial \Delta_2}{\partial S} > 0, \frac{\partial \Delta_3}{\partial S} > 0 \text{ and } \frac{\partial \Delta_4}{\partial S} > 0 \end{aligned}$$

$$\begin{aligned} & \frac{\Delta_1 - \Delta_2 + \Delta_3 + \Delta_4}{-9(37b-16b^2-48b^3-31)+\phi(2b+5)(328b-64b^2-157)+\phi^2(b-2)(448b+176b^2-385)} \\ & + \frac{32\phi^3(b-2)^3-4\sqrt{3(27-47\phi)+48b\phi(b+2)+16\phi^2(b-2)^2}(3(16b+2b^2-13)+10\phi(b-2)^2(2-\phi))}{32} > 0 \text{ if } b \gtrsim 1.7 \end{aligned}$$

$$\begin{aligned} & \text{case II. } S = S^{\max} = \frac{b+1}{2}, \max(\Delta_1), \max(\Delta_2), \max(\Delta_3) \text{ and } \max(\Delta_4) \text{ for } S^{\max}, \text{ as } \frac{\partial \Delta_1}{\partial S} > 0, \\ & \frac{\partial \Delta_2}{\partial S} > 0, \frac{\partial \Delta_3}{\partial S} > 0 \text{ and } \frac{\partial \Delta_4}{\partial S} > 0 \end{aligned}$$

$$\begin{aligned} & \frac{\Delta_1 - \Delta_2 + \Delta_3 + \Delta_4}{3(5b+22b^2+9b^3+10)+-2\phi(b+4)(-22b+4b^2+1)+\phi^2(b-2)(52b+11b^2-13)} \\ & + \frac{2\phi^3(b-2)^3-2\sqrt{9(1-\phi)+3b\phi(b+4)+\phi^2(b-2)^2}(3(12b+b^2-4)+5\phi(b-2)^2(2-\phi))}{2} > 0 \text{ if } b \gtrsim 1.5 \\ & \frac{\partial p_H^{NC}}{\partial \phi} < 0 \text{ if } b \gtrsim 1.7 \end{aligned}$$

$$\frac{\partial q_H^{NC}}{\partial \phi} = 4S \frac{\Delta_1 - \Delta_2 - \Delta_3}{9(\phi-1)^2\Omega},$$

$$\text{with } \Delta_1 = S(3\phi+5), \Delta_2 = 2\Omega, \Delta_3 = (1-\phi)(b-2)$$

$$\Delta_1 = S(3\phi+5) > 0, \frac{\partial \Delta_1}{\partial S} > 0$$

$$\Delta_2 = 2\Omega > 0, \frac{\partial \Delta_2}{\partial S} > 0$$

$$\Delta_3 = (1-\phi)(b-2) > 0, \text{ if } b > 2$$

$$\text{case I } S = S^{\min} = \frac{4b+1}{8}, \min(\Delta_1) \text{ and } \min(\Delta_2) \text{ for } S^{\min}, \text{ as } \frac{\partial \Delta_1}{\partial S} > 0, \frac{\partial \Delta_2}{\partial S} > 0,$$

$$\begin{aligned} & \frac{\Delta_1 - \Delta_2 - \Delta_3}{3(4b+7)+\phi(20b-13)-16\Omega^{\min}} > 0, \end{aligned}$$

if $\phi < 1$

$$\text{case II. } S = S^{\max} = \frac{b+1}{2}, \max(\Delta_1) \text{ and } \max(\Delta_2), \text{ as } \frac{\partial \Delta_1}{\partial S} > 0, \frac{\partial \Delta_2}{\partial S} > 0$$

$$\begin{aligned} & \frac{\Delta_1 - \Delta_2 - \Delta_3}{3(b+3)+\phi(5b-1)-4\Omega^{\max}} > 0 \end{aligned}$$

$$\begin{aligned}\frac{\partial q_H^{NC}}{\partial \phi} &> 0 \\ \frac{\partial q_L^{NC}}{\partial \phi} &= -\frac{\partial q_H^{NC}}{\partial \phi} \\ \frac{\partial q_L^{NC}}{\partial \phi} &< 0\end{aligned}$$

Welfare

No Regulation

$$\begin{aligned}\pi_H &= \pi_L = \frac{3}{16} \\ CS &= \frac{16b^2 - 16b - 23}{64} \\ R &= -\left(\eta - \alpha q_H s_H^2 - \alpha q_L s_L^2\right) = \frac{\alpha(-16b + 16b^2 + 13)}{64} - \eta \\ W &= \pi_H + \pi_L + CS + R \\ &= \frac{3(-16b + 16b^2 + 41) + 16\alpha(-16b + 16b^2 + 13)}{1024} - \eta\end{aligned}$$

Minimum Quality Standard and Compliance

$$\begin{aligned}\pi_H^C &= \frac{4(b-2S+1)^3}{243} \\ \pi_L^C &= \frac{(b-2S+1)(4S-2b+7)^2}{243} \\ CS^C &= \frac{2(b+1)(22b+2b^2-61) + S(3(98b-29-8b^2) + 2S(24b-147-16S))}{486} \\ R^C &= \frac{\alpha(2(b+1)^3 + S^2(32S-24b+57))}{81} - \eta \\ W^C &= \frac{S(2S(-80S+120b+171\alpha+96S\alpha-72b\alpha-195) - 3(-130b+40b^2+73)) + 4(b+1)^2(5b-4+3\alpha(b+1))}{486} - \eta \\ &= -\frac{\pi_H^C - \pi_H}{3888} (8S-4b+5)(8S(8S-8b-17)+68b+16b^2+133) < 0 \\ \pi_L^C - \pi_L &= -\frac{(8S-4b+5)(8S(8S-8b+19)-76b+16b^2-11)}{3888} < 0 \\ CS^C - CS &= \frac{(8S-4b+5)(548b-32b^2+337-8S(16S-16b+137))}{15552} > 0, \\ \text{if } S < S^* &= \frac{b + \frac{9}{16}\sqrt{265} - \frac{137}{16}}{2} \\ R^C - R &= \frac{\alpha(64S^2(32S-24b+57) + 128b^3 - 912b^2 + 1680b - 925)}{5184} > 0 \\ W^C - W &= \frac{512S(2S(-80S+120b+171\alpha+96S\alpha-72b\alpha-195) - 3(-130b+40b^2+73))}{248832} \\ &+ \frac{5520b+624b^2+10240b^3-38081+48\alpha(8b-37)(4b-5)^2}{248832} > 0 \\ \text{if } \alpha > \alpha^* &= -\frac{(5520b-512S(-390b+2S(80S-120b+195)+120b^2+219)+624b^2+10240b^3-38081)}{48(4b-5)^2(8b-37)+1024S^2(96S-72b+171)} \\ \frac{\partial \pi_H^C}{\partial S} &= -\frac{8(b-2S+1)^2}{81} < 0 \\ \frac{\partial \pi_L^C}{\partial S} &= -\frac{2(4S-2b+1)(4S-2b+7)}{81} < 0 \\ \frac{\partial CS^C}{\partial S} &= -\frac{4S(8S-8b+49)+8b^2+29-98b}{162} < 0 \\ \frac{\partial R^C}{\partial S} &= \frac{2S\alpha(16S-8b+19)}{27} > 0 \\ \frac{\partial W^C}{\partial S} &= \frac{(4S(-40S+40b-65+3\alpha(16S-8b+19)) - (-130b+40b^2+73))}{162} > 0\end{aligned}$$

