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LIMITED SELF-CONTROL AND LONG-RUN GROWTH

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Limited Self-Control and Long-Run Growth

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Abstract. This paper integrates imperfect self-control into the standard model of endogenous growth. Individuals are conceptualized as "dual-selves" consisting of a long-run planner and a short-run doer. The long-run self can partly control the short-run self's strife for immediate gratification. It is shown that the solution is structurally equivalent to the one of the standard endogenous growth model as long as self-control is sufficiently strong. Within a certain range of self-control an investment subsidy can be useful in order to reduce consumption and to increase investment, growth, and welfare of the long-run self. A consumption tax, perhaps surprisingly, is counterproductive. It induces individuals with limited self-control to consume even more.

Keywords: temptation, self-control, consumption, investment, endogenous growth.
JEL: D91; E21; 040.

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1. Introduction

The conventional theory of economic growth is based on exponential (sometimes also called geometric) discounting of future utility. This setup, introduced by Samuelson (1937), implies a constant rate of time preference, a fact that considerably simplifies the analysis of the underlying problem of intertemporal choice. Simplicity, however, is bought by abstracting from real economic behavior, which limits the power of the underlying model. Samuelson, for his part, worried that the level of abstraction may be too great for the model to be useful for welfare analysis.

Research in psychology and behavioral economics documents that time preference rates are indeed not constant but declining over time, presumably in a hyperbolic fashion (see Frederick et al., 2002; DellaVigna, 2009, for surveys). The present-bias of preferences implies – without further assumptions – that intertemporal plans are time-inconsistent (Strotz, 1956, Pollak, 1968). In particular, it has been argued that individuals save and invest too little when they discount the future hyperbolically (e.g. Laibson, 1996, 1998). It is straightforward to see that reduced savings are harmful for long-run growth.

A couple of papers have investigated how the consideration of hyperbolic preferences modifies the predictions of standard models of economic growth. Barro (1999) and Krusell (2002) studied the neoclassical growth model with quasi-hyperbolic preferences when individuals make time-inconsistent decisions or, respectively, "solve" the time inconsistency problem by following a linear Markovian consumption strategy. Strulik (2013) investigates the basic model of endogenous growth (Romer, 1986) for the case of hyperbolic preferences. A common result from these studies is that the solution is structurally equivalent to the solution of the respective standard models. The equivalence results make it difficult to draw any policy conclusion, in particular since the underlying parameters of the discounting function are not known but only imprecisely estimated (Frederick et al., 2002). Drawing policy conclusions is furthermore complicated by the time-inconsistency of individual plans, which means that policy is differently evaluated in any period such that it is not a priori clear which period-self of an individual should be used for welfare analysis.[1]

The behavioral approach to intertemporal choice has recently been modified by a couple studies integrating self-control or impulse control into economics (Gul and Pesendorfer, 2001, 2004); Benhabib and Bisin, 2004; Fudenberg and Levine, 2006). These studies take into account insights from neurology showing that different areas of the brain are occupied with short-run (impulsive) behavior and and long-run (planned) behavior. Individuals are conceptualized as neither just "cold" long-run planners nor just "hot" affective consumers. The present paper relates mostly to Fudenberg and Levine's (2006) dual-self model consisting of a planner who can partly control the (impulsive) actions of a short-run doer. Self-control, however, comes at a utility cost, which is increasing in the deviation of the constrained optimal solution from the unconstrained optimal solution preferred by the short-run self. The solution of Samuelson's discounted utility model is included as a special case of perfect self-control.

The Gul-Pesendorfer (2001, 2004) theory of self-control avoids the distinction between short- and long-run selves and develops instead the notion of temptation utility and commitment utility, in which deviating from unconstrained-optimal temptation-consumption incurs a cost. In the context of the present paper the Gul-Pesendorfer and Fudenberg-Levine approach are identical in setup and solution of the problem. The interpretation of the solution, however, deviates. The present paper follows Fudenberg and Levine by arguing that welfare is represented by the long-run self's utility because short- and long-run self share the same utility function. Krusell et al. (2010) investigate the neoclassical growth model when individuals are equipped with Gul-Pesendorfer preferences and show (for the log-utility case) that an optimal subsidy of wealth exist.

Compared with the earlier literature on hyperbolic preferences the new approach has not only the advantage of integrating further aspects of human psychology. It provides also

[1]There exists also a larger literature on the consumption-savings choice under generally time-variant discounting to which the present paper contributes, see e.g. Epstein and Hynes (1983), Obstfeld (1990), Drugeon (1996), Becker and Mulligan (1997), Palivos et. al. (1997), Das (2003), Schumacher (2009), Strulik (2012).

a time-consistent solution, even if the short-run self has time-inconsistent (and perhaps hyperbolic) preferences. This feature, together with the feature that welfare is uniquely defined, allows a straightforward discussion of welfare improving economic policies.

The present paper integrates the dual-self model of Fudenberg and Levine into the standard model of endogenous growth (Romer, 1986). The long-run self can be conceptualized as the optimizing agent of the standard model of economic growth. The short-run self shares the same utility function but displays a preference for immediate gratification or, more generally, for consumption in the near future. The next section sets up the model for generally iso-elastic period utility and investigates the central mechanism at play. The general model, unfortunately, cannot be solved in closed form.

Section 3 derives the closed-from solution for the log-utility case and shows that limited self-control leads to overconsumption and reduced economic growth. It extends the list of structural equivalence results (see above) by showing that any degree of limited self-control can be equivalently represented by adjusting the rate of time preference in the conventional model of economic growth. It is shown that a consumption tax, perhaps surprisingly, exacerbates the problem of overconsumption due to limited self-control. Similarly, an investment subsidy, raises the incentive to consume and reduces investment through the self-control channel. But an investment subsidy also raises the incentive to consume through the conventional channel of a higher net return on investment. It is shown that the conventional channel dominates as long as the self-control problem is sufficiently small. In this case an investment subsidy reduces consumption and may even be used to implement the consumption plan of perfect self-control under laissez faire policy or the welfare maximizing consumption plan.[2]

Section 4 investigates the general case and shows that the policy conclusions from the basic model are robust. Structural equivalence, however, breaks down when the self-control problem gets sufficiently large. Comparing reasonable numerical specifications of the model with recent estimates of the degree of limited self-control in consumption

[2]In the present context the individual savings decision under perfect self-control and laissez faire does not coincide with the welfare maximum because endogenous growth is based on knowledge externalities.

(Bucciol, 2012; Huang et al., 2013) leads to the conclusion that self-control problems are likely to be sufficiently small for structural equivalence to be preserved.