Minimum Quality Standard and Non-Compliance

Surveillance cost F are normalized to zero.

$$\begin{aligned}
\pi_H^{NC} &= \frac{(8S-2\Omega-(1-\phi)(4b+1))((1-\phi)^2(2b+5)(b-2)+2S(4S(1-3\phi)-(1-\phi)(4b+1))+\Omega(4S-(1-\phi)(2b+5)))}{486(1-\phi)^2} \\
\pi_L^{NC} &= \frac{2(4S-\Omega-2(1-\phi)(b-2))((\phi-1)^2(b-2)^2+4S(S(1-3\phi)-(1-\phi)(b-2))+\Omega(2S-(1-\phi)(b-2)))}{243(1-\phi)} \\
CS^{NC} &= \frac{(1-\phi)^3(b-2)(37+26b-2b^2)-\Omega(4S((1-\phi)(2b-13)-2S(3\phi+1))+(1-\phi)^2(37+26b-2b^2))}{486(1-\phi)^2} \\
&\quad + \frac{S(3(1-\phi)^2(122b+4b^2-71)+2S(8S(1-9\phi)-3(1-\phi)(4b+61-3\phi(4b+7))))}{486(1-\phi)^2} \\
R^{NC} &= \\
\alpha &\frac{2S(-9S+9S\phi-36Sb+40S^2+36Sb\phi+6(\phi-1)^2(b-2)^2)+\Omega(-12S\Omega+\Omega^2+12S^2-3(\phi-1)^2(b-2)^2)+2(1-\phi)^3(b-2)^3}{162(\phi-1)} - \eta \\
W^{NC} &= \pi_H^{NC} + \pi_L^{NC} + CS^{NC} + R^{NC} \\
\pi_H^{NC} - \pi_H^C|_{b=2} &= \frac{S(8S(6S+72\phi-26S\phi-45\phi^2+4S\phi^2-27))+297(1-\phi)^2-108(1-\phi)^2}{243\phi^2-486\phi+243} \\
&\quad + \frac{\sqrt{S^2(3\phi+1)(8S(2S+9\phi+6S\phi-9)+81(1-\phi)^2)}}{243\phi^2-486\phi+243} > 0 \\
\pi_H^{NC} - \pi_H^C|_{b=5} &= \frac{2S(16S(3S+72\phi-13S\phi-45\phi^2+2S\phi^2-27)+1269(\phi-1)^2)+27(25\phi-89)(\phi-1)^2}{486(1-\phi)^2} \\
&\quad + \frac{\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2(8S(2S+15\phi+6S\phi-15)+225(\phi-1)^2)}}{486(1-\phi)^2} > 0 \\
\pi_H^{NC} - \pi_H^C|_{b=10} &= \frac{S(8S(6S+264\phi-26S\phi-165\phi^2+4S\phi^2-99))+4329(\phi-1)^2+4(625\phi-1956)(\phi-1)^2}{243(\phi-1)^2} \\
&\quad + \frac{\sqrt{S(S+8\phi+3S\phi-8)+16(\phi-1)^2(8S(2S+25\phi+6S\phi-25)+625(\phi-1)^2)}}{243(\phi-1)^2} > 0 \\
\pi_L^{NC} - \pi_L^C &: \text{ see incentive to non-comply} \\
CS^{NC} - CS^C|_{b=2} &= \frac{S(4S(6S+36\phi-26S\phi-9\phi^2+4S\phi^2-27))+81(\phi-1)^2+27(1-\phi)^2}{243(1-\phi)^2} \\
&\quad + \frac{\sqrt{S^2(3\phi+1)(4S(2S-9\phi+6S\phi+9)-81(\phi-1)^2)}}{243(1-\phi)^2} < 0 \\
CS^{NC} - CS^C|_{b=5} &= \frac{2S(8S(3S+54\phi-13S\phi-27\phi^2+2S\phi^2-27))+567(\phi-1)^2-27(13\phi+31)(\phi-1)^2}{486(\phi-1)^2} \\
&\quad + \frac{\sqrt{4S(S+3\phi+3S\phi-3)+9(\phi-1)^2(4S(2S-3\phi+6S\phi+3)-117(\phi-1)^2)}}{486(\phi-1)^2} < 0 \\
CS^{NC} - CS^C|_{b=10} &= \frac{S(4S(6S+228\phi-26S\phi-129\phi^2+4S\phi^2-99))+2097(\phi-1)^2-(388\phi+3561)(1-\phi)^2}{243(\phi-1)^2} \\
&\quad + \frac{\sqrt{S(S+8\phi+3S\phi-8)+16(\phi-1)^2(4S(2S+7\phi+6S\phi-7)-97(\phi-1)^2)}}{243(\phi-1)^2} < 0 \\
R^{NC} - R^C|_{b=2} &= 2\alpha \frac{4S^2(-6S-9\phi+13S\phi+9)-2S^2\sqrt{3S^2\phi+S^2(3\phi+4)-27(1-\phi)}}{81(1-\phi)} < 0 \\
R^{NC} - R^C|_{b=5} &= \alpha \frac{4S^2(-12S-45\phi+26S\phi+45)-27(1-\phi)(-2\phi+\phi^2+17)}{81(1-\phi)} \\
&\quad - \alpha \frac{\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2(2S(4S+3\phi+3S\phi-3)-9(\phi-1)^2)}}{81(1-\phi)}
\end{aligned}$$

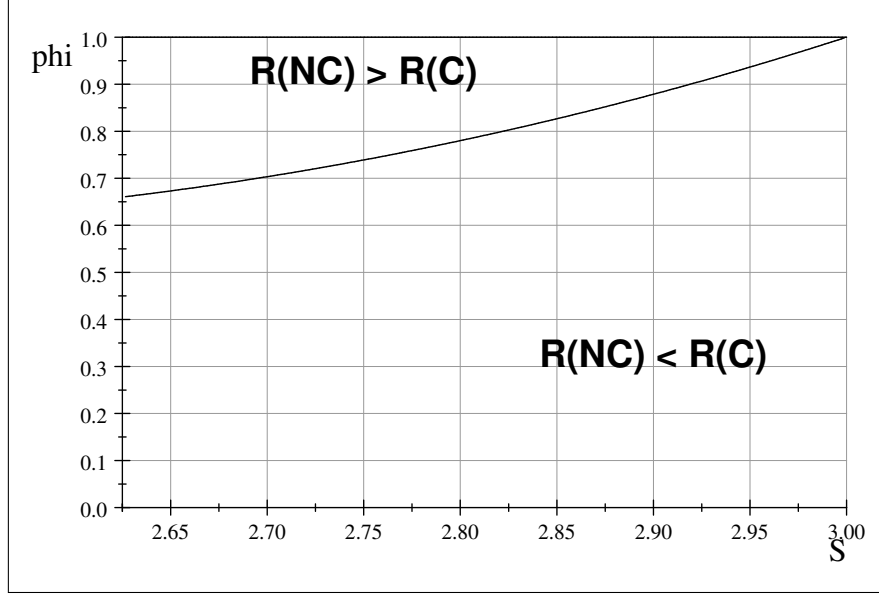


Figure 1: $R^{NC} - R^C$, $b = 5$, $\alpha = 1$

$$R^{NC} - R^C \Big|_{b=10} = 2\alpha \frac{4S^2(-6S-45\phi+13S\phi+45) - (1-\phi)(-512\phi+256\phi^2+1587)}{81(1-\phi)} - 2\alpha \frac{2\sqrt{S(S+8\phi+3S\phi-8)+16(\phi-1)^2(S(4S+8\phi+3S\phi-8)-32(\phi-1)^2)}}{81(1-\phi)}$$

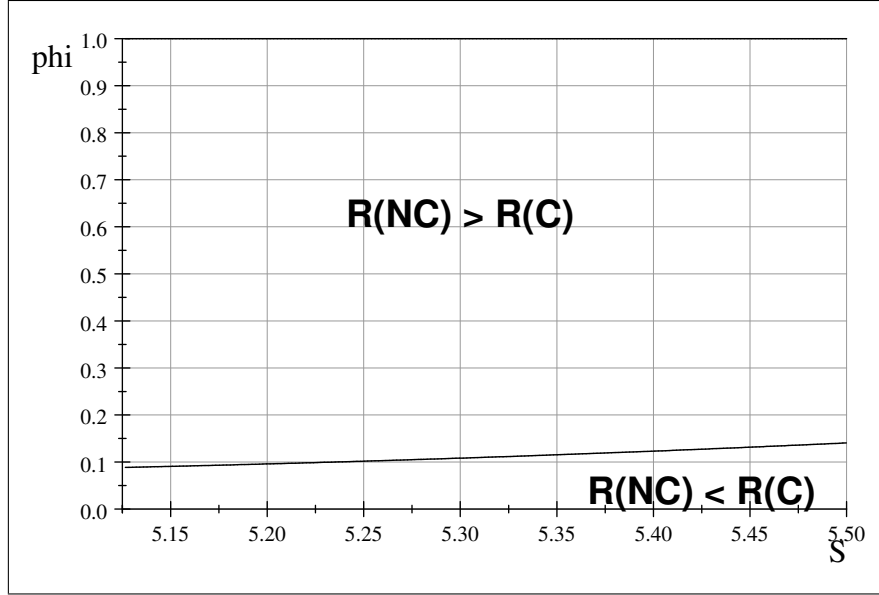


Figure 2: $R^{NC} - R^C$, $b = 10$, $\alpha = 1$

$$W^{NC} - W^C \Big|_{b=2, \alpha=1} = \frac{-4S(S(6S-72\phi+20S\phi+45\phi^2+22S\phi^2+27)-81(\phi-1)^2)}{243(\phi-1)^2} + \frac{4S\sqrt{S^2(3\phi+1)(S(-3\phi^2+29\phi-2)-9(1-\phi))-270(\phi-1)^2}}{243(\phi-1)^2} < 0$$

$$W^{NC} - W^C \Big|_{b=5, \alpha=1} = \frac{-2S(2S(2S(10\phi+11\phi^2+3)+9(1-\phi)(-23\phi+6\phi^2+9))-27(39-2\phi)(\phi-1)^2)-27(1-\phi)^2(5\phi^2-16\phi+115)}{243(\phi-1)^2}$$

$$+ \frac{\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2} (2S(S(29\phi-3\phi^2-2)-3(1-\phi)(10-\phi))+9(\phi-1)^2(11-5\phi))}{243(\phi-1)^2}$$