2. The Model

2.1. **Individuals.** Consider an economy populated by a large number of infinitely living individuals of measure one. Individuals experience utility from consumption c. All variables are time-variant but whenever this causes no confusion the time index is omitted. In deviation from the conventional growth model the consumption – savings decision is subject to a problem of self-control. Following Fudenberg and Levine (2006) the self-control problem can be represented as a game between an impulsive short-run self seeking for immediate gratification and a long-run self planning ahead in the way known from conventional growth theory. The short-run self takes the action but is partly controlled by the long-run self. Self-control is costly in terms of utility. The costs of self-control are increasing in the deviation of actual utility from the utility that could be obtained by following the unconstrained impulsive consumption choice. Welfare is evaluated according to the long-run preferences.

We assume that both selves face the same iso-elastic utility function, $u(c) = [c^{1-\theta} - 1]/(1-\theta)$ for $\theta \neq 1$ and $u(c) = \log(c)$ for $\theta = 1$. The difference between the selves consists solely of their different evaluation of the future. The long-run self discounts future consumption at a constant rate ρ. Let the solution preferred by the constrained short-runs self be called c_s. The long-run self then maximizes

$$\int_0^\infty \mathrm{e}^{-\rho t} \Big\{ u(c) - \gamma \big[u(c_s) - u(c) \big] \Big\} \, \mathrm{d}t \ , \qquad (1)$$

The parameter γ measures the cost of self-control, which is assumed to be proportional to the difference between the utility from actual consumption c and short-run optimal consumption c_s.

Households supply one unit of labor and earn wages w and hold wealth (capital) k. They face an interest rate r and a depreciation rate δ on capital holdings k and decide on how to divide their capital labor income between consumption c and savings. In order to better manage their self-control problem and improve their consumption decisions it could be in the interest of individuals to mandate a government to set up taxes and subsidies. We thus assume that individuals potentially pay a linear tax τ on consumption and receive a linear subsidy σ on savings (investment). The government runs a balanced budget and rebates excess revenues as transfers g, which would be a lump tax if subsidies exceed tax revenues. Summarizing, the individuals' budget constraint is given by

$$\dot{k} = (1+\sigma)\left[rk + w + g - (1+\tau)c - \delta k\right]. \tag{2}$$

2.2. **Firms.** This part of the model follows strictly Romer (1986). The economy is populated by a large number of firms of measure one. Any firm i uses capital input k_i and labor input $\ell(i)$ to produce output $y(i) = \tilde{A}(i)k(i)^{\alpha}\ell(i)^{1-\alpha}$. Firms operate under perfect competition such that factor prices are given by $r = \alpha\tilde{A}(i)k(i)^{\alpha-1}\ell(i)^{1-\alpha}$ and $w = (1-\alpha)\tilde{A}(i)k(i)^{\alpha}\ell(i)^{-\alpha}$. As in Romer (1986) there is learning-by-doing such that the technology available to any firm is a positive function of aggregate capital $\tilde{A}(i) = A\left[\int_0^1 k(j)\mathrm{d}j\right]^{1-\alpha} = Ak^{1-\alpha}$. Using the normalization of firms ($k(i) = k$) this provides the aggregate production function $y = Ak$, wages $w = (1-\alpha)Ak$, and the interest rate $r = \alpha A$. We assume $\alpha A > \delta$ for investment to be worthwhile.

2.3. **Perfect Self-Control.** The conventional endogenous growth model is included as a special case when there are no costs of controlling the short-run self. With perfect self-control, individuals maximize $\int_0^{\infty} \mathrm{e}^{-\rho t}u(c)\mathrm{d}t$ subject to (2) which provides the familiar first order conditions (3) and (4), in which λ is the costate variable (the shadow price of capital). Inserting wages, interest rates, and the government budget constraint $g = \tau c - \sigma\left[(r-\delta)k + w - (1+\tau_c)c + g\right]$, into the individuals' budget constraint (2) provides the equation motion (5).

$$\mathrm{e}^{-\rho t}c^{-\theta} = \lambda(1+\sigma)(1+\tau) \tag{3}$$

$$(1+\sigma)\lambda(r-\delta) = -\dot{\lambda} \tag{4}$$

$$\dot{k}/k = A - \delta - c. \tag{5}$$

As is well known, equations (3) and (4) can be summarized to the Ramsey rule $\dot{c}/c = [(1+\sigma)(r-\delta)-\rho]/\theta$. Guessing that the solution has the form $c = ak$, such that c and k grow at equal rates, inserting c into the Ramsey rule, and equating it with(5) and solving for a provides the optimal consumption rate out of wealth

$$a = A - \delta - \frac{(1+\sigma)(\alpha A - \delta) - \rho}{\theta}. \tag{6}$$

Notice that τ and g dropped out such that only the investment subsidy σ affects consumption. The neutrality of consumption taxes in this context is a well known result. The solution in (6) provides the consumption rate of out wealth. The more familiar consumption rate out of income is given by $c/y = ak/(Ak) = a/A$, and the savings rate is $1 - a/A$.

2.4. **The Short-Run Self's Preferred Consumption.** We assume that the short-run self prefers to consume a greater share of wealth, $c_s = a_s k$ with $a_s > a$. This approach includes several interesting special cases. Setting $a_s = 1$ captures the case investigated (in partial equilibrium) by Fudenberg and Levine (2006), i.e. the case in which the short run self prefers to consume all wealth immediately. Another interesting special case can be constructed by assuming that the short-run self has access to a commitment technology for savings. This reduces the problem from self-control such that wealth (for example, deposits at banks) is not available for impulse consumption. Accordingly, the short-run self prefers to spend current income on consumption, which is captured by setting $a_s = A$.

More generally we allow the short-run self to plan ahead by assuming that it discounts the future more heavily than the long-run self. The short-run plans may be time-consistent or time-inconsistent. In the first case we assume that the short-run self discounts the future exponentially as well albeit at a (much) higher rate $\rho_s > \rho$. In this case we arrive immediately at the solution by replacing ρ by ρ_s in (3). Proceeding as above we obtain the short-run self's consumption rate

$$a_s = A - \delta - \frac{(1+\sigma)(\alpha A - \delta) - \rho_s}{\theta} \tag{7}$$

Consumption of the short-run self is decreasing in the investment subsidy and increasing in the time preference rate ρ_s and the curvature parameter of the utility function θ. There exists a natural upper bound of ρ_s, at which the short-run self wants to consume all wealth immediately, given by $\rho_s = (1 - A + \delta)\theta + (1+\sigma)(\alpha A - \delta)$.

Finally the short-run self could discount the future hyperbolically and make time-inconsistent consumption plans, which are then continuously revised. In this case life time utility is given by $\int_0^\infty u(c(t))D(t)\mathrm{d}t$ with discount factor $D(t) = [1 + \rho_0 \beta t]^{-\frac{1}{\beta}}$. This problem exhibits a closed form solution in the case of log-utility. As shown by Strulik (2013) the optimal consumption rate is given, in this papers notation, by

$$a_s = A - (1+\sigma)(\alpha A - \delta) + \rho_0(1-\beta). \tag{8}$$

The case of hyperbolic discounting is thus conveniently captured (for the log-utility case) by setting $\rho_s = \rho_0(1-\beta)$ in (7).