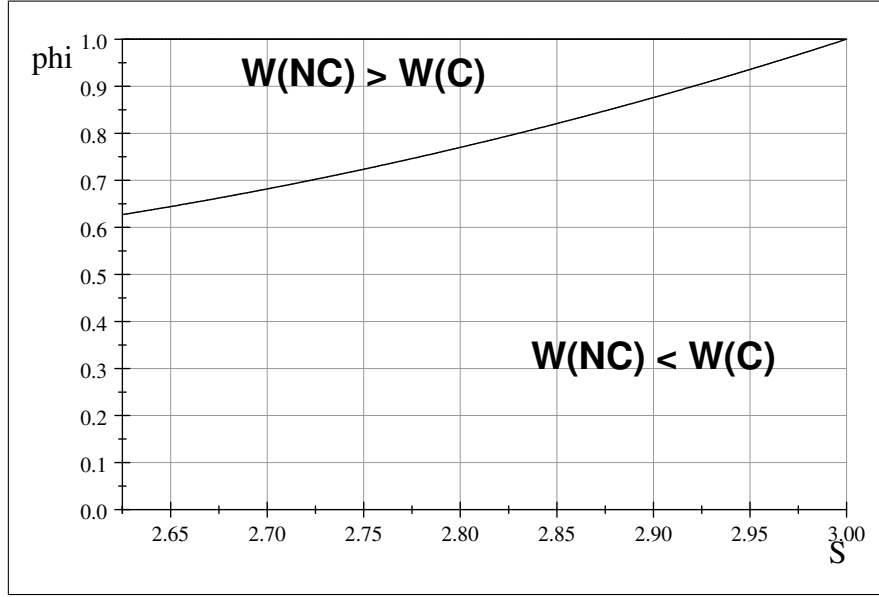


Figure 3: $W^{NC} - W^C$, $b = 5$, $\alpha = 1$

$$W^{NC} - W^C \Big|_{b=5, \alpha=5} = \frac{-2S(2S(2S(-218\phi+167\phi^2+75)-9(\phi-1)(37\phi+6\phi^2-51))+27(2\phi-39)(\phi-1)^2)-27(\phi-1)^2(17\phi^2-40\phi+319)}{243(\phi-1)^2}$$

$$+ \frac{\sqrt{12S^2\phi+4S^2+12S\phi-12S+9\phi^2-18\phi+9} (2S(41S\phi-39\phi-50S+33\phi^2+33S\phi^2+6)-9(\phi-1)^2(17\phi-23))}{243(\phi-1)^2}$$

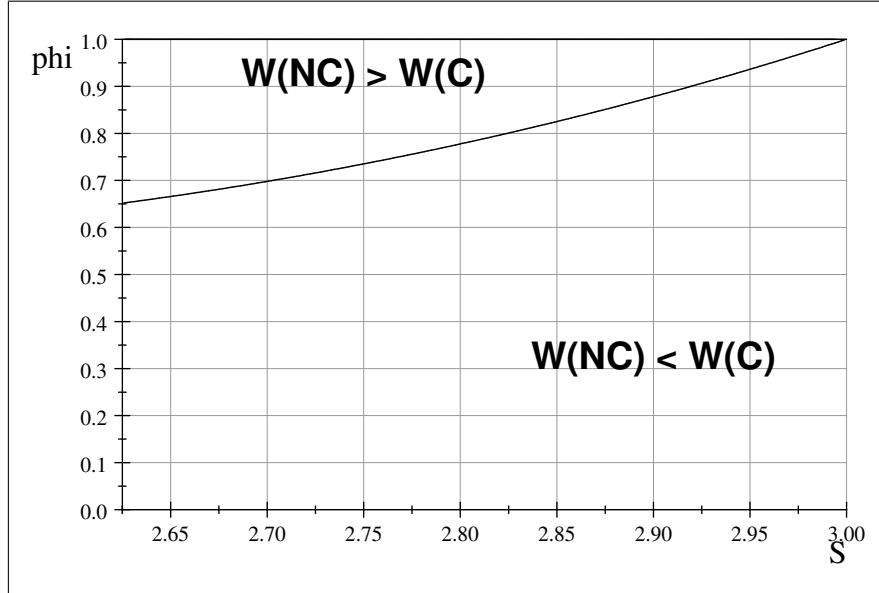


Figure 3: $W^{NC} - W^C$, $b = 5$, $\alpha = 5$

$$W^{NC} - W^C \Big|_{b=10, \alpha=1} = \frac{-4S(S(6S-648\phi+20S\phi+621\phi^2-144\phi^3+22S\phi^2+171))+3(64\phi-723)(\phi-1)^2)}{243(\phi-1)^2}$$

$$\begin{aligned}
& - \frac{2(\phi-1)^2(1280\phi^2-3616\phi+11895)}{243(\phi-1)^2} \\
& + \frac{4\sqrt{S(S+8\phi+3S\phi-8)+16(\phi-1)^2(S(-2S+73\phi+29S\phi-8\phi^2-3S\phi^2-65)-4(40\phi-73)(\phi-1)^2)}}{243(\phi-1)^2}
\end{aligned}$$