The present problem of self-control is isomorph to the commitment problem set up by Gul and Pesendorfer (2001, 2004) and discussed in context of the neoclassical growth model by Krusell et al. (2010). According to their methodology, the term $\tilde{v}(t) \equiv \gamma \cdot \int_0^\infty u(c(t)) \cdot D(t)\ \mathrm{d}t$ represents temptation utility and the term $\tilde{u} \equiv \int_0^\infty \mathrm{e}^{-\rho t} u[c(t)]\mathrm{d}t$ represents commitment utility. The consumer maximizes the sum of commitment and temptation utility net of the cost of self-control, which is computed as the difference between unconstrained and constrained temptation utility.

2.5. **The Long-Run Self's First Order Conditions.** Using (7) the Hamiltonian for problem (1) and (2) reads

$$H = \mathrm{e}^{\rho t}\left[(1+\gamma)\frac{c^{1-\theta}-1}{1-\theta} - \gamma\frac{(a_s k)^{1-\theta}-1}{1-\theta}\right] + \lambda(1+\sigma)\left[(r-\delta)k + w + g - (1+\tau)c\right] \tag{9}$$

with co-state variable λ. The Hamiltonian is not necessarily concave in states and controls. Below we check whether the maximized Hamiltonian is strictly concave in the state variable such that the first order conditions indeed describe the optimal solution. The first order condition with respect to consumption is the same as for the unconstrained individual (3), which is for convenience displayed again as (10). The costate equation takes into account that a higher capital stock implies higher consumption and lower marginal utility from consumption for the short-run self. It is given by (11), which replaces (4).

$$\mathrm{e}^{-\rho t} c^{-\theta} = \lambda(1+\sigma)(1+\tau) \tag{10}$$

$$\lambda(1+\sigma)(r-\delta) - \mathrm{e}^{-\rho t}\gamma a_s^{1-\theta} k^{-\theta} = -\dot{\lambda}. \tag{11}$$

The first term on the left hand side of (11) is the well-known return on investment measured in present value utils. The presence of the second term on the left hand side causes the shadow price of capital, λ, to decline at a slower rate than in the conventional model. This is so because more capital entails more consumption desires of future short-run selves and thus leads to a higher future cost of self-control. The anticipated rising costs of self-control reduce the incentive to save. This is so because consumption grows optimally in proportion to the rate at which the shadow price of capital declines. As usual, this can be seen by log-differentiating (10), which provides $\theta\dot{c}/c = -\dot{\lambda}/\lambda$. Inserting this information in (11) and substituting $\mathrm{e}^{-\rho t}$ from (10) provides the "modified Ramsey rule" (12).

$$\theta\frac{\dot{c}}{c} = (1+\sigma)(r-\delta) - \rho - \frac{\gamma}{1+\gamma}(a_s)^{1-\theta}(1+\sigma)(1+\tau)\left(\frac{c}{k}\right)^{\theta}. \tag{12}$$

The last term in (12) reflects the negative impact of limited self-control on consumption growth. It captures the effect that increasing capital accumulation leads to higher future consumption desire of the short-run self, which leads to more pain from self-control, which diminishes the net return on savings. Lower consumption growth means less savings, i.e. more current consumption. Ceteris paribus, the reduction in consumption growth is increasing in the strength of self-control problems γ. For $\gamma = 0$ the term disappears and the ordinary Ramsey rule is obtained. As before, inserting wages, interest rates and the government budget constraint into the individuals' budget constraint leads to (5). The optimum is thus fully described by (5) and (12). Unfortunately, a closed form solution exists only in the case of log-utility, which we discuss next. Section 4 is dedicated to the general iso-elastic case.

3. The Benchmark Case

3.1. **Consumption and Growth with Limited Self-Control.** In this section we discuss the log-utility case by setting $\theta = 1$. For convenience we replicate the solution under prefect self-control (i.e. the solution of the conventional growth model) for $\theta = 1$ from (6) as (13).

$$a = A - \delta - (1+\sigma)(\alpha A - \delta) + \rho. \tag{13}$$

Suppose the solution for limited self-control has the form $c = bk$, implying that c and k grow at equal rates and (3) simplifies to $\dot{k}/k = A - \delta - b$. After inserting $\dot{c}/c = A - \delta - b$, $c/k = b$, $\theta = 1$, and $r = \alpha A$ into (12) the optimal consumption rate for the self-control afflicted individual is obtained as

$$b = \Big[A - \delta - (1+\sigma)(\alpha A - \delta) + \rho\Big] \cdot \mu, \quad \mu \equiv \frac{1+\gamma}{1-\gamma(\sigma+\tau+\sigma\tau)}. \tag{14}$$

Observe that the solution under limited self-control (14) equals the solution of perfect self-control times a multiplier μ. Subsidies and taxes must not be too high for a meaningful positive solution to exist, $1/\gamma > \sigma+\tau+\sigma\tau$, which is assumed to hold henceforth. Inserting the solution $c = bk$ into (9) verifies that the maximized Hamiltonian is strictly concave in the state variable (see Appendix). The solution is a maximum.

Proposition 1. *Given log-utility, the presence of self-control problems reduces long-run growth. The reduction is increasing in the cost of self-control.*

For the proof observe that the multiplier μ is larger than unity such that $b > a$ and that the multiplier is increasing in γ. Next compare growth under unlimited self-control, $A-\delta-a$, with growth under limited self-control, $A-\delta-b$, to see that self-control problems reduce growth by $\Delta g = b - a$, which is increasing in γ because b is increasing in γ.

Comparing (14) with (13) leads to the observation of structural equivalence.

PROPOSITION 2. *In case of log-utility any solution of the model of limited self-control can be represented as a solution of the model of perfect self-control.*

For the proof replace ρ in (13) by $\bar{\rho}$ and equate (13) and (14) to obtain

$$\bar{\rho} = \mu\rho + (\mu - 1)\left[A - \delta - (1+\sigma)(\alpha A - \delta)\right].$$

Thus, any solution of the problem of limited self-control can equivalently be stated in terms of the conventional model of endogenous growth by adjusting the time preference rate to $\bar{\rho}$.

With respect to consumption policy we observe the following, perhaps surprising, result.

PROPOSITION 3. *Given log-utility a consumption tax cannot reduce overconsumption due to limited self-control. In fact, overconsumption could be reduced by a consumption subsidy.*

For the proof compute

$$\frac{\partial\mu}{\partial\tau} = \frac{\gamma(1+\gamma)(1+\sigma)}{[1-\gamma(\sigma+\tau+\sigma]^2} > 0.$$

and conclude that the consumption multiplier is increasing in τ.

This is explained as follows. An increase of the consumption tax leaves consumption under perfect self-control (13) unaffected. This is so because a parametric change of the tax does not affect the relative price of future consumption. The consumption tax, ceteris paribus, reduces the shadow price of capital λ. This can be seen from (10) for constant c. A lower shadow price means a lower value of the (future) capital stock because wealth can buy less after-tax consumption. With future capital and consumption of the long-run self being less worthwhile, the pain from unfulfilled short-run consumption desires gets relatively more important for overall utility such that individuals save less and indulge more in present consumption.

PROPOSITION 4. *Given log-utility an investment subsidy has a generally ambiguous effect on consumption. It leads to lower consumption through the conventional "return to investment channel". It leads to higher consumption through the "limited self-control channel".*

For the proof begin with observing that the first term in (14) responds negatively to rising σ. This is the conventional "return to investment channel", which is also present in the standard growth model (in equation (13)). Then notice that the multiplier responds positively:

$$\frac{\partial \mu}{\partial \sigma} = \frac{\gamma(1+\gamma)(1+\tau)}{[1-\gamma(\sigma+\tau+\sigma]^2} > 0.$$

The intuition for the positive self-control effect is similar as the one derived for increasing τ. A larger investment subsidy makes a unit of savings less precious for the long-run self. With an increasing investment subsidy the same future marginal utility from consumption can be realized by lower investment today. The fact that the value of capital accumulation declines, makes it easier for the individual to give in and allow the short-run self more consumption today.

In other words, an investment subsidy is not capable to reduce the self-control problem. Its effect on self-control would, taken for itself, raise consumption. Nevertheless an investment subsidy may be useful to raise investment and growth, namely when the usual return on investment channel dominates. A particularly interesting question in this regard is whether an investment subsidy could be used to establish the laissez-faire solution under perfect self-control, i.e. the solution given by (13) for $\sigma = 0$.

PROPOSITION 5. *If the self-control problem is sufficiently small, i.e. for $\gamma < \gamma_1 \equiv (\alpha A - \delta)/[(1-2\alpha)A + \rho + \delta]$, there exists an investment subsidy that implements the laissez-faire consumption plan under perfect self-control. It is given by*

$$\sigma^* = \frac{\gamma\left[(1-\alpha)A + \rho\right]}{(1+\gamma)(\alpha A - \delta) - \gamma\left[(1-\alpha)A + \rho\right]}. \tag{15}$$

For the proof set $\tau = 0$ and $\sigma > 0$ in b from (14) and equate it with consumption rate a from (13) for $\sigma = 0$. Solving for σ yields $\sigma = \sigma^*$ in (15). As γ goes to γ_1 from below, the denominator of (14) goes to zero and the subsidy needed to implement the first best goes to infinity. Taking the respective derivatives of (14) verifies that σ^* is not only increasing in the severity of self-control problems, γ, but also in the rate of time preference ρ. Moreover, σ^* is decreasing in A, indicating that overconsumption due to self-control problems is, ceteris paribus, easier reduced when the return on investment is higher.

If σ^* cannot be implemented, it may still be possible to use subsidies to move investment closer to the laissez faire solution of perfect control.

PROPOSITION 6. *When self-control problems are of intermediate strength, i.e., for* $\gamma \in [\gamma_1,\ \gamma_2]$, $\gamma_2 \equiv (\alpha A - \delta)/[(1-\alpha)A + \rho]$, *an investment subsidy is helpful to reduce consumption, i.e.* $\mathrm{d}b/\mathrm{d}\sigma < 0$, *but not powerful enough to implement the laissez faire consumption of perfect control.*

The proof inspects the derivative $\partial b/\partial \sigma = -(1+\gamma)\left[\alpha A(1+\gamma) - \gamma(A+\rho) - \delta\right]/(1-\gamma\sigma)^2$ and concludes that it is negative as long as γ is smaller than γ_2. The lower limit γ_1 follows from Proposition 3. Notice that $\gamma_1 < \gamma_2$ because $\alpha A > \delta$.

Laissez faire, however, is not the first best policy in case of perfect self-control because individual investment decisions do not take into account the knowledge externality. Individuals invest too little even in case of perfect self-control, implying that the subsidy σ^* does not implement the welfare optimum. In order to derive the welfare maximizing policy in case of limited self-control we first consider the first best policy and consumption decision under perfect self-control.

LEMMA 1. *The optimal investment subsidy in case of perfect self-control is* $\tilde{\sigma} = (1-\alpha)A/(\alpha A - \delta)$ *and the implied welfare maximizing consumption plan* c/k *is given by* $\tilde{a} = \rho$.

For the proof see Appendix.

Proposition 7. *The welfare maximizing subsidy is given by*

$$\sigma^{**} = \frac{(1-\alpha)A(1+\gamma)+\gamma\rho}{(\alpha A-\delta)(1+\gamma)-\gamma\rho}.$$

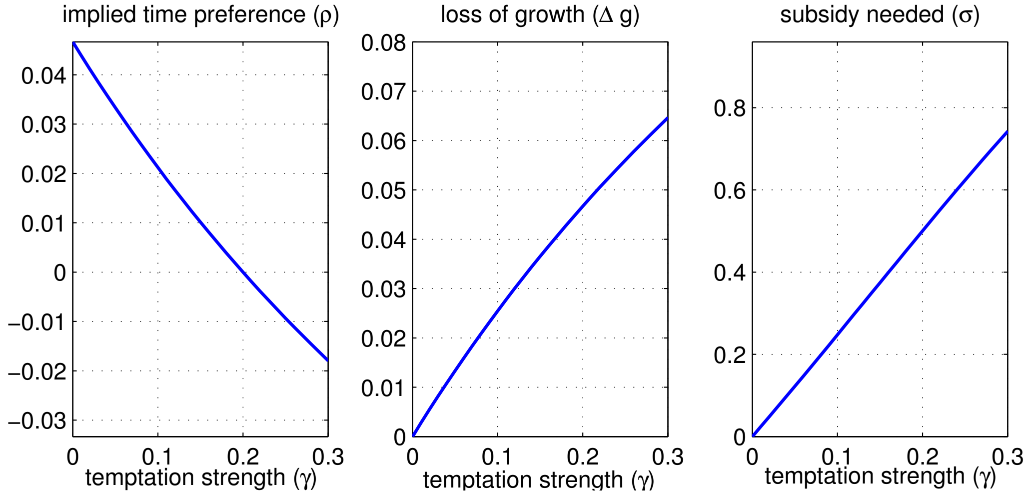
It is increasing in the cost of self-control γ and the time preference rate of the long-run self ρ and decreasing in productivity A.