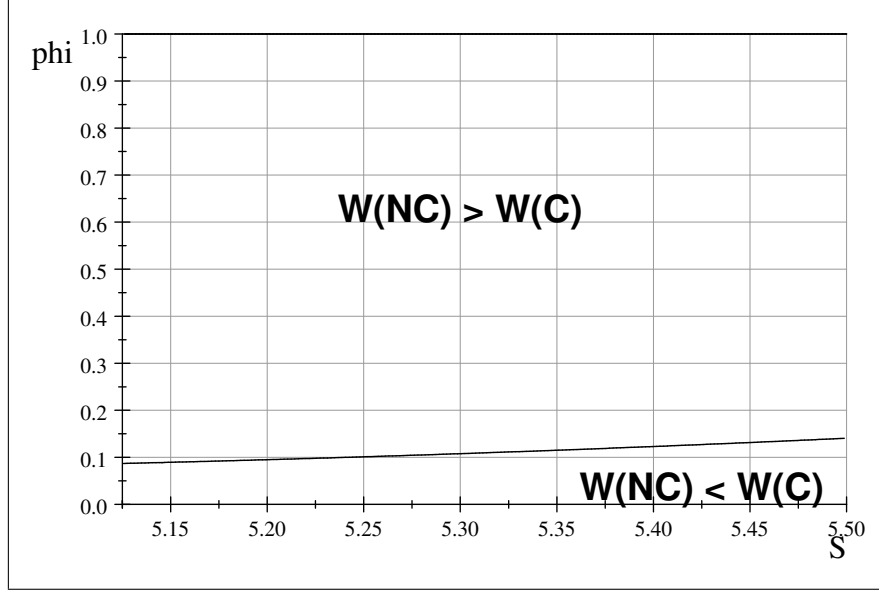


Figure 4: $W^{NC} - W^C$, $b = 10$, $\alpha = 1$

5.0.2 Incentive to non-comply

$$\begin{aligned}
& \pi_L^{NC} - \pi_L^C \\
& = \frac{64S^3(1-2\phi)-48S^2(1-\phi)(2-\phi)(b-2)+8S^2\Omega(3\phi+1)+6S(1-\phi)(-32b+8b^2+23-4\phi(b-2)^2)}{243(1-\phi)} \\
& + \frac{2\Omega(1-\phi)(b-2)((1-\phi)(b-2)+\Omega)-4S\Omega(2(1-\phi)(b-2)+\Omega)-(1-\phi)((69b-48b^2+8b^3+17)+4\phi(b-2)^3(\phi-2))}{243(1-\phi)} \\
& \pi_L^{NC} - \pi_L^C|_{b=2} = \frac{16S^2\sqrt{S^2(3\phi+1)}(3\phi+1)-27(1-\phi)-2S(88S^2\phi-24S^2+27(1-\phi))}{243(1-\phi)} > 0, \\
& \text{if } \phi < \phi^* = \frac{(1458S+\frac{1}{2}\sqrt{(54S-80S^3+27)^3(54S-464S^3+27)+1458S^2-4752S^3-9504S^4+8576S^6+\frac{729}{2}})}{6912S^6}
\end{aligned}$$

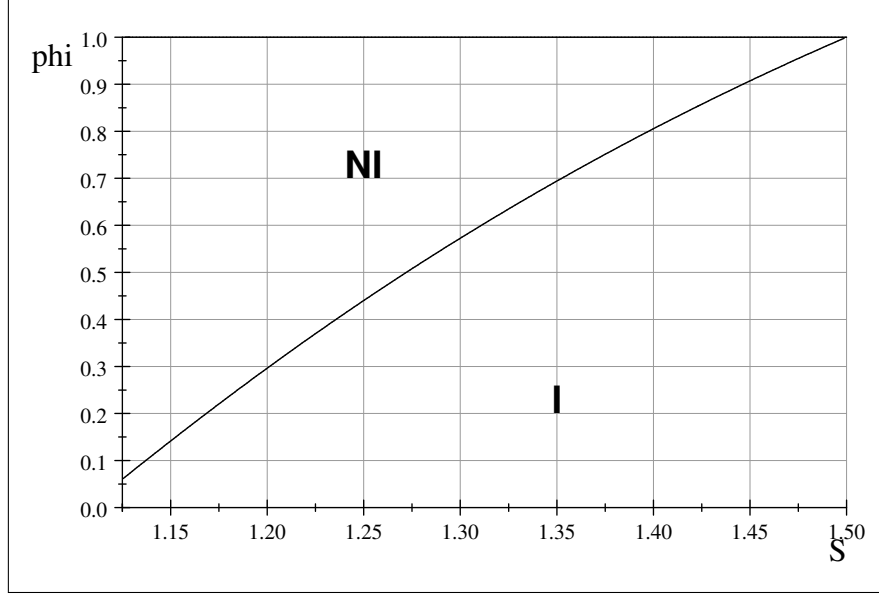


Figure 6: $\pi_L^{NC} - \pi_L^C$, $b = 2$

Figure 6 shows the incentive for the low quality firm to comply or to not comply with the minimum quality standard depending on combinations of values of S and ϕ for a given maximum willingness to pay for quality $b = 2$. In the area I the firm has an incentive for non-compliance, as $\pi_L^{NC} - \pi_L^C > 0$. In the area NI the firm has no incentive for non-compliance, as $\pi_L^{NC} - \pi_L^C < 0$. Given the market size of $b = 2$, a higher level of S requires a higher detection probability ϕ in order to avoid non-compliance behavior of the low quality firm.

$$\pi_L^{NC} - \pi_L^C \Big|_{b=5} = \frac{-2S(108S+189\phi-216S\phi-54\phi^2+108S\phi^2+88S^2\phi-24S^2-135)+54(\phi-1)(-2\phi+\phi^2+2)}{243(1-\phi)} + \frac{2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2(-12S-18\phi+12S\phi+9\phi^2+12S^2\phi+4S^2+9)}}{243(1-\phi)}$$

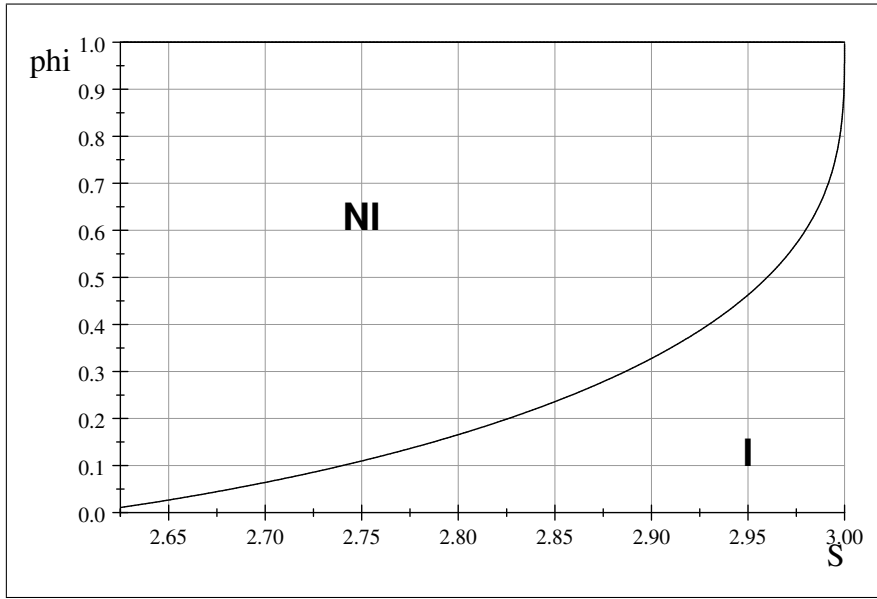


Figure 7: $\pi_L^{NC} - \pi_L^C$, $b = 5$

Figure 7 is similar. The market size is now set to $b = 5$. Irrespective of the detection probability a higher level of the minimum quality standard is needed to trigger non-compliance.