The proof equates b from (14) with ρ, using Lemma 1, and obtains σ^{**} by solving for σ. The second part of the proof follows from inspection of the derivatives of σ^{**} with respect to γ, ρ, and A. A positive solution for σ^{**} requires that $\gamma > \gamma_3 \equiv -(\alpha A-\delta)/(\alpha A-\delta-\rho)$. Noting that the term $\alpha A-\delta-\rho$ is the laissez-faire growth rate of an economy populated by individuals of perfect self-control leads to the conclusion that the welfare maximum can always be implemented when there is positive long-run growth under perfect self-control. Noting furthermore that in this case $\gamma_3 < \gamma_2 < \gamma_1$, leads to the conclusion that the upper bounds in Proposition 5 and 6 are not restrictive.

3.2. **Quantitative Implications.** The observation of structural equivalence is useful for the design of a "controlled" numerical experiment. For that purpose I set the capital share α to 1/3, the depreciation rate δ to 0.05, and the consumption rate in terms of GDP, given by bA, to 80 percent. The latter provides the estimate $A = 0.35$, implying a net return on investment of $\alpha A - \delta = 0.066$. I then set the two remaining parameters γ and ρ, such that any solution of the model provides a growth rate of GDP per capita of 2 percent annually, which means that the model is roughly consistent with the U.S. growth experience in the 20th century (Maddison, 2003).

Figure 1 shows the results for alternative strengths of the self-control problem γ. The panel on the left hand side shows the associated long-run time preference rate ρ that implements an annual growth rate of 2 percent. For example, a growth rate of two percent could be equivalently realized by perfect self-control and $\rho = 0.047$ or imperfect self-control with $\gamma = 0.1$ and $\rho = 0.02$. A moderate self-control problem allows to explain observable growth rates by moderate rates of time preference. Strong self-control problems (γ above 0.2), however, are associated with negative time preference rates. They would imply implausible human behavior within the framework of endogenous economic growth in the sense that eliminating the self-control problem would lead to the experience of infinite bliss (an infinite utility integral).

Figure 1: The Impact of Limited Self-Control (Log-Utility Case)



Parameters: $\alpha = 1/3$ and $\delta = 0.05$. A and ρ are set such that (gross) investment rate is 0.2 and the growth rate is 0.02. This implies $A = 0.35$ and ρ depending on γ as shown in the left panel. The middle panel shows the entailed loss of growth compared to perfect self-control. The panel on the right hand side shows the subsidy needed to implement the laissez-faire consumption plan of perfect control.

The center panel of Figure 1 shows the hypothetical loss of economic growth caused by limited self-control. It is obtained by computing $a - b$ for any given (γ, ρ) from the left panel. For example, for $\gamma = 0.1$ and $\rho = 0.02$ the economy could grow at a rate of 4.5 percent if γ were zero, implying a loss of growth of 2.5 percent. Overall the predicted loss of growth is quite dramatic, even for moderate values of γ.

The panel on the right hand side shows the subsidy needed in order to implement consumption and growth under perfect self-control and laissez faire, i.e. it displays σ^* from (15). For all plausible values of γ there exists a subsidy that could implement σ^* but the subsidy needed is steeply increasing and already quite high when the self-control problem is relatively small.

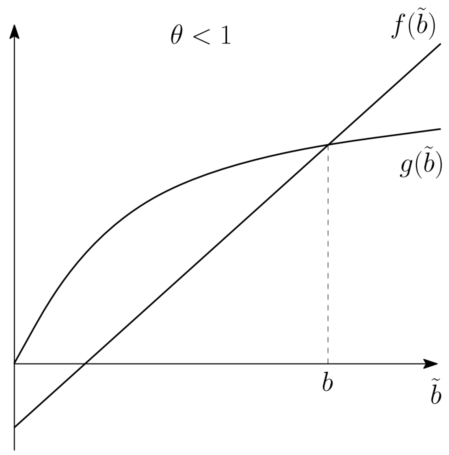
4. The General Case

In the general case the optimal solution can no longer be obtained in closed form. It seems reasonable (and is verified below) that consumption continues to be a constant share of wealth. Otherwise, with a perpetually declining or rising investment rate, the economy would implode or explode. Guessing $c = bk$ such that $\dot{c}/c = \dot{k}/k = A - b - \delta$. and inserting this information in (12) we see that the solution fulfils

$$f(b) \equiv \theta b + (1+\sigma)(\alpha A - \delta) - \theta(A - \delta) - \rho = \frac{\gamma}{1+\gamma}(1+\sigma)(1+\tau)(a_s)^{1-\theta} b^{\theta} \equiv g(b). \quad (16)$$

Notice that for the left hand side of (16), $f(b) = \theta(b - a)$. Diagrammatically $f(b)$ it is thus represented by a straight line originating from $-a$. The right hand side of (16) is represented by a curve through the origin, which is concave for $\theta < 1$ and convex for $\theta > 1$.

Figure 2: Optimal Consumption for $\theta < 1$



The case of $\theta < 1$ is shown in Figure 2. There is a unique intersection of the $f(\tilde{b})$ and $g(\tilde{b})$ curves, which identifies the optimal solution b. Observe that $f(b)$ is independent from τ and that increasing τ bends the $g(\tilde{b})$-curve upwards, implying that a higher tax on consumption increases the consumption rate. A higher investment subsidy shifts the $f(\tilde{b})$ curve upwards, implying an intersection at lower value of b. The higher return on investment leads, taken for itself, to a lower consumption rate. But a higher investment rate pulls the $g(\tilde{b})$-curve upwards as well, implying, taken for itself, an intersection a higher value of b because the decreasing value of future consumption for the long-self raises the pain from suppressing consumption desires of the short-run self. This means that, as for the log-utility case, the outcome is ambiguous in the aggregate.

Figure 3: Optimal Consumption for $\theta > 1$

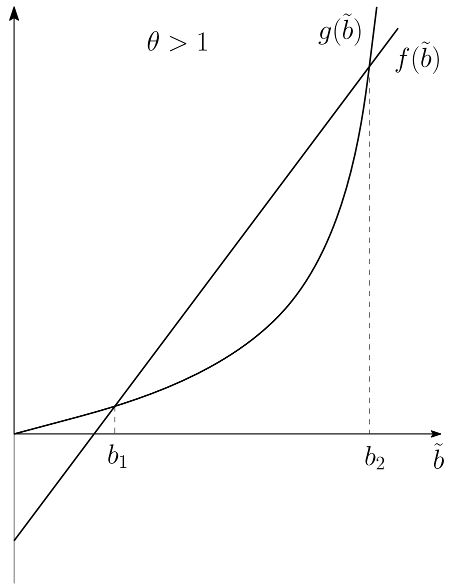


Figure 3 shows the case of $\theta > 1$. Aside from the degenerate case where $f(\tilde{b})$ and $g(\tilde{b})$ are tangent to each other, the curves intersect either twice or not at all. In the later case there is no equilibrium (with constant consumption rate). After plugging in the first order conditions and the guess $c = bk$ into the Hamiltonian, we find that the slope $\frac{\partial H}{\partial k}$ is positive for small b and negative for large b (See Appendix for details). This means that the optimal solution is found at the lower intersection of the two curves (and that the upper intersection identifies a minimum).

At the lower intersection, all comparative static result remain unchanged. A larger γ or τ draws the $g(\tilde{b})$ curve upwards and leaves the $f(\tilde{b})$-curve unchanged, leading thus to a higher value of b_1. A larger σ draws the $g(\tilde{b})$ curve upwards and shifts the $f(\tilde{b})$ curve upwards, such that the movement of b_1 is a priori ambiguous. Altogether this means that the results from Proposition 1 – 3 carry over to the general case.

PROPOSITION 8. *Given an iso-elastic utility function, the presence of self-control problems reduces long-run growth. The reduction is increasing in the cost of self-control. A consumption tax cannot reduce overconsumption due to limited self-control. An investment subsidy has a generally ambiguous effect on consumption. It leads to lower consumption through the conventional "return to investment channel". It leads to higher consumption through the "limited self-control channel".*

The result of structural equivalence, however, is not robust. For sufficiently high γ the self-control model predicts a degenerate response of consumption to an increasing curvature parameter θ. To see this, recall that the conventional growth model predicts that rising θ causes the consumption rate to increase (see (6)). The reason is that an increasing curvature of the utility function entails an increasing desire to smooth consumption and thus a declining desire to save.

In order to obtain the response of the consumption rate to varying θ under limited self-control define the implicit function $G = f(b) - g(b)$ with $\mathrm{d}b/\mathrm{d}\theta = -(\partial G/\partial\theta)/(\partial G/\partial b)$ and $\partial G/\partial b = \theta + \theta b^{\theta-1} a_s^{1-\theta}(1+\sigma)(1+\tau)\gamma/(1+\gamma) > 0$. The derivative with respect to θ is obtained as

$$\frac{\partial G}{\partial \theta} = -(A - \delta - b) - \left\{ \frac{\gamma}{1+\gamma}(1+\sigma)(1+\tau)\left(\frac{b}{a_s}\right)^{\theta} \right\} \left\{ a_s\left[\log b - \log a_s\right] + (1-\theta)\frac{\partial a_s}{\partial \theta} \right\}. \tag{17}$$

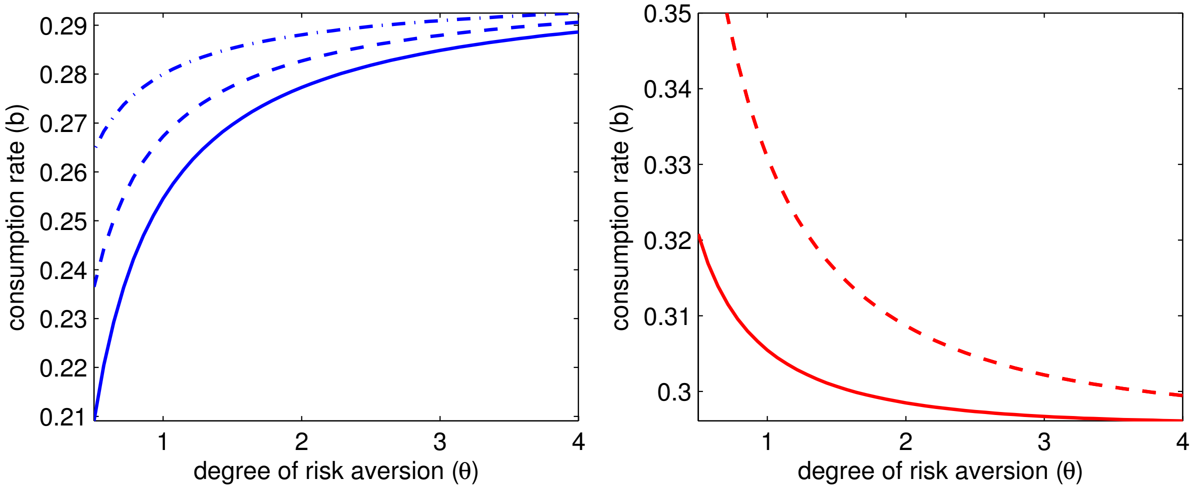
The first term in (17) is negative and in absolute terms equal to the rate of economic growth, i.e. it is a quantitatively small term. It is also present without self-control and would, taken for itself, produce the "normal" response $\mathrm{d}b/\mathrm{d}\theta > 0$. The first expression in curly parenthesis is positive and strictly increasing in the cost of self-control γ. The second term in curly parenthesis is potentially negative. To see this, recall that $a_s > b$ if there is any self-control. A sufficient, not necessary condition for the second term to be negative is thus $\theta > 1$. But the second term may turn negative even for $\theta < 1$, as it does, for example, for the special cases $a_s = 1$ (consume all wealth) and $a_s = A$ (consume all income) for which $\partial a_s / \partial \theta = 0$. In these cases the product of the terms in curly parenthesis is negative and absolutely large for large γ such that the response of consumption is negative, $\mathrm{d}b/\mathrm{d}\theta < 0$. The reason is that the marginal utility from consumption experienced by the short-run self, $a_s^{-\theta}$ is declining in θ, implying that the marginal cost of self-control is declining in θ, which taken for itself, means a lower desire to consume and a higher savings rate when θ is large.

Figure 4 shows results for the set of parameters from Figure 1 ($\alpha = 1/3$, $A = 0.35$, $\rho = 0.021$) and alternative values of θ. Consumption desires of the short-run self are set to $a_s = A$, implying that the uncontrolled short-run self would consume all income. These parameters provide a consumption rate out of GDP of 0.8 and a growth rate of 2 percent for $\theta = 1$ and $\gamma = 0.1$. The solid line in the left hand side panel shows the consumption rate b for $\gamma = 0$ and alternative θ. For a better quantitative assessment recall that $c/y = b/A$, implying that c/y runs from 0.60 to 0.82 as b runs from 0.21 to 0.29. The result replicates the outcome from the standard model of endogenous growth. Dashed lines show the consumption rate for $\gamma = 0.05$ and and dash-dotted lines show results for $\gamma = 0.1$. Consumption rises with increasing cost of self-control (upwards shift of the curves) but structural equivalence with the standard model is preserved: the consumption rate is increasing in curvature of the utility function θ, i.e. it is increasing in the desire to smooth consumption.

The panel on the right hand side shows the breakdown of structural equivalence. The solid line shows consumption behavior for $\gamma = 0.2$. The consumption rate out of GDP runs from 0.91 to 0.84 as b run from 0.32 to 0.296. The consumption rate adjusts from the "wrong side". More curvature of the utility function leads to lower consumption (a higher savings rate). This degenerate behavior cannot be displayed by the conventional model. Dashed lines show consumption rates for $\gamma = 0.3$.