$$\pi_L^{NC} - \pi_L^C|_{b=10} = \frac{-2S(8S(-3S-72\phi+11S\phi+36\phi^2+36))-3(1-\phi)(375-128\phi)-(1-\phi)(-2048\phi+1024\phi^2+2883)}{243(1-\phi)} + \frac{16\sqrt{S(S+8\phi+3S\phi-8)+16(\phi-1)^2(S(S+8\phi+3S\phi-8)+16(\phi-1)^2)}}{243(1-\phi)}$$

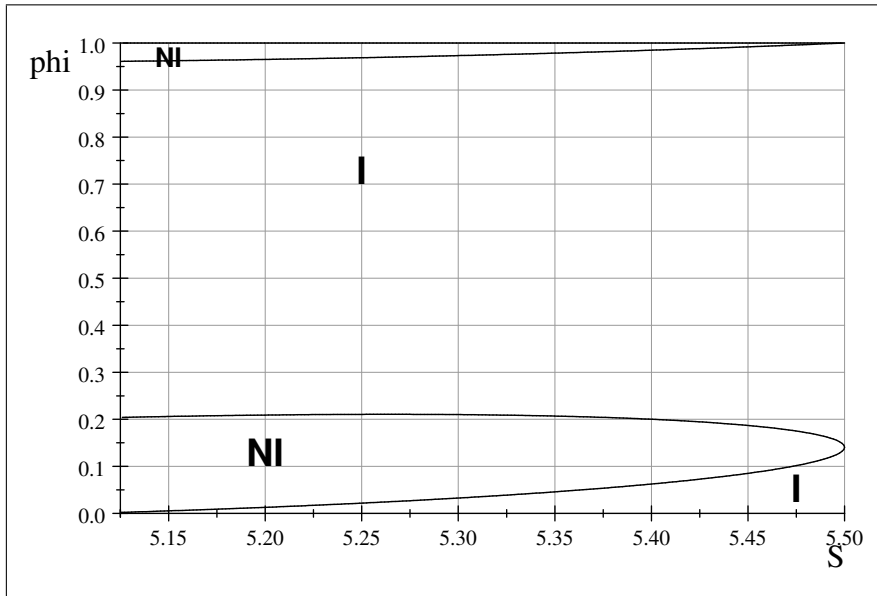


Figure 8: $\pi_L^{NC} - \pi_L^C$, $b = 10$

For a market size of $b = 10$, the picture becomes more complex (see Figure 8). For most combinations of ϕ and S there is an incentive for non-compliance. Compared to lower levels of

b a higher minimum quality level S is needed to trigger non-compliance. Only for very modest values of ϕ and for very strict levels of government enforcement combinations of ϕ and S exist, where compliance is the best strategy for the low-quality firm.

Increase in Standard

$$\begin{aligned} \left. \frac{\partial \pi_H^{NC}}{\partial S} \right|_{b=2} &= \frac{6S(3\phi+1)(16S(S+3\phi+3S\phi-3)+27(\phi-1)^2) - 6\sqrt{3S^2\phi+S^2}(16S(-S-12\phi+9S\phi+9\phi^2+3) - 27(\phi-1)^2)}{486(\phi-1)^2\sqrt{S^2(3\phi+1)}} < 0 \\ \left. \frac{\partial \pi_H^{NC}}{\partial S} \right|_{b=5} &= -\frac{-6S(8S(3\phi+1)(2S+15\phi+6S\phi-15)+9(33\phi+31)(\phi-1)^2)+1215(1-\phi)^3}{243(1-\phi)^2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\ &\quad - \frac{3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}(16S(-S-24\phi+9S\phi+18\phi^2+6) - 135(\phi-1)^2)}{243(1-\phi)^2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} < 0 \\ \left. \frac{\partial \pi_L^{NC}}{\partial S} \right|_{b=2} &= \frac{16}{81} \frac{S^2(S(3\phi+1)^2 + \sqrt{S^2(3\phi+1)}(1-9\phi))}{(1-\phi)\sqrt{S^2(3\phi+1)}} > 0 \\ \left. \frac{\partial \pi_L^{NC}}{\partial S} \right|_{b=5} &= \frac{4(6S(2S(3\phi+1)(2S+9\phi+6S\phi-9)+27(\phi+1)(\phi-1)^2) - 81(1-\phi)^3)}{243(1-\phi)\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\ &\quad - \frac{4(3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}(4S(-S-12\phi+9S\phi+9\phi^2+3) - 9(\phi-1)^2))}{243(1-\phi)\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \end{aligned}$$