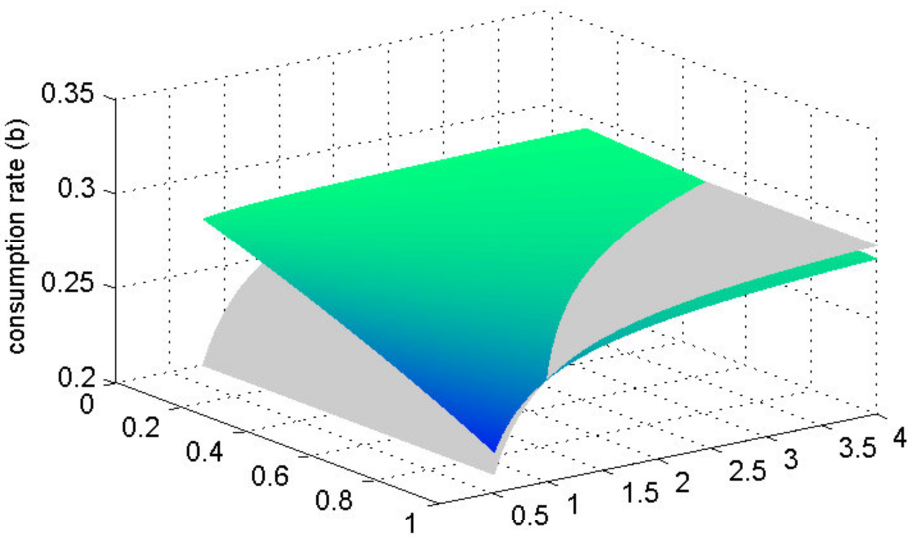
Finally, we investigate the power of investment subsidies for the general case. Figure 5 shows the result for benchmark parameters and alternative θ when $\gamma = 0.15$. The

Figure 4: Consumption Rate c/y for Alternative θ and γ



Parameters: Parameters as for Figure 1 and $a_s = A$. Left: solid line: $\gamma = 0$; dashed: $\gamma = 0.05$; dash-dotted: $\gamma = 0.1$. Right: solid: $\gamma = 0.2$; dashed: $\gamma = 0.3$.

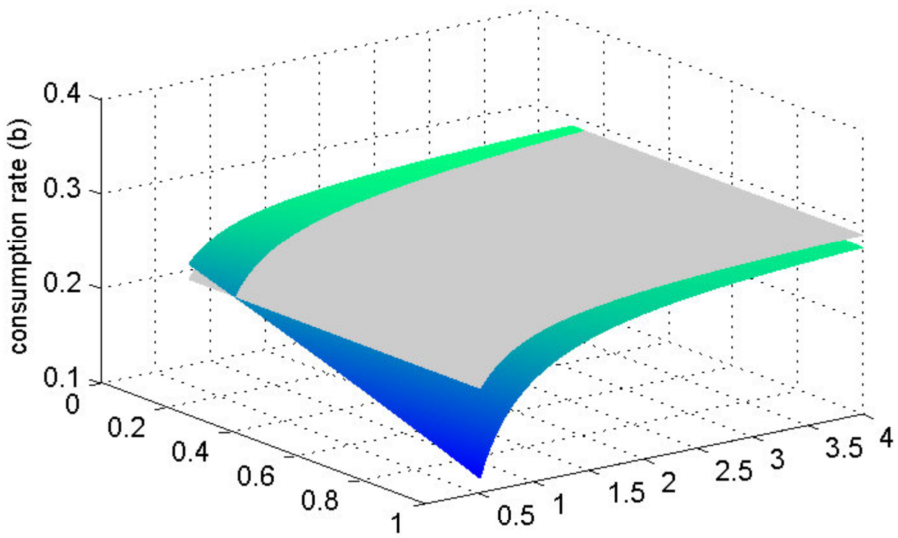
Figure 5: Consumption Rates for $\gamma = 0.15$ and Alternative θ and σ



Parameters as for Figure 1 and $\theta \in [0.5,\ 4]$; $\sigma \in [0,\ 1]$. Light gray plane: laissez faire consumption under perfect self-control.

curvature parameter θ runs from 0.5 to 4 and the subsidy σ runs from 0 to 1. The light grey plane shows consumption under perfect self-control and laissez faire policy. The concavity of the consumption rate with respect to θ (the declining savings rate) is clearly visible. With limited self-control, in contrast, the consumption profile is almost flat when there is no investment subsidy (the upper rim of the green plane is almost horizontal). As explained above, this means that then tendency for increasing consumption of the long-run self is (almost) balanced by the declining marginal utility of the short-run self, which makes self-control easier and which, taken for itself, leads to lower consumption.

Figure 6: Consumption Rates for $\gamma = 0.05$ and Alternative θ and σ



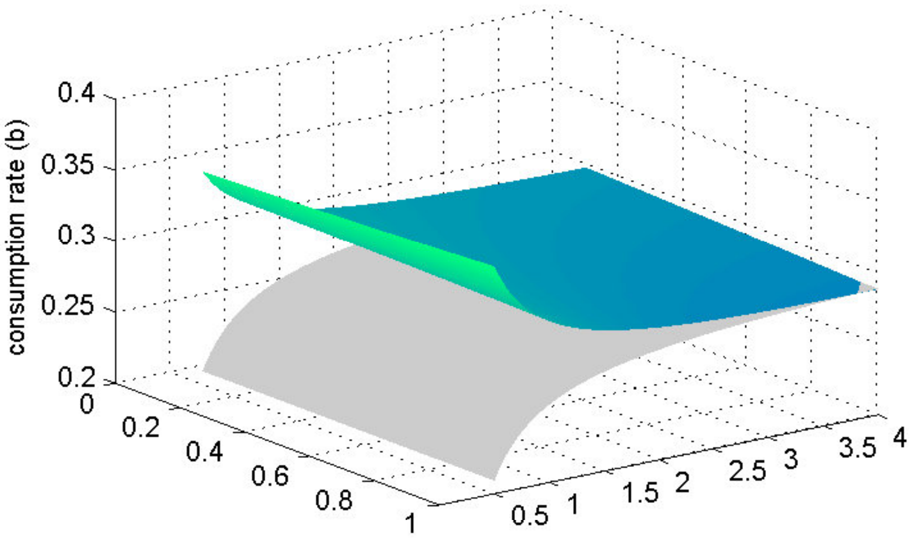
Parameters as for Figure 1 and $\theta \in [0.5,\ 4]$; $\sigma \in [0,\ 1]$. Light gray plane: laissez faire consumption under perfect self-control.

This observation provides the intuition for the result that an investment subsidy is more powerful in reducing consumption when θ is high. When θ is unity a subsidy of about unity is needed to move consumption toward the perfect self-control value (visible by the intersection between the two consumption planes). When θ is 4 a subsidy of about 0.5 is sufficient to generate the perfect self-control outcome.

Figure 6 shows results for $\gamma = 0.05$. In this case the subsidy needed to compensate self-control effects is much lower, around 0.1 to 0.2, and the sensitivity with respect to θ is largely diminished.

Figure 7 shows the results for $\gamma = 0.25$, i.e. an example of the degenerate case in which the consumption rate responds inversely to rising θ. It can be seen that policy is

Figure 7: Consumption Rates for $\gamma = 0.25$ and Alternative θ and σ



Parameters as for Figure 1 and $\theta \in [0.5, \ 4]$; $\sigma \in [0, \ 1]$.Gray plane: laissez faire consumption under perfect self-control.

not capable do implement the perfect self-control outcome unless θ is around 4 and the subsidy is around 100 percent (visible at the lower right corner of the plane).

5. Conclusion

This paper has introduced insights from economic psychology into the workhorse model of endogenous growth. Individuals were conceptualized as consisting of dual selves, a short-run self representing the (impulsive) desire for gratification through immediate or short-run consumption and a long-run self, planning ahead and partly controlling the desires of the short-run self. Within this framework it has been shown that limited self-control leads to overconsumption and reduced long-run growth. For sufficiently low costs of self-control the solution is structurally equivalent to the one from the conventional model of economic growth (albeit with a higher time discount rate). Structural equivalence breaks down when self-control problems become sufficiently severe. In that case the model predicts that the savings rate increases with increasing curvature of the utility function (increasing degree of relative risk aversion), a behavior that cannot be generated by the standard model.

It has been shown and explained that a consumption tax is not helpful to reduce impulsive consumption desires. In fact, it induces even higher consumption and lower growth. Similarly an investment subsidy leads to more consumption through the self-control channel. But an investment subsidy also leads to a lower consumption through the usual price channel by raising the net return on investment. For self-control costs within an empirically plausible range it is thus possible through an appropriate choice of investment subsidies to implement the consumption plan of perfect self-control under laissez faire policy or the welfare maximizing consumption plan. The needed subsidy depends quite strongly on the severity of the self-control problem. For empirically plausible costs of self-control in the range between 5 and 15 percent of the unfilled consumption desires the subsidy varies between 10 and 100 percent when the utility curvature parameter varies between 1 and 4.

In order to arrive at an analytically tractable solution for life satisfaction the growth framework was chosen to be deliberately simple, built upon the structure of the Ak growth model. Based on Rebelo (1991), it has been argued by Carroll et al. (2000) in a similar context that the linear Ak production function is the ultimate structure of all endogenous growth models. One straightforward extension would be to allow individuals to invest in physical and human capital, as proposed by Uzawa (1965) and Lucas (1988). The expected outcome is underinvestment in education due to short-run consumption desires and the expected fiscal policy conclusion would be a subsidy on education.

The present analysis has also shown that fiscal policy may cure the symptoms (overconsumption) but does not help to cure the causes (pain from unfulfilled short-run desires). In fact an investment subsidy raises the pain from unfulfilled short-run desires and compensates the individual by higher future returns. In this sense the analysis speaks in favor of regulation, in line with the original contributions by Gul and Pesendorfer (2001) and Fudenberg and Levine (2006). Reducing the scope of consumption possibility of the short-run self seems to be the only policy that gets down to the root of the problem.

On a broader level it would be interesting to explore the generality of the result that manipulating the price of consumption is not helpful in order to reduce short-run consumption desires. For example, would a fat tax lead to more overweight, given that long-run planning individuals face problems of impulse control? Integrating limited self-control into dynamic problems of health demand and health behavior seems to be a particularly interesting object for future research.

Appendix

The Maximized Hamiltonian. After inserting the first order condition (10), factor prices, and the first order condition, the Hamiltonian reads $H = \mathrm{e}^{-\rho t}\tilde{H}$ with

$$\tilde{H} = (1+\gamma)\frac{c^{1-\theta}-1}{1-\theta} - \gamma\frac{(a_s k)^{1-\theta}-1}{1-\theta} + \frac{1+\gamma}{1+\tau}c^{-\theta}\left[Ak - \delta k - c\right]$$

Suppose the solution takes the form $c = bk$ such that

$$\tilde{H} = k^{1-\theta}\left\{\frac{1+\gamma}{1-\theta}b^{1-\theta} - \frac{\gamma a_s^{1-\theta}}{1-\theta} - \frac{1}{1-\theta} + \frac{1+\gamma}{1+\tau}b^{\theta}\left[A-\delta-b\right]\right\}$$

and

$$\frac{\partial^2\tilde{H}}{\partial k^2} = -\theta k^{-\theta-1}(1+\gamma)b^{1-\theta}\left\{1 - \frac{\gamma}{1+\gamma}\left(\frac{a_s}{b}\right)^{1-\theta} + \frac{1-\theta}{1+\tau}b^{-1}\left[A-\delta-b\right]\right\}.$$

In the log-utility case the expression in curly parenthesis collapses to $1-\gamma/(1+\gamma) > 0$. The Hamiltonian is strictly concave in the state variable. The first order conditions provide a Maximum. In case of $\theta < 1$, since $a_s > b$, non-negative growth, i.e. $A - \delta - b \geq 0$ is a sufficient, non-necessary condition for a strictly concave Hamiltonian.

In the case of $\theta > 1$ the Hamiltonian is strictly concave for small values of b. This can best be seen by taken the limit $b \to A - \delta$ such that the term curly parenthesis equals $1-\gamma/(1+\gamma)(b/a_s)^{\theta-1} > 0$, since $b < a_s$. For large b the term in curly parenthesis becomes negative, implying a convex Hamiltonian. This means that the smaller b satisfying the first order conditions identifies a maximum while the larger b identifies a minimum.

Proof of Lemma 1. Let g_c denote the growth rate of consumption when individuals have perfect self-control such that life time utility is given by

$$V^p = \int_0^\infty \left[\log c(0) + g_c t\right]\mathrm{e}^{-\rho t}\mathrm{d}t = \frac{g_c}{\rho^2} + \frac{\log c(0)}{\rho}.$$

Consumption under perfect self-control is taken from (13), that is $c = [A - \delta - (1+\sigma)(\alpha A - \delta) + \rho]k$. The implied growth rate is $g_c = (1+\sigma)(\alpha A - \delta) - \rho$. Inserting this information, life time utility becomes

$$V^p = \frac{(1+\sigma)(\alpha A-\delta)-\rho}{\rho^2} + \frac{\log\left[A-\delta-(1+\sigma)(\alpha A-\delta)+\rho\right]}{\rho} + \frac{\log k(0)}{\rho}.$$

Solving the first order condition $\partial V^p/\partial\sigma = 0$ for σ provides the welfare maximizing policy $\sigma = \tilde{\sigma} = A(1-\alpha)/(\alpha A - \delta)$. Re-inserting $\sigma = \tilde{\sigma}$ into the consumption rate (13) provides

$$a = A - \delta - \left[1 + \frac{1-\alpha)A}{\alpha A - \delta}\right](\alpha A - \delta) + \rho = \rho.$$

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