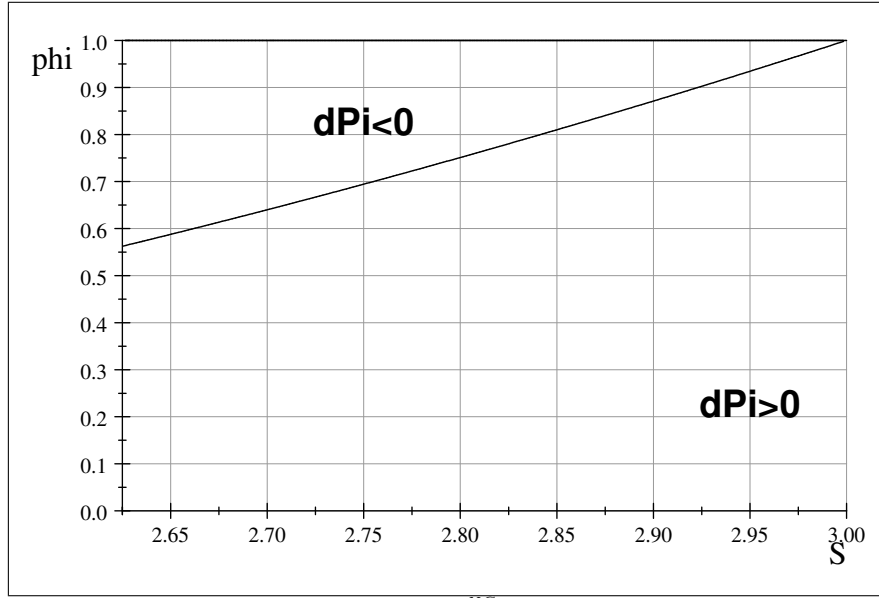


Figure 9: $\frac{\partial \pi_L^{NC}}{\partial S}$, $b = 5$

$$\begin{aligned} \left. \frac{\partial C_S^{NC}}{\partial S} \right|_{b=2} &= -\frac{6\sqrt{S^2(3\phi+1)}(4S(-4S-114\phi+36S\phi+45\phi^2+69) - 189(1-\phi)^2)}{972(1-\phi)^2\sqrt{S^2(3\phi+1)}} \\ &\quad + \frac{12S(3\phi+1)(8S(S-3\phi+3S\phi+3) - 27(\phi-1)^2)}{972(1-\phi)^2\sqrt{S^2(3\phi+1)}} < 0 \\ \left. \frac{\partial C_S^{NC}}{\partial S} \right|_{b=5} &= -\frac{12S(4S(3\phi+1)(2S+3\phi+6S\phi-3) - 9(9\phi+5)(\phi-1)^2) - 810(1-\phi)^3}{486(1-\phi)^2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\ &\quad - \frac{3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}(4S(-4S-162\phi+36S\phi+81\phi^2+81) - 639(\phi-1)^2)}{486(1-\phi)^2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} < 0 \end{aligned}$$

$$\begin{aligned}
\left. \frac{\partial R^{NC}}{\partial S} \right|_{b=2, \alpha=1} &= 12S \frac{(\sqrt{S^2(3\phi+1)}(-8S-27\phi+36S\phi+27)-2S^2(3\phi+4)(3\phi+1))}{162(1-\phi)\sqrt{S^2(3\phi+1)}} > 0 \\
\left. \frac{\partial R^{NC}}{\partial S} \right|_{b=5, \alpha=1} &= \frac{-24S(8S-3\phi+30S\phi+27\phi^2+18S\phi^2-24)+18(\phi-1)^2}{(162(1-\phi))\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\
&+ \frac{12S\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}(-8S-39\phi+36S\phi+39)}{(162(1-\phi))\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} > 0 \\
\left. \frac{\partial W^{NC}}{\partial S} \right|_{b=2, \alpha=1} &= \frac{-6\sqrt{3S^2\phi+S^2}(16S(S-12\phi+14S\phi+9\phi^2+9S\phi^2+3)-243(\phi-1)^2)-48S^2(3\phi+1)(2S-6\phi-29S\phi+3S\phi^2+6)}{972(\phi-1)^2\sqrt{S^2(3\phi+1)}}
\end{aligned}$$

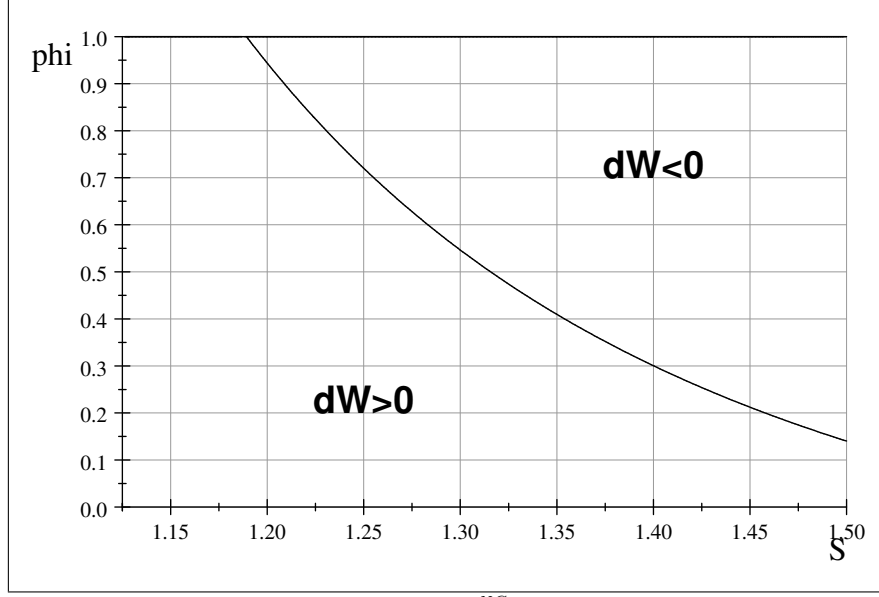


Figure 10: $\frac{\partial W^{NC}}{\partial S}$, $b = 2$

$$\begin{aligned}
\left. \frac{\partial W^{NC}}{\partial S} \right|_{b=5, \alpha=1} &= \frac{-24S(S(4S+231\phi-46S\phi-288\phi^2+27\phi^3-168S\phi^2+18S\phi^3+30))+9(-18\phi+6\phi^2-13)(\phi-1)^2)-324(7-2\phi)(1-\phi)^3}{486(\phi-1)^2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\
&- \frac{3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}(16S(S-60\phi+14S\phi+69\phi^2-18\phi^3+9S\phi^2+9))+9(8\phi-109)(\phi-1)^2)}{486(\phi-1)^2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} > 0
\end{aligned}$$

Increase in Government Enforcement

$$\begin{aligned}
\left. \frac{\partial \pi_H^{NC}}{\partial \phi} \right|_{b=2} &= \frac{S^2(8S(2S(3\phi+13)(3\phi+1)-9(3\phi+5)(1-\phi))+243(1-\phi)^3-32\sqrt{S^2(3\phi+1)}(7S+9\phi+9S\phi-9))}{486(1-\phi)^3\sqrt{S^2(3\phi+1)}} > 0 \\
\left. \frac{\partial \pi_H^{NC}}{\partial \phi} \right|_{b=5} &= \frac{2S(S(-1032S-3321\phi+144S^2\phi^2+528S\phi+2673\phi^2-675\phi^3+504S\phi^2+672S^2\phi+208S^2+1323))+675(1-\phi)^3)-2025(1-\phi)^4}{486(1-\phi)^3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\
&- \frac{\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}(32S^2(7S+18\phi+9S\phi-18)-675(1-\phi)^3)}{486(1-\phi)^3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} > 0 \\
\left. \frac{\partial \pi_L^{NC}}{\partial \phi} \right|_{b=2} &= -\frac{8S^4(3\phi+1)(11-3\phi)+128S^3\sqrt{S^2(3\phi+1)}}{243(\phi-1)^2\sqrt{S^2(3\phi+1)}} < 0 \\
\left. \frac{\partial \pi_L^{NC}}{\partial \phi} \right|_{b=5} &= -\frac{2(2S(S(120S+189\phi+36S^2\phi^2-192S\phi-297\phi^2+135\phi^3+72S\phi^2-120S^2\phi-44S^2-27)-135(1-\phi)^3)+162(\phi-1)^4)}{243(\phi-1)^2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\
&- \frac{2(2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}(S(-54S-54\phi+108S\phi+27\phi^2-54S\phi^2+32S^2+27)-27(1-\phi)^3))}{243(\phi-1)^2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}}
\end{aligned}$$

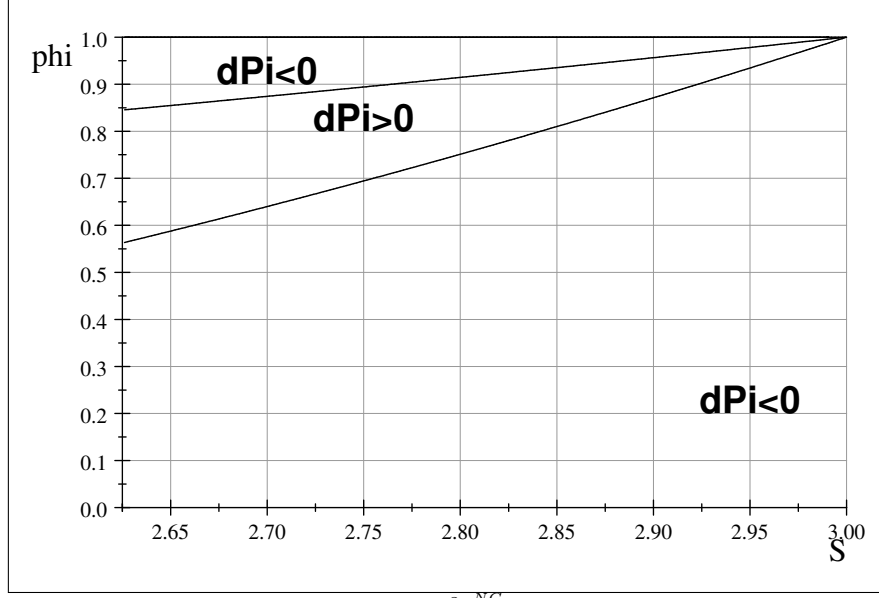


Figure 11: $\frac{\partial \pi_L^{NC}}{\partial S^-}$, $b = 5$

$$\begin{aligned}
& \left. \frac{\partial CS^{NC}}{\partial \phi} \right|_{b=2} \\
&= - \frac{S^2 \left(16\sqrt{S^2(3\phi+1)}(7S-9\phi+9S\phi+9)+243(1-\phi)^3-4S(2S(3\phi+13)(3\phi+1)+9(3\phi+5)(1-\phi)) \right)}{486(1-\phi)^3\sqrt{S^2(3\phi+1)}} < 0 \\
& \left. \frac{\partial CS^{NC}}{\partial \phi} \right|_{b=5} \\
&= - \frac{2S \left(-243S+1053\phi+36S^2\phi^2+72S^3\phi^2+837S\phi-1053\phi^2+351\phi^3-945S\phi^2+120S^2\phi+351S\phi^3+336S^3\phi-156S^2+104S^3-351 \right)}{486(1-\phi)^3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\
&+ \frac{\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2} \left(1053\phi-1053\phi^2+351\phi^3-144S^3\phi-112S^3-351 \right) - 1053(\phi-1)^4}{486(1-\phi)^3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} < 0 \\
& \left. \frac{\partial R^{NC}}{\partial \phi} \right|_{b=2} = \frac{4S^4(-39\phi+9\phi^2-26)+112S^3\sqrt{S^2(3\phi+1)}}{162(1-\phi)^2\sqrt{S^2(3\phi+1)}} > 0 \\
& \left. \frac{\partial R^{NC}}{\partial \phi} \right|_{b=5} = \frac{4S(-81\phi+18S^2\phi^2+9S^3\phi^2-27S\phi+81\phi^2-27\phi^3+54S\phi^2-57S^2\phi-27S\phi^3-39S^3\phi+39S^2-26S^3+27)-162(\phi-1)^4}{81(\phi-1)^2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\
&+ \frac{2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}(-81\phi+81\phi^2-27\phi^3+28S^3+27)}{81(\phi-1)^2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} > 0 \\
& \left. \frac{\partial W^{NC}}{\partial \phi} \right|_{b=2} \\
&= - \frac{2}{243} \frac{S^2 \left(-S(S(289\phi+42\phi^2+9\phi^3+44))-9(3\phi+5)(1-\phi)+4\sqrt{S^2(3\phi+1)}(16S+9\phi+32S\phi-9) \right)}{(1-\phi)^3\sqrt{S^2(3\phi+1)}} < 0 \\
& \left. \frac{\partial W^{NC}}{\partial \phi} \right|_{b=5} = \frac{4S \left(-300S-918\phi+42S^2\phi^2+9S^2\phi^3+186S\phi+1161\phi^2-756\phi^3+216\phi^4+96S\phi^2+289S^2\phi+18S\phi^3+44S^2+297 \right)}{243(1-\phi)^3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\
&+ \frac{4S(+27(8\phi-11)(\phi-1)^3)+162(5\phi-8)(\phi-1)^4}{243(1-\phi)^3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \\
&- \frac{2\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}}{243(1-\phi)^3\sqrt{4S(S(1+3\phi)-3(1-\phi))+9(1-\phi)^2}} \times \\
&\left(2S(-126S-81\phi+234S\phi+81\phi^2-27\phi^3-162S\phi^2+64S^2\phi+54S\phi^3+32S^2+27)-27(5\phi-8)(\phi-1)^3 \right) \\
&> 0
\end{aligned}$